INDIAN AGRICULTURAL
RESEARCH INSTITUTE, NEW DELHI.

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MGIPC—S1—51 AR/57—3-4-58—5,000.

PROCEEDINGS

OF THE

ROYAL SOCIETY OF EDINBURGH

PROCEEDINGS

OF

THE ROYAL SOCIETY OF EDINBURGH

VOL. LV.

1934-1935

EDINBURGH
PRINTED BY NEILL AND COMPANY, LIMITED
MCMXXXVI

CONTENTS

PAGE

*1. The Duration of Life in an Albino Rat Population. By B. P. Wiesner, D.Sc., and N. M. Sheard, Macaulay Laboratory, Institute of Animal Genetics, University of Edinburgh. (With Five Text-figures.) Issued separately February 23, 1935, ... 1

2. Dating of Late-Glacial Clay Varves in Scotland. By Gerard De Geer. *Communicated by* Dr J. B. Simpson. (With One Text-figure.) Issued separately February 25, 1935, ... 23

3. The Invariant Theory of the Correlation. By Professor H. W. Turnbull, F.R.S., University of St Andrews. Issued separately February 25, 1935, . 27

4. On Least Squares and Linear Combination of Observations. By A. C. Aitken, M.A., D.Sc., Mathematical Institute, University of Edinburgh. Issued separately March 6, 1935, ... 42

5. The Mathematical Representation of the Energy Levels of the Secondary Spectrum of Hydrogen.—II. By Ian Sandeman, Ph.D., Late Carnegie Research Scholar in the University of St Andrews. Issued separately May 30, 1935, ... 49

6. The New Crystallography. By W. L. Bragg, F.R.S., Langworthy Professor of Physics, University of Manchester. (Bruce-Preller Lecture, delivered on February 4, 1935.) (With Four Text-figures.) Issued separately May 31, 1935, ... 62

7. The Mathematical Representation of the Energy Levels of the Secondary Spectrum of Hydrogen.—III. By Ian Sandeman, Ph.D., Late Carnegie Research Scholar in the University of St Andrews. Issued separately August 1, 1935, ... 72

8. Some Series and Integrals involving Associated Legendre Functions, regarded as Functions of their Degrees. By Professor T. M. MacRobert, M.A., D.Sc. Issued separately August 1, 1935, ... 85

9. The Lunar Atmospheric Pressure Inequalities at Glasgow. By R. A. Robb, M.A., M.Sc., and T. R. Tannahill, M.A., B.Sc., University Observatory, Glasgow. Issued separately August 1, 1935, ... 91

PAGE

10. Charles Darwin as a Student in Edinburgh, 1825–1827. (An Address delivered on October 28, 1935.) By Professor J. H. Ashworth, F.R.S., Department of Zoology, University of Edinburgh. (With Three Plates.) Issued separately December 19, 1935, ... 97

11. On Fourfold Sampling with and without Replacement. By A. C. Aitken, D.Sc., and H. T. Gonin, Ph.D., Mathematical Institute, University of Edinburgh. Issued separately December 20, 1935, ... 114

12. The Experimental Analysis of the Growth of an Insect Population. By D. Stewart MacLagan, D.Sc., F.R.E.S., and Edward Dunn, B.Sc., Natural History Department, University of Aberdeen. *Communicated by* Professor James Ritchie, D.Sc. (With Five Figures.) Issued separately January 11, 1936, ... 126

OBITUARY NOTICES: Francis James Blight; Henry Briggs; David Lawrence Bryce; Cecil Carus-Wilson; James Chatham; Shepherd Dawson; T. Cuthbert Day; James Main Dixon; Rev. William Galloway Donaldson; Sir James Alfred Ewing; James Gordon Gray; William Richard Hodgkinson; Sir Horace Lamb; Hector Munro Macdonald; Sir Leslie Mackenzie; John James Rickard Macleod; James Musgrove; Robert Alexander Robertson; Sir John (Smith) Samuel; Sir Edward Albert Sharpey-Schafer; Theobald Smith; Robert Rattray Tatlock; Hugo de Vries; Sir James Walker; Alfred C. Wilson; Bertram Martin Wilson; Johnstone Christie Wright, . . . 140–179

APPENDIX—

Proceedings of the Statutory General Meeting, October 1934, ... 183
Proceedings of the Ordinary Meetings, Session 1934–1935, ... 186
Proceedings of the Statutory General Meeting, October 1935, ... 191
The Keith, Makdougall-Brisbane, Neill, Gunning Victoria Jubilee, James Scott, Bruce, and David Anderson-Berry Prizes, and the Bruce-Preller Lecture Fund, ... 194
    Awards, ... 198
Accounts of the Society, Session 1934–1935, ... 206
Voluntary Contributors under Law VI (end of para. 3), ... 214
The Council of the Society at October 28, 1935, ... 215
Fellows of the Society at October 28, 1935, ... 216
Honorary Fellows of the Society at October 28, 1935, ... 242
Changes in Fellowship during Session 1934–1935, ... 244
Fellows of the Society elected during Session 1934–1935, ... 244
Laws of the Society, ... 245
Additions to the Library—Presentations, etc.—1934–1935, ... 253
Index, ... 258
Index, under Authors' Names, of Papers published in *Transactions*, . . 260

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* The thanks of the Society are due to the Carnegie Trust for the Universities of Scotland for grants towards the cost of the illustrations and tables in this paper, and towards the cost of the plate in the paper by Dr Jessie A. R. Wilson in *Proceedings*, Vol. LIV, 1934, pp. 188–192.

PROCEEDINGS

OF THE

ROYAL SOCIETY OF EDINBURGH.

VOL. LV. 1934-35.

I.—The Duration of Life in an Albino Rat Population.

By **B. P. Wiesner**, D.Sc., and **N. M. Sheard**, Macaulay Laboratory, Institute of Animal Genetics, University of Edinburgh.

(MS. received June 2, 1934. Read November 5, 1934.)

NUMEROUS authors have been concerned with phenomena and problems of senescence, duration of life, order of dying-out, and related subjects. With the exception of studies on *Drosophila* (Pearl and Parker, 1921; Pearl, 1922), other insects (for literature see Korschelt, 1924) and, in some respects, human subjects (see actuarial literature), most of the research of previous authors was embarrassed by the lack of sufficiently large and sufficiently homogeneous material; many conclusions rest on an insecure basis, since they refer only to isolated specimens or to experiments carried out on small numbers of animals.

Five years ago the writers founded and have since maintained a rat colony in this laboratory, and a number of experimental and observational data relating to problems of senescence were collected. This communication presents the data relating to duration of life and order of dying-out in the colony. The data are presented without further attempt at actuarial treatment, to which they will be subjected by other workers; to the present writers they serve only as material for the calculation of standards required for certain other purposes.

ANIMAL MATERIAL.

(a) *Origin of the Colony.*

In order to establish the colony one of the writers selected three pairs of rats at the Wistar Institute, Philadelphia, Pa., in the summer of 1929.

One pair had a litter of fourteen (7/7); the other two females had smaller litters. The great majority of the animals bred in the colony were derived from the seven pairs which formed the largest litter, and from subsequent litters of the same parental pair; a small number of animals were derived from another pair.

The total number of weaned animals considered in this communication is 2927. Originally it was planned to maintain the colony at a level of about 3000 (including young under continuous observation). However, the plan had to be curtailed.

(b) *Breeding Methods.*

Generally speaking, continuous brother-sister mating was employed. However, during the autumn of 1929 and the early part of 1930, *i.e.* the period during which the colony was developed—cousins were not infrequently mated. This proved necessary because it seemed essential for the purpose of certain experiments and for general reasons to build up the colony quickly and avoid selection by using all litters born in the first months. The proportion of males and females in any one litter varies, of course, widely; since animals were kept in pairs cousin-mating had to be employed where the number of males or females respectively was not large enough for brother-sister mating. Animals bred by continuous brother-sister mating since the foundation of the colony form the A Stock; animals in whose ancestry a cousin-mating occurred form the B Stock.

In the great majority of cases litters remained with their parents until the young had passed the weaning age. In a limited number of instances the litter number was reduced to or raised to six, by exchange of young, so as to enable the writers to study the influence on duration of life of intra- and extra-uterine litter-size respectively. In the present study all litters, irrespective of size, are considered.

The young were removed from their parents at ages varying from 28 to 35 days. They were paired and the pairs were isolated in standard cages. The pairs remained together till one mate died; the survivor was then remated with an animal of the recorded stock and of about equal age; when this became impossible at a later stage of the experiment because of the reduced size of the colony, a somewhat younger or older animal, which sometimes had to be taken from the D Stock (see below), was used for remating.

An exception to the general procedure of pairing and isolating had to be made during a short period in the early history of the colony. At that time the supply of cages was insufficient for the colony, then growing fast, and for a time several animals had to be kept in one cage. However, by appropriate observation, recording and isolating of pregnant females, the general procedure of breeding (brother-sister mating, etc.) was maintained.

After the colony had reached its desired size, surplus young were permitted to reach an age of 28 to 35 days, and then either killed, sold, or used to form the D Stock. Animals of this stock were not recorded as fully as those of the A or B Stock, and only used for experimental purposes or else for mating with survivors of pairs of the A and B Stock for which no appropriate mate could be found within the recorded stock.

(c) *Methods of Recording.*

Individual records were kept for each animal of the A and B Stocks by means of a Shannon-Recording System. However, a small number of record cards were lost; also a small number of animals have had to be excluded from consideration through faults or omissions in recording. The progeny of these animals was not used for the continuation of the colony.

(d) *Husbandry.*

During the first year of its existence the colony was housed in a large outhouse heated electrically. The animals were then transferred to a rat-proof building erected for the purpose, a description of which has been published previously (Wiesner and Sheard, 1933). It may be repeated, though, that thermostatic central heating and roof-cooling combined to maintain the temperature between 65° and 70° F. (with the exception of a few days every summer when the temperature rose occasionally above 80° F., and rare winter days when it fell to 60° for a few hours through incidental causes). The temperature inside the cages was slightly higher than the room temperature.

Standard cages, built in this laboratory to the design of the writers, were employed for all animals. Their inside measurements are 16 × 8 × 15 inches.* The cages were built of pine wood impregnated with creosote. The floor of each pen was formed by a perforated zinc tray, underneath which another zinc tray filled with sawdust was provided. Water and milk dishes were of stone-ware, and were cleaned daily in a hot solution of washing-soda followed by high-pressure spray; perforated trays were soaked once weekly in a hot-water bath and afterwards brushed with revolving brushes constructed for the purpose. They were dried before use. The cages were periodically creosoted. This sufficed to keep them free from parasites for the period of observation under discussion. During the last year, however, the common bed-bug invaded the colony and against this parasite creosote proved of little avail. If the presence of this parasite should have any influence on duration of life, etc., it would yet only affect a small proportion of the animals dealt with in this communication. Of other parasites the colony proved to be singularly free during the first three years of its existence. Later on some animals suffered from the Meguin-mite; but no serious case was observed. Mites and lice (*Polyplax*) attacked, in the first instance, almost exclusively aged animals, and their development and spreading was prevented by applying to the affected animals a mixture of liquid paraffin and rectified spirit.

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* Through a misprint wrong measurements were given in a previous publication (*loc. cit.*).

Straw was used for bedding and was changed once or twice weekly according to the size of the litter present. In the first year hay was used in the place of straw, but proved unsatisfactory.

(e) *Feeding.*

It would have been desirable to employ a synthetic diet, but this proved impracticable because of the expense involved. Nor was it possible to keep the animals on the scrap diet used by many authors working with Wistar rats. A mixed diet, of which *ad lib.* rations were fed daily (except Sundays; see Table I), was therefore employed. The main constituents of the diet were: flaked maize, bran, whole wheat, barley, whole oats, steamed potatoes, milk cake (dried skimmed milk fresh from the roller), fish meal, bone meal, dried yeast and liver mixture, whole milk powder, lettuce (during the season), apples (in winter), stale bread. The animals also received cod-liver oil (in mixture) once weekly, and for a time chopped raw ox liver. They were given daily rations of skimmed milk with the exception of Saturday, when only water was given.

As an example the diet list of one week is appended (Table I).

TABLE I.—EXAMPLE OF DIET DURING A WEEK IN THE WINTER SEASON.

Monday	Maize, yeast and liver powder, milk.
Tuesday	Soaked wheat, bone meal, milk.
Wednesday	Maize, bran and liver, apples, milk.
Thursday	Maize, bran, milk.
Friday	Steamed potatoes, bread, milk.
Saturday	Maize, bran, cod-liver oil. Quantities fed sufficient for two days.
Sunday	No solid food; milk.

All cages were inspected twice daily (during cleaning in the morning and feeding in the early afternoon), when birth of litters, death of animals, etc., were recorded, and dead animals were removed for post-mortem examination.

For the purpose for which the colony was founded, it was essential that a large number of animals should be permitted to reach their maximum age without interference, since only in this way could reliable standards and control data for various experiments be established. Diseased stock animals were therefore not treated, but permitted to recover or die as the case might be. Only few exceptions to this rule were made. In the first place, it was necessary to treat some animals, particularly aged animals, for parasites, since the writers feared spreading of the infection; in the second place, some animals developed large tumours which sometimes reached a weight greater than that of the animal itself. These tumours were frequently situated abdominally and the animal had to drag the tumour along. In some cases these tumours, or parts of them, were removed for histological or other purposes. But the number of animals thus treated was small and omission of these animals from the material of the present study would only relate to the last phases of life and not change the results to an appreciable degree. Of other "therapeutical" interferences, the cleaning of superficial abscesses and the periodic cutting of hypertrophic incisors should be mentioned.

It seemed essential for various reasons to avoid directed selection, both during the foundation of the colony, and during later stages when animals were chosen for experimental purposes. Selection was easily avoided during the earliest phase when all surviving young were used for breeding, with the exception of some new-born and small young used for experimental purposes. At later stages whenever litters had to be chosen or animals had to be removed for experiments or histological purposes, random selection was practised, the animals being chosen, for instance, by reference to the cage number only. Since the number of cages was 1326, directed selection by unconscious knowledge of the contents of cages seems unlikely. This method of breeding without directed selection was abandoned in the summer of 1933 when the required complement of animals had been established; since then the colony has been maintained by selecting the strongest young of any litter. It is hoped that any change in the constitution of the colony produced by such selection will reveal itself within the next few years.

(f) *General History of the Colony.*

No epidemic, and no accident involving more than one animal at a time, occurred during the period under discussion. The absence of epidemics, which are not infrequent in colonies of the Wistar rat, may perhaps be ascribed to the fact that animals which had left the house were not returned to it and no strangers were introduced (see Topley, 1921), that the house was kept rat-proof, and that the cages and the house were kept as clean as possible.

Generally speaking, the stock is satisfactory for most experimental purposes, though it does not equal in body-size and growth-rate the Wistar rats bred by Mr A. D. Bacharach of London, if one may judge from some of his animals kept under observation by the writers. The stock has maintained the original characteristics of the Wistar rat, which are too well known to need description. Among about 30,000 animals born in the colony during the period under discussion, only about fifteen abnormal individuals were noted. Two of these were tailless; the abnormality did not recur in subsequent generations. Two cases were micro-ophthalmia; various abnormalities of the genital organs were exhibited by eleven animals. In a number of males a curious malformation developed in later life; a perforation developed at the base of the sheath of the penis, and the penis protruded from this perforation either continuously or else during erection.

RESULTS.*

(a) *Order of Dying-out in Rat Nurslings.*

For the purpose of presenting the data, the life span of the rat has been divided into two phases. The first phase comprises the span of life spent by the rat in greater or lesser dependence on its mother; this phase ends during the fourth week post-partum. The second phase comprises life after weaning.

Mortality in the young of our stock was very high. It would, however, serve no useful purpose to enumerate at this juncture the whole mortality experience in the more than 5000 litters born in the colony. It is very difficult to establish in any given case whether the death of either a litter or a single young was due to low viability or to accidents, such as cannibalism, or squashing of the young by the mother because the latter fails to assume the appropriate "nursing posture." These factors are, of course, eliminated once the young have been separated from their parents.

As an example, data relating to 250 litters chosen at random from our records are presented here (Table II), from which it will be seen that the total number of young in these litters was 1607. Of these, a total of 492 animals died, or were killed by their mothers before the age of 30 days; while 1115 survived up to or beyond this age. This would correspond to a death-rate during this first phase $\left(\frac{\text{Number of dying}}{\text{Number of animals born}}\right)$ of about 30 per cent. (·306161).

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* The calculations on which the following paragraphs are based were performed on a ten-place Munroe calculating machine or by Chambers's tables.

TABLE II.—SURVIVAL OF NURSLINGS TO AGE 30 DAYS.

		Born.			Died.			Survived.		
	No. of Litters.	Total.	Males.	Females.	Total.	Males.	Females.	Total.	Males.	Females.
Whole sample	250	1607	812	795	492	241	251	1115	571	544
Litters with one or more than one survivor	152	1187	604	583	71	39	32	1115	569	546

However, we believe that an approximation to the "real" death-rate can be achieved by the following procedure. Generally speaking, it can be held that cannibalism on the part of the mother is not restricted to any single young, but leads to the destruction of the whole litter. Again, a mother (particularly a young one) that has squashed one or two young will frequently, by her inadequate maternal activities, bring about the death of the whole litter. This is by no means generally true, as the writers have mentioned in a previous publication, but it does hold true in the great majority of cases. Conversely, survival of at least one young indicates that conditions of maternal behaviour were not wholly unfavourable to survival. If, therefore, we take into consideration only the number of litters in which at least one young survived up to the age of 30 days, there remain 152 of the 250 litters chosen for analysis. These litters comprised 1187 young (see Table II). Of these 71 died before attaining the 30th day of life; 1115 survived up to or beyond the 30th day of life; this corresponds to a total death-rate of ·05986. The death-rate for the males $q^{M}_{1-30} = \cdot 06457$; the corresponding figure for the females $q^{F}_{1-30} = \cdot 05489$. It will be seen that in litters in which the chance of death was reduced by the absence of, or the incomplete development of, cannibalism, the death-rate was very considerably lowered compared with the corresponding chance in the group as a whole. The death-rate in the case of both males and females is fairly low, since during the period 0–30 days only 64 and 54 animals respectively would die of 1000 assumed to be alive at the beginning of the period. At the same time, it should be emphasised once more that the significance of this result is impaired, inasmuch as the exclusion of litters without survivors may also imply a selection for viability, and thus fail to produce results as valid as those obtained for animals which have passed the weaning age and were kept isolated from their parents. The writers are, however, under the impression that the degree of viability of the litter has but little to do with the incidence of cannibalism.

(b) *Order of Dying-out in Females Older than* 30 *Days*

In the following paragraphs the writers omitted from consideration all animals of the D Stock, all animals which were interfered with experimentally, and animals whose records were incomplete. This may be done safely, since inspection of these records shows, first that the number is small, and secondly that the cases are widely distributed as to age at death.

In the first instance the records were only those of animals which had been under uninterrupted observation from birth to death, had belonged to the recorded stock (A or B), had been paired off in the standard manner, and had not been subjected to any interference.

There remain then 843 females. They were born and died between August 1929 and December 1, 1933; this date will be referred to as "closing day."

The relevant statistical constants have been entered in Table III. It will be seen that the average duration of life was less than two years, viz. 693 days (22·79 months). The maximum duration of life was 1269 days; but only one animal attained this age.

TABLE III.—CONSTANTS (±STANDARD ERRORS) FOR MALES AND FEMALES (CONTROLS) OBSERVED TILL DEATH.*

	Males.	Females.
Number of animals	1,005	843
Number of days lived	672,717	584,293
Mean duration of life	669·37 ±3·57	693·1 ±7·33
Standard deviation	113·23 ±2·53	212·83 ±5·18
Difference of means	−23·73 ±8·15	23·73 ±8·15
$\frac{\text{Difference of means}}{\text{S. E. of difference of means}}$	2·91	

A picture of the order of the dying-out and distribution of ages at death can be obtained from fig. 1 where the animals have been grouped according to age at death, at intervals equalling 50 days each. It will be seen that a very definite mode is exhibited between the ages 675 and 775. Since appropriate actuarial treatment of the data will be undertaken separately, further discussion of the nature of the distribution is unnecessary here.

If consideration of our data were restricted to females which died during the period of observation, much material of some value would be lost. A large number of females was actually exposed to risk of death during the period of observation, but was used experimentally at some time or failed to die before or on the closing day, so that they do not appear in the material for Table III. The reliability of the values of the death-rate for any period would, of course, be enhanced considerably if the total number of animals that were exposed to risk of death during that period are considered. The total number of females which should be studied is 1482 (including 843 animals mentioned above, but omitting animals of unrecorded stock, etc.). In order to use the available data and to secure their future usefulness, the following procedure was adopted: the span of life was divided into periods of five days, and all recorded animals were counted which lived during the years under discussion (1929–33), each animal being counted once for each period at the *beginning* of which it was (*a*) still alive, (*b*) had not been used for experimentation, and (*c*) had been kept under standard conditions. As mentioned before, a considerable number of animals was used for experimental purposes, a small number was given to other workers; thus the number of animals alive at the beginning of any given period does not necessarily represent the number exposed to risk of death during the whole of the period. For instance, 1169 females were alive at the beginning of the period 481–485 days; but between the days 481–485, 7 of the females were removed for experimental purposes, so that only 1162 animals were exposed to risk of death throughout the whole of the period. The observed deaths (5) thus refer to a total of 1162, not 1169, and $q_{481}=\frac{5}{1169}$. To be more accurate

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* Referring to animals alive at age 31 days.

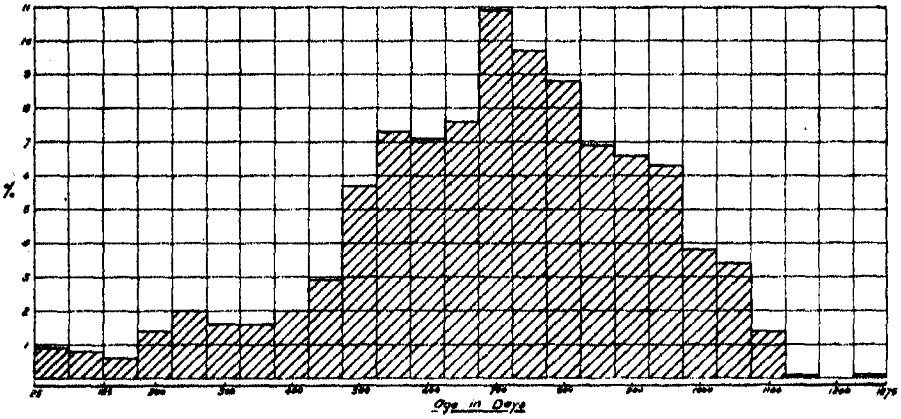


FIG. 1.—Frequency diagram showing distribution of ages at death in 843 females observed from birth to death. The number of animals dying during any interval is given in per cent. of the total number.

it might have seemed advisable to deduct only half the number of females removed during any given period so as to allow for the distribution of the removals over the period; however, the periods are very short and any error introduced is probably negligible; it is balanced by the greater ease in calculation (avoiding fractions of animals) and is further justified by the fact that the majority of the animals eliminated at any time were removed at ages which correspond to the beginning of periods. It was necessary, of course, to allow in the same manner for animals still alive on the closing day. For instance, one animal was aged 736 days on the closing day; it was included in the total number of animals considered but not entered in any groups beyond period 736–740.

A "foundation table" was constructed on these principles. This table represents a collection of all the available data relating to the order of dying-out in females. It shows the total number of animals alive at the beginning of each period of 5 days; the number removed during that particular period; the number actually exposed to risk of death during the period in question; the number of deaths which occurred at ages falling in the corresponding age period; it also shows the probability (q_x) of an animal aged x days dying between ages (x) and ($x+5$) days.

For reasons of space the foundation table has not been printed here; it has been deposited with the Royal Society of Edinburgh for the use of future students.

In the form in which the data are presented in the foundation table they are not suitable for obtaining the general trend of the order of dying-out. The periods are very short and involve a high sampling error; also, they are too numerous to display trends clearly. The data of the foundation table were therefore used to construct an abbreviated life table (Table IV).

For the purpose of the abbreviated table the whole life span from 31 days to the terminal age was divided into periods of 50 days each, except for the first period. The number of animals (t_x) alive at the beginning of the periods was taken, all animals (whether they died before, or lived beyond, the closing day) being included in this Table. But some animals were either removed at ages falling within the period, or were eliminated from further consideration, having reached the corresponding ages on the closing day, and allowance must be made for these animals. However, since the removal of these animals was distributed over the whole of the period more or less evenly, only half the number of animals (k_x) (rounded up to the next integer in cases of odd numbers) was deducted from the total number alive at the beginning of the period. Column (q_x) shows the death-rate. Two columns (l'_x and d'_x) have been added showing the order of dying-out in an assumed population of 100,000 females alive at age 31 days.

TABLE IV.—ABBREVIATED LIFE TABLE: FEMALE RATS.
Based on mortality experience in 1456 females.

Age, Days.	t_x.	k_x.	D_x.	$10^8 . q_x$.	l'_x.	d'_x.
51–	1456	7, 4, 6, 0, 2, 6, 8, 3, 1, 5	1, 0, 2, 1, 0, 3, 0, 0, 0, 0	484429	99728	483
101–	1407	4, 3, 3, 5, 6, 3, 3, 2, 5, 1	0, 1, 0, 2, 1, 1, 1, 2, 0, 1	647948	99245	643
151–	1363	2, 8, 1, 2, 5, 1, 0, 2, 0, 0	0, 0, 0, 0, 0, 0, 0, 3, 3, 0	443787	98602	438
201–	1336	1, 0, 0, 0, 0, 0, 1, 0, 0, 0	2, 1, 2, 0, 1, 1, 1, 0, 2, 3	973784	98164	956
251–	1321	2, 0, 1, 1, 0, 0, 0, 0, 0, 0	1, 4, 0, 0, 5, 0, 0, 2, 3, 2	1288855	97208	1253
301–	1300	1, 3, 7, 4, 1, 4, 4, 0, 0, 0	2, 0, 1, 3, 1, 1, 3, 1, 1, 2	1164596	95955	1117
351–	1261	1, 0, 0, 1, 0, 1, 0, 0, 1, 2	2, 0, 2, 1, 1, 1, 3, 1, 1, 2	1112878	94838	1055
401–	1241	8, 8, 2, 0, 0, 1, 1, 0, 0, 0	2, 2, 2, 2, 1, 1, 4, 3, 2, 3	1787164	93783	1676
451–	1199	0, 0, 5, 4, 2, 3, 7, 4, 3, 2	3, 2, 1, 3, 3, 4, 5, 5, 5, 7	3209460	92107	2956
501–	1131	4, 4, 11, 4, 2, 1, 0, 2, 1, 2	4, 5, 7, 4, 2, 4, 4, 7, 10, 3	4484305	89151	3998
551–	1050	1, 0, 0, 0, 4, 1, 2, 4, 1, 1	12, 8, 1, 4, 9, 4, 4, 4, 9, 5	5752637	85153	4899
601–	976	4, 10, 1, 4, 9, 13, 4, 4, 4, 2	8, 3, 8, 9, 6, 4, 7, 2, 9, 6	6540084	80254	5249
651–	859	2, 2, 4, 1, 1, 1, 6, 13, 3, 5	8, 7, 7, 10, 4, 7, 5, 7, 14, 11	9523809	75005	7143
701–	741	3, 3, 3, 8, 9, 1, 6, 19, 16, 11	5, 12, 12, 7, 12, 9, 8, 8, 15, 6	13409415	67862	9100
751–	568	8, 7, 13, 18, 6, 13, 10, 22, 12, 10	8, 8, 9, 5, 6, 10, 5, 8, 10, 11	15748031	58762	9254
801–	369	14, 7, 3, 2, 6, 2, 1, 0, 1, 0	6, 9, 5, 6, 4, 7, 9, 6, 8, 2	17663818	49508	8745
851–	271	5, 1, 3, 1, 0, 2, 0, 1, 1, 4	3, 5, 3, 10, 5, 6, 6, 8, 5, 6	21755725	40763	8868
901–	196	1, 2, 1, 1, 1, 1, 1, 1, 0, 0	5, 6, 3, 7, 4, 6, 9, 4, 7, 5	29319372	31895	9351
951–	131	0, 1, 1, 1, 0, 1, 2, 0, 1, 1	2, 6, 4, 4, 6, 4, 4, 2, 4, 2	29921260	22544	6745
1001–	85	0, 0, 2, 2, 0, 0, 0, 3, 2, 1	4, 3, 2, 4, 3, 2, 4, 2, 3, 1	35000000	15799	5530
1051–	47	0, 1, 1, 1, 0, 0, 0, 1, 0, 0	4, 3, 4, 4, 2, 1, 2, 1, 2, 0	51111111	10269	5249
1101–	20	0, 0, 0, 0, 0, 1, 5, 0, 1, 0	0, 3, 1, 0, 2, 0, 1, 0, 0, 0	43750000	5020	2196
1151–	6	1, 0, 1, 2, 0, 0, 0, 0, 0, 0	0, 0, 0, 0, 0, 0, 0, 0, 0, 0	0	2824	0
1201–	2	0, 0, 1, 0, 0, 0, 0, 0, 0, 0	0, 0, 0, 0, 0, 0, 0, 0, 0, 0	0	2824	0
1251–	1	0, 0, 0, 0, 0, 0, 0, 0, 0, 0	0, 0, 0, 1, 0, 0, 0, 0, 0, 0	10000000	2824	2824

The time covered is divided in periods of 50 days each (first column). The second column (t_x) shows the number of animals alive at the beginning of each period. The number of animals removed during any period is shown in column 3—not as a total, but according to the succession of removals in each of the ten successive periods of 5 days each which constitute the whole of the period of 50 days. Thus there were removed, *e.g.* 4 animals between ages 601–605 days, 10 animals between ages 606–610 days, etc. The number of animals removed (k_x) includes the number of animals which had reached the corresponding age on the closing day. The (D_x) column shows, in similar manner, the incidence of death during successive periods of 5 days each. The q_x column shows the death-rate for each period and is calculated as $\frac{\Sigma D_x}{t_x - \frac{1}{2}\Sigma k_x}$. The sixth column shows the order of dying-out in a hypothetical population of 100,000 females aged 31 days in which the observed q_x's are operating (l'_x). (d'_x) shows the number of animals dying, during any given period, in such a population: $d'_x = l'_x q_x$. The data for the period 31–50 days have not been entered since they do not cover a span of 50 days. These data, and the q_x's for individual periods of 5 days each, will be found in the foundation table.

The values of l'_x have been plotted on an arithmetic grid (fig. 2); the graph shows accordingly the falling off in number of a female rat population. Figs. 3 and 4 demonstrate the values of q_x at the respective ages on an arithmetic and arithlog grid respectively. The terminal values of q_x have not been plotted; they are quite unreliable in view of the small number of cases.

Inspection of the graphs shows that the mortailty is very low at ages following the weaning period. After a slight rise in death-rate and a subsequent equally slight and possibly insignificant fall, the death-rate rises continuously, till very near the end of the life span when the rise becomes less pronounced—a phenomenon of doubtful significance in view of the small number of survivors at that age and the consequent large sampling error. It is characteristic of the mortality curves that they remain very nearly horizontal for the early ages (up to approximately 200 days) before the death-rate begins to increase definitely.

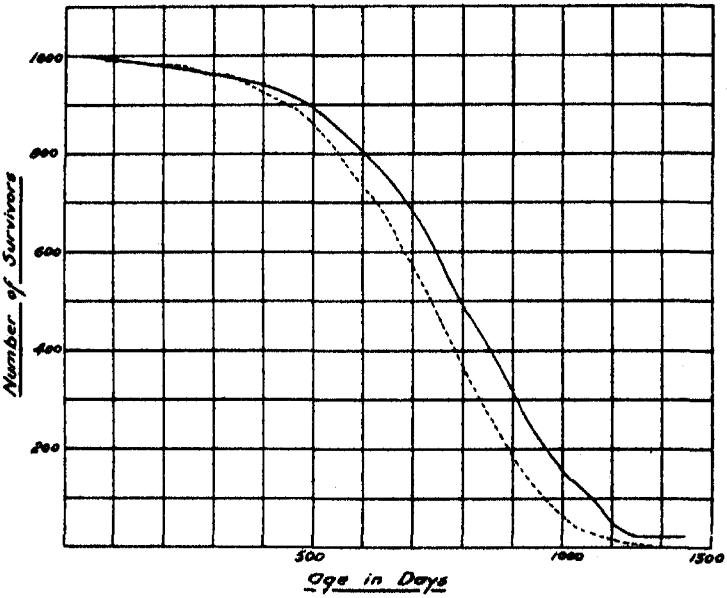


FIG. 2.—Values of l'_x on arithmetic grid. The graphs demonstrate the order of dying-out in an assumed population of 1000 animals aged 31 days, on which the observed death-rates are operating. · · · · males; —— females.

It should be remembered when considering Table IV and the graphs constructed on the basis of this Table that the periods have a duration of 50 days each, and begin at multiples of 50 days. In view of the procedure adopted, the values of q_x have been entered in the graph at the central day of each period.

As has been mentioned before, the writers did not wish to, and are not in a position to, discuss the more intricate actuarial problems arising from the data. In particular they did not attempt to study the question whether the "laws of mortality" believed to be exhibited by human life tables can also hold good for the rat stock under discussion. They therefore did not undertake to fit expressions, such as the Gompertz or Makeham, to the data. However, a graduation of the data involving no hypothesis as to the character of the mortality curve seemed of interest for obvious reasons. The writers fitted a polynomial of the fifth order to the data of Table IV, q_x being taken as the variable, also another polynomial of the second order (by Whittaker's method). Neither of these polynomials produced anything approaching a close fit over the whole range of data.

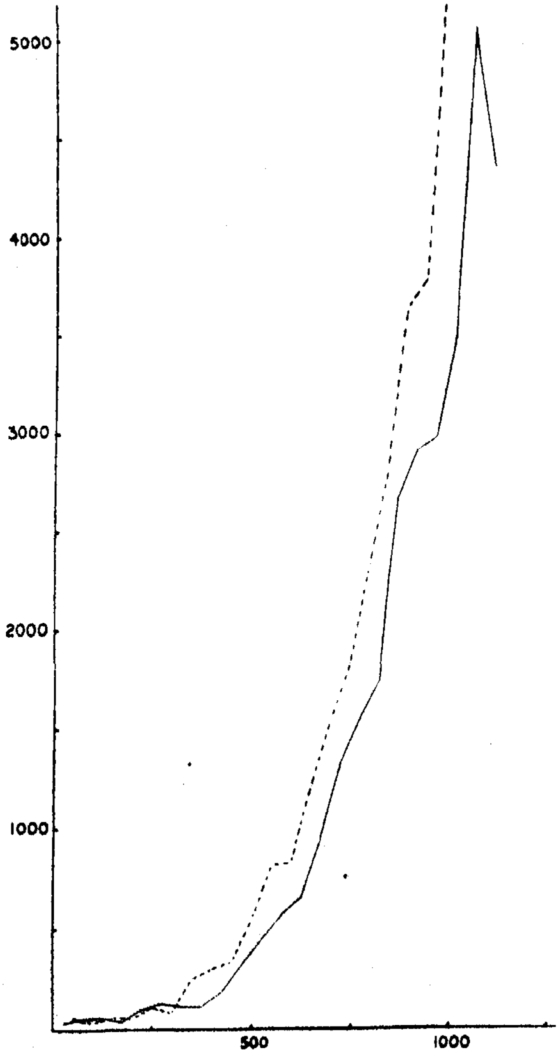


FIG. 3.—Values of q_x at successive ages. (Arithmetic grid.) Abscissa shows age in days; the ordinates indicate how many animals (out of 10,000 assumed to be alive) will die during the following 50 days. - - - - males; ——— females.

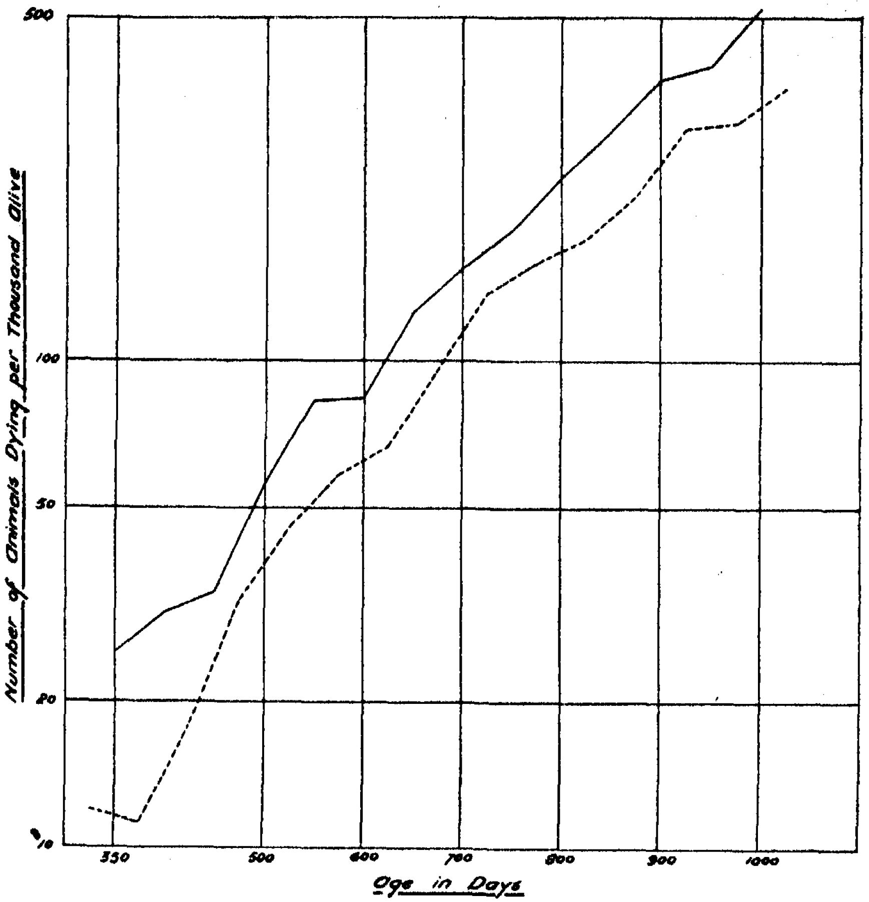


FIG. 4.—Values of q_x at successive ages. (Arithlog grid.) Abscissa shows age in days; the ordinates indicate how many animals (out of 1000 assumed to be alive) will die during the following 50 days. - - - - females; —— males.

(c) *Order of Dying-out in Males.*

In the case of males, the same procedure was adopted as in females. 1005 fully recorded males reached the age of 31 days before, and died before or on, the closing day (see column 1 of Table III). The average duration of life in these males was 22·12 months. The distribution of ages at death is demonstrated by the histogram of fig. 5.

Reference has been made before (p. 9) to the manner in which the mortality data, relating to animals which were still alive on the closing day or were removed from the colony at any time before this date, can be utilised in the construction of a life table. This method was applied to males; thus a male foundation table (deposited with the Royal Society of Edinburgh) was constructed. An abbreviated life table was derived from the foundation table and is reproduced here (Table V).

Table V corresponds to Table IV; it gives the abbreviated data, so that q_x in this Table refers to periods of 50 days. It should be noted, however, that the periods of 50 days in the case of the male table are centred at multiples of 50 days, *i.e.* the periods did not end at multiples of 50 days, but at multiples of 25 days (75, 125, etc.). This difference should not embarrass the comparison of Tables IV and V, and is expected to be advantageous for certain further studies in progress, where experimental results have to be evaluated. The graphs permit direct comparison of male and female data respectively.

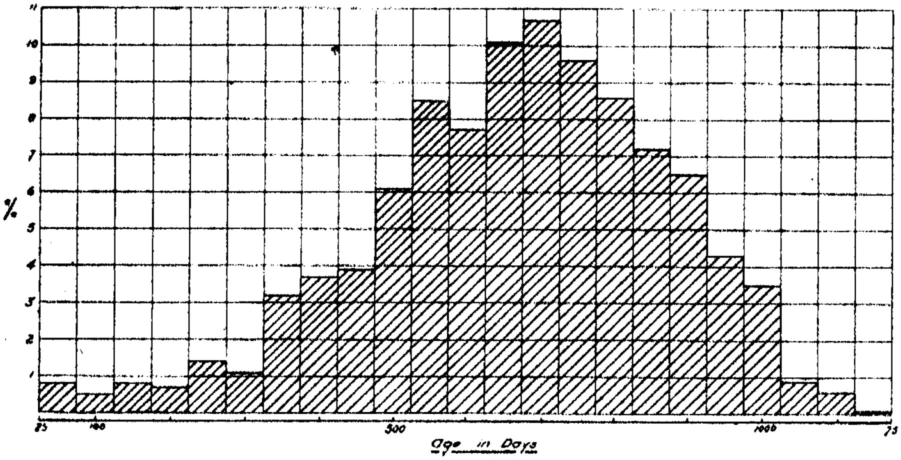


FIG. 5.—Frequency diagram showing distribution of ages at death in 1005 males observed from birth to death. The number of animals dying during any interval is given in per cent. of the total number.

Fig. 2 shows the order of dying-out in a population of 1000 males alive at age 31 days and exposed to the successive death-rates shown in Table V. The successive values of q_x of Table V have been plotted on an arithmetic and arithlog grid respectively (figs. 3 and 4).

The male data were graduated by fitting a polynomial of the second order. The resulting fit was unsatisfactory.

TABLE V.—ABBREVIATED LIFE TABLE: MALE RATS.
Based on mortality experience in 1399 males.

(For explanation see legend of Table IV.)

Age, Days.	t_x.	k_x.	D_x.	$10^8 . q_x$.	l'_x.	d'_x.
76–	1399	0, 0, 2, 0, 2, 3, 0, 2, 6, 7	0, 0, 0, 1, 1, 1, 0, 1, 1, 0	360230	99439	359
126–	1372	5, 0, 1, 3, 0, 10, 6, 0, 1, 0	2, 0, 1, 0, 1, 0, 2, 1, 0, 1	588668	99080	583
176–	1338	0, 0, 1, 1, 0, 0, 0, 0, 0, 0	0, 1, 0, 0, 0, 0, 2, 1, 2, 1	523560	98497	516
226–	1329	0, 0, 2, 0, 1, 3, 0, 0, 0, 0	3, 0, 2, 1, 4, 1, 2, 0, 0, 1	1055807	97981	1034
276–	1309	0, 0, 0, 0, 0, 1, 3, 5, 4, 0	2, 0, 0, 1, 2, 1, 1, 2, 1, 1	844854	96947	819
326–	1285	4, 5, 0, 0, 0, 0, 0, 0, 1, 0	3, 1, 1, 1, 2, 5, 4, 3, 3, 9	2500000	96128	2403
376–	1243	2, 0, 1, 0, 1, 7, 9, 1, 0, 0	2, 4, 3, 5, 7, 2, 2, 2, 4, 6	3003264	93725	2815
426–	1185	0, 0, 0, 0, 0, 0, 0, 8, 5, 1	5, 5, 6, 6, 2, 3, 1, 6, 2, 3	3310696	90910	3010
476–	1132	3, 6, 2, 2, 1, 4, 3, 9, 3, 1	4, 8, 4, 5, 5, 7, 9, 3, 10, 6	5470880	87900	4809
526–	1037	2, 0, 1, 1, 0, 1, 0, 2, 0, 1	5, 7, 7, 7, 11, 10, 9, 10, 9, 10	8228461	83091	6837
576–	944	0, 0, 2, 1, 3, 3, 9, 1, 2, 8	9, 5, 7, 7, 8, 8, 8, 11, 6, 8	8288482	76253	6320
626–	838	9, 6, 4, 2, 0, 2, 0, 4, 1, 1	10, 5, 5, 10, 9, 14, 14, 11, 14, 9	12272175	69933	8582
676–	708	0, 6, 12, 3, 1, 2, 1, 5, 3, 5	10, 10, 10, 8, 6, 8, 16, 15, 11, 13	15529753	61351	9528
726–	563	0, 3, 8, 12, 7, 4, 4, 9, 22, 15	12, 10, 7, 8, 11, 10, 7, 11, 11, 9	18426104	51823	9549
776–	383	3, 5, 17, 5, 0, 1, 2, 2, 2, 2	11, 7, 6, 6, 8, 13, 6, 16, 6, 7	23691460	42274	10015
826–	258	2, 1, 1, 1, 2, 1, 0, 0, 0, 0	10, 8, 8, 11, 8, 2, 9, 5, 6, 5	28346457	32259	9144
876–	178	0, 0, 0, 0, 0, 0, 0, 0, 0, 0	8, 4, 10, 4, 7, 5, 8, 6, 11, 2	36516854	23115	8441
926–	113	0, 0, 0, 0, 0, 0, 0, 0, 0, 0	2, 5, 10, 3, 4, 5, 6, 3, 3, 2	38053097	14674	5584
976–	70	0, 0, 0, 1, 1, 1, 1, 0, 0, 1	4, 4, 7, 5, 3, 1, 2, 4, 3, 2	52238806	9090	4749
1026–	30	0, 0, 1, 0, 0, 2, 2, 2, 2, 0	2, 0, 2, 0, 1, 2, 0, 1, 1, 0	36000000	4341	1563
1076–	12	2, 0, 0, 0, 0, 0, 0, 0, 0, 1	0, 1, 0, 1, 0, 1, 1, 1, 0, 1	60000000	2778	1667
1126–	3	0, 0, 0, 0, 1, 1, 0, 0, 0, 0	0, 0, 0, 0, 1, 0, 0, 0, 0, 0	50000000	1111	556

DISCUSSION.

Consideration of the data and the graphs derived from them reveals the general properties of the mortality curves of the rats of our stock during the period in question. During the period following upon weaning very few animals died under the conditions in which they were maintained. Thus the death-rate remains steady for some time after age 30 days. After this initial period the death-rate rises; and it continues to rise in the case of both males and females (between ages 200 and 300 days), up to nearly the terminal age in the vicinity of which the numbers are too small to permit significant conclusions. In other words, a rat population that has reached the age of 31 days dies out very slowly during the first eight months, but begins to die out more and more rapidly as life progresses. The number of animals dying out of 1000 alive at any time has been plotted in fig. 4 on an arithlog grid. While neither the graph for the females nor that for the males represents a straight line on an arithlog grid, it approximates such a line over a considerable expanse of time indicating that the force of mortality increases in geometrical progression during the period during which the majority of animals die.

The writers confess that the very low mortality exhibited during the early life is a matter of surprise to them. It should be remembered that the low level of the mortality in the case of the females is maintained at a time when in most animals pregnancy occurs for the first time. A considerable number of cases in which parturition ended in the death of the mother were also observed, and one might expect the physiological upheaval of the first pregnancy to produce adverse results.

It is possible that selection is partly responsible for low mortality in early life. Such selection was not practised in our colony, but it was applied extensively throughout the years preceding this experiment in the Wistar Institute from whose colony the present stock is derived (Greenman and Duhring, 1923). We understand that in the first instance the Wistar rat was selected for certain general characteristics (growth-rate, fertility, etc.). Now if in a human population mothers die at an early age, this does not, in our present state of civilisation, imply the death of the progeny; where early parental death was due to genetic causes, the responsible factors may be transmitted to subsequent generations. In the case of the rat, death of the mother implies death of the progeny in most cases. Also, since reproduction in the rat continues over a relatively longer span of life than is the case in man (as we shall show in a subsequent publication), it seems not improbable that any genetic constitution reducing the chance of early death will become predominant in the stock—always supposing that certain specific (and improbable) genetic conditions can be excluded.

Such automatic selection may be responsible for the similarity between the data obtained by the present writers and the corresponding data calculated by Greenwood (1928) for Hill's and Murray's mice respectively.

Whatever the explanation, the fact remains that the death-rate during youth and early maturity is very low, and increases rapidly with age. The comparative scarcity of old animals indicates the necessity for large numbers of animals with which any study on senescence has to begin, and may represent some justification for the somewhat circumstantial plan of the studies undertaken by the writers. It is obviously difficult to obtain animals which have reached the vicinity of the terminal age and can be described as senile on purely statistical grounds, and without reference to their somatic constitution. This is true whatever statistical term one would prefer to choose for a definition of senility. It is of interest to note in this connection that only little more than 500 males out of 100,000 males aged 30 days would survive to the terminal age observed in this colony during the past four years.

Attention must be drawn to the differences in order of dying-out in males and females respectively. In the first place, the mean duration of life is almost 24 days shorter in the case of males than in the case of females, and the difference equals nearly three times the standard error, so that it may be regarded as significant. Not only is the mean duration shorter in males, but the terminal age reached by females exceeds the terminal age of males by 116 days. On the other hand, it is possible that a slightly higher terminal age would have been obtained in the case of males had we not killed one very old male for anatomical purposes when it was nearly 1160 days old.

Reference to the tables will show that more females than males reached the vicinity of the respective terminal ages. The difference between the orders of dying-out in males and females respectively is further elucidated by reference to the q_x graph. It will be seen that the curve for females begins to rise later than the curve for males, and that its rise, over a period, is less steep.

The absolute values of the terminal age are decidedly different from the few other comparable data available for the rat. Slonaker (1912) states, for instance, that one of his groups of rats reared on a vegetable diet reached the average age of 1222 days. It must be remembered, however, that his rats belonged to a different stock, that there were only three individuals which may have received far greater individual attention than the writers were able or willing to give to their animals, and that the general conditions under which these animals were kept no doubt varied considerably from those of the present experiment. There is certainly nothing in our figures approaching Slonaker's result, one of whose rats attained an age of 45 months. Donaldson (1924) states that an albino rat "used for show purposes was kept by its owner for four years and eight months and was already about a year old when purchased." He also mentions that "during the last few years the oldest rats of both sexes in the Institute Colony (Wistar Institute) have attained three years and somewhat over. . . ." This would correspond more closely to our data; but unfortunately no details have yet been published.

The significance of the data presented in this communication is limited in various respects. It must be emphasised that they cannot be regarded as more than standards for one particular stock, kept under particular conditions during a particular period of time. There can be but little doubt that genetic conditions which, as is well known, generally influence longevity to a great extent (Pearson, 1902; Pearl, 1922, and many others) determine the results obtained; nor is it inconceivable that, for instance, climatic conditions prevailing during the years in question also influenced the duration of life, although everything possible was done to minimise the influence of such factors. Also, the results would probably not be fully relevant for a stock of a mixed origin. For the present writers' purpose the data are, of course, adequate, since they can serve as a basis of comparison with the duration of life obtained under experimental conditions during the years in question, and in relating somatic changes to age.

If it be assumed, for the moment, that the rat stock used represented a genetically uniform material, and if it be conceded that the environmental conditions were really approaching the constancy the writers attempted to secure, then it would appear that even comparative genetic uniformity and comparative constancy of environment did not prevent the distribution of the incidence of death over a wide range of ages. If one were dealing with absolutely constant environment and absolutely uniform genetical constitution, one would expect all animals to die at the same age. If only one of these conditions were fulfilled, the measure of dispersion would represent an indication to what degree environmental or genetical conditions respectively are responsible for the duration of life. Now it might be maintained that the rat stock in question was fairly homogeneous genetically, in view of the selection practised in the Wistar Institute for years previous to the founding of the present colony, and in view of the writers' attempt to prevent increase in the initial degree of heterozygosity by continued inbreeding. It may also be held that the environmental conditions were such as to exclude major variations; it would follow, then, that a comparatively low degree of genetical diversity combined with small variations in environment would suffice to produce very great differences in the duration of life. Some derivative of the standard deviation might be used to indicate to what degree this combination of minor genetical diversity and minor variations in environment can scatter the incidence of death over a range.

However, the writers are reluctant to grant themselves the premises for this application of, say, the coefficient of variation. Their diffidence is influenced by their doubt as to whether they really may speak of constant environment in the stock. It is quite true that the main conditions (housing, heating, light, food, etc.) were, comparatively speaking, con stant; but these factors do not constitute the whole of the environmental conditions.

The life of the mammal begins *in utero*, and litter-size, both *intra* and *extra uterum*, determines very largely weight at birth and growth during infancy. The young of small litters grow faster, conditions being otherwise equal, than young of large litters, and there must be considerable environmental difference between the existence of young in small and large litters. Now it has been shown recently by Kermack, McKendrick and McKinlay (1934), that in man the historical improvement established at any particular time in the death-rates of the various age groups depends primarily upon the date (*i.e.* actual year) of birth of the individuals concerned and only indirectly upon the particular year under consideration. They show that these results "are consistent with the hypothesis that the important factor from the point of view of health of the individual during his whole life is his environment up to the age of 15 years and that improved conditions at later ages have little direct effect." The authors thus draw attention to the importance of good environmental conditions during childhood.

If it is true for the rat that duration of life is influenced by conditions prevailing during the developmental period rather than by later environment, then differences in litter-size gain an added significance. It does not necessarily follow that small litter-size with its consequent high growth-rate will procure a higher degree of longevity. McCay (1933) has indeed pointed out that the inverse relation may hold and that low growth-rate may result in an increase in longevity. She compares her own rats kept on a diet procuring high growth-rate with those of Slonaker, whose animals grew slowly; her rats died at a mean age of only 503 ± 12 days; Slonaker's reached a mean age of 1222 days.

Whatever the rôle of the growth-rate during childhood in the rat may be, and whether a high rate affects the duration of life adversely or beneficially, it must be realised that the assumption of constant environment does not hold for the early phases of life which, for all we know at present, may affect duration of life very considerably. Thus it is doubtful if one is justified in regarding the variations in environment as minor.

While the writers are presenting these data for their own interest and have collected them for purposes connected with their experiments, it seems yet of interest to compare, even at the present juncture, the span of life of the Wistar rat with that of human beings. Donaldson concludes that a rat aged 3 years corresponds to a man of 90 years, but he does not say how he has arrived at this conclusion. Greenwood, basing his calculations on the data on Hill's and Murray's mice, equates the points at which q_x is at a minimum in man and mouse respectively, and concludes that 55 human years (which represent the expectation of life at the age of 11 years) are equivalent to 636·5 mouse-days. Both he and Pearl have also used another method, taking the interval elapsing before the survivors are reduced to a certain percentage of the entrants. The latter method gives a value of just over ·08 human year corresponding to 1 mouse-day.

For the present purpose a similar method is appropriate; assuming 100,000 viable male fœtuses being born in litters of which at least one animal survived to the age of 31 days, it would appear that these 100,000 animals will be reduced to 50,000 after 733 days, so that one rat-day would correspond to ·0955 human year. This calculation is based on the figures of Hunter's life table for the male peerage (1913); the use of this table is perhaps more appropriate than that of a life table relating to an unselected human population living under widely different conditions; also it does not require extrapolation in order to obtain l_0 as is the case in the life tables for insured population. Performing a similar calculation on the basis of the English Life Table No. 9, one rat-day corresponds to 0·08 human year.

SUMMARY.

(1) The method of breeding of a colony of the Wistar rat is described.

(2) Data are given relating to infantile mortality.

(3) Data are given relating to the order of dying-out in males and females respectively.

(*a*) The mean duration of life (in animals under observation from birth to death):

AL♀=693·1 days
AL♂=669·37 „

females thus dying later than males.

(*b*) The terminal age of females exceeds that of males.

(*c*) The force of mortality rises after the ninth month of life in geometrical progression.

The expenses of this investigation were defrayed by grants made by T. B. Macaulay, Esq., LL.D., and by additional grants from the funds of the University of Edinburgh.

The writers are also indebted to Dr Macaulay for his constant interest in their work. They wish to acknowledge the valuable advice given by Lieut.-Col. A. G. McKendrick, Dr W. O. Kermack, and Dr Macaulay on the actuarial treatment of the data here presented.

[REFERENCES.

REFERENCES TO LITERATURE.

DONALDSON, H. H., 1924. *The Rat: Data and Reference Tables*, Philadelphia.

GREENMAN, J. M., and DUHRING, F. L., 1923. *Breeding and Care of the Albino Rat for Research Purposes*, Philadelphia.

GREENWOOD, MAJOR, 1928. "Laws of Mortality from the Biological Point of View," *Journ. of Hygiene*, vol. xxviii, No. 3.

HUNTER, R. M., 1913. "Peerage Males: Statistics of Mortality, First Marriage and Issue," *Trans. Fac. Act.*, vol. vi, p. 364.

KERMACK, W. O., MCKENDRICK, A. G., and MCKINLAY, P. L., 1934. "Death-Rates in Great Britain and Sweden," *Lancet*, March 31, p. 698.

KORSCHELT, E., 1924. *Lebensdauer, Altern und Tod*, Jena.

McCAY, C. M., 1933. "Is Longevity Compatible with Optimum Growth?" *Science*, vol. lxxvii, p. 410.

PEARL, R., 1922. *The Biology of Death*, Philadelphia.

PEARL, R., and PARKER, S. L., 1921. "Experimental Studies on the Duration of Life," *Amer. Nat.*, vol. lv.

PEARSON, K., 1902. "On the Change in Expectation of Life in Man during a Period of *circa* 2000 Years," *Biometrika*, vol. i, p. 261.

SLONAKER, J. R., 1912. "The Effects of a Strictly Vegetable Diet on the Spontaneous Activity, the Rate of Growth, and the Longevity of the Albino Rat," *Leland Stanford Jr. Univ. Publ.*, Univ. Series.

TOPLEY, W. W. C., 1921. "The Spread of Bacterial Infection," *Journ. of Hygiene*, vol. xx, p. 103.

WIESNER, B. P., and SHEARD, N. M., 1933. *Maternal Behaviour in the Rat*, Edinburgh.

(*Issued separately February* 23, 1935.)

II.—Dating of Late-Glacial Clay Varves in Scotland. By **Gerard De Geer.** *Communicated by* Dr J. B. SIMPSON. (With One Figure.)

(MS. received December 5, 1934. Read January 7, 1935.)

DURING a visit to Scotland this autumn I had the opportunity of meeting Dr John B. Simpson and of discussing with him the desirability of definitively dating, by varve-measuring, the interesting ice-oscillation which he had described from the southern slope of the Scottish Highlands (1933).

I received recently from Dr Simpson such a varve-measurement from near Dunning in the valley of the River Earn, about 12 km. south-west of Perth.

The construction of a diagram (p. 25) showed that the thickness variation of the varves was very marked, affording some hope of identification, though the series so far measured did not comprise more than 59 varves. After a systematic comparison I succeeded in finding a strikingly corresponding curve from that part of the Swedish time-scale which we had extended to the Danish islands, viz. a varve-measurement carried out in 1919 by E. Antevs at Lyngby, about 6 km. N.N.W. of Copenhagen. This identification was confirmed by a comparison with the earlier teleconnected excellent measurement by Chester A. Reeds (1926) at Hackensack, N.J., near New York. The last-named curve shows a few local deviations, but on the other hand some striking similarities with the Scottish curve even better than and completing the Danish one, especially at the years 4324–25, 4334–39, 4345–48, and 4365–68. Such similarities appearing at exactly the right intervals and in the same way so strikingly completing the curve from Lyngby certifies the teleconnection of the Scottish curve practically as to all of its varves.

Thus it is evident that the Scottish curve from Dunning is now dated to represent the years from 4313 to 4371 before the zero or ending year of the Ice Age which, as may be known, occurred about 8700 years before our century. This means that the Scottish varves measured were deposited during the earlier part of the Gotiglacial subepoch when the Scandinavian land-ice still filled the whole of the Baltic valley, having its limit at Öresund and not very far from the outermost stage of the

Gotiglacial Baltic ice-lobe. An account of the extension of this ice-lobe around the western and southern parts of the Baltic depression is now being printed in *Geogr. Ann.*, Stockholm.

Now that the first varve series in Scotland, at Dunning, representing some decades of the Gotiglacial subepoch, has been dated, it will probably be possible to find out when, in Scotland, the Gotiglacial ice-oscillation arrived at its greatest extension. Dr Simpson has found varved clay in the proximal as well as in the distal parts of the valley in question. Continued varve-measurements will show if the varve-deposition was going on during the extension or the recession of the ice-border. In the latter case it is possible that the Scottish ice-oscillation had its maximum more correspondent to that in other countries, though, of course, for each country different topography and different climatic situation must always cause some differences in the date when a climatic oscillation could be registered at the very ice-border.

That the varved clay in the Perth region often is moraine-covered may possibly be explained by oscillations of the ice-border during its recession. But this question may, no doubt, obtain its solution by continued clay-measurements.

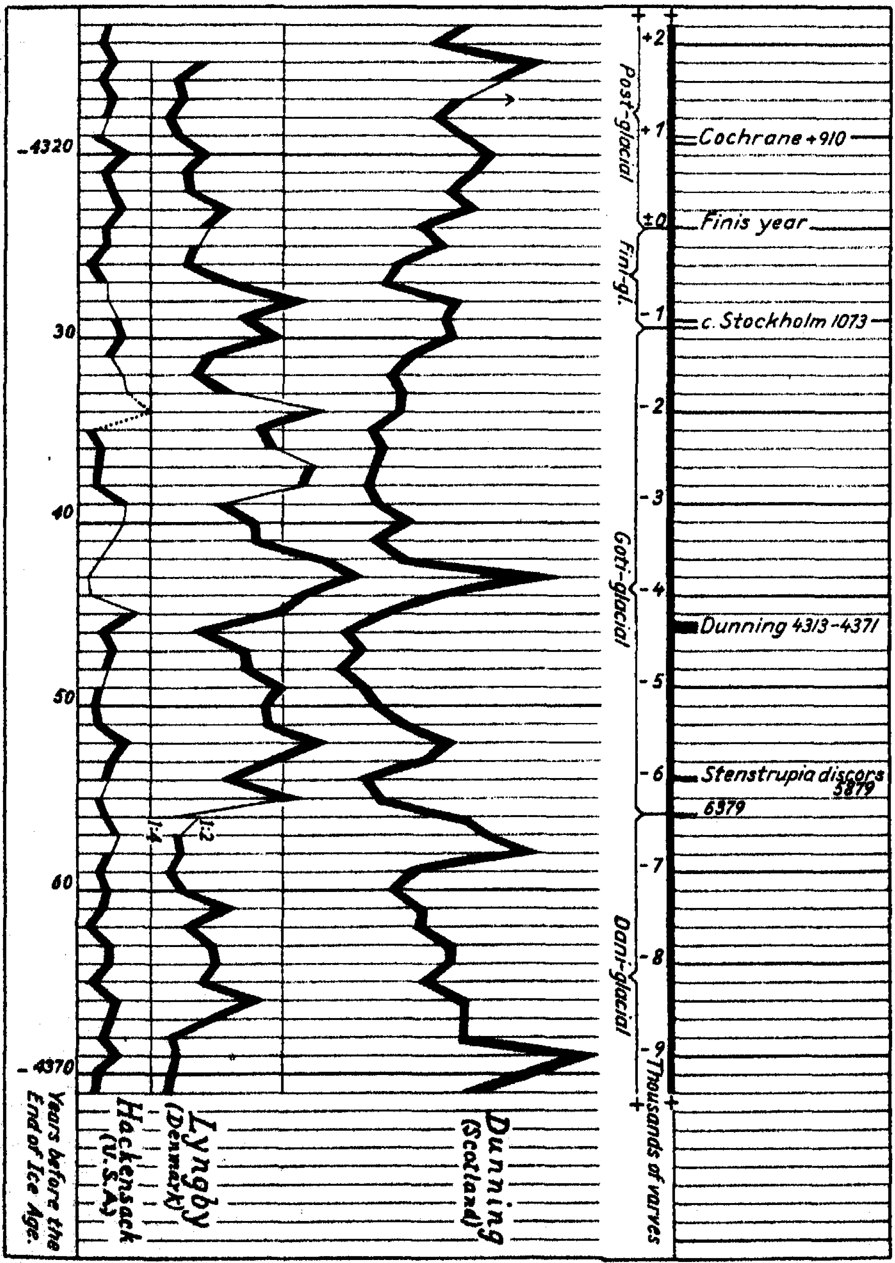
At present it is possible only to state that the clay at Dunning is of Gotiglacial age and was deposited during the earlier part, though considerably later than the beginning, of that subepoch.

The identification of the Dunning varves with the Swedish time-scale affords a new example of how misleading correlations can be when based only on an assumed general analogy in the utterly varying and, so far, as a rule utterly unexplored recession of the ice-border, without any geochronological support of varve-identification.

As to the suggestion of Simpson that the Perth oscillation might correspond to an ice-oscillation, assumed by Antevs (1928) to have possibly occurred simultaneously at Cochrane in Canada and at the Fenno-Scandinavian moraines a little south of the Stockholm ice-border, I may refer to a paper which elucidates the rather fundamental discrepancy between the views on the parallelisation of European and American Quaternary events as suggested by Antevs and those founded on real geochronological varve-connections (De Geer, 1934).

It is remarkable that Antevs, who has executed such a considerable and highly valuable amount of varve-measurements, has been opposed to their logical conclusions and instead suggested rather vague and conditional correlations.

In fact, varve-teleconnections, founded on very long series of corresponding thickness variations, have shown that the two ice-oscillations



DATING OF GOTIGLACIAL VARVES IN SCOTLAND

G. DE GEER 1934.

named, which Antevs assumed to be synchronous, in reality are separated by a time-interval of no less than two millenniums and belong to different subepochs, as indicated on the diagram. The teleconnection with the Swedish time-scale furthermore has shown that the varve series at Dunning is more than five thousand years older than the ice-border at Cochrane, and has nothing in common with either of those two oscillations.

It follows from this conclusion that these varves may possibly be distinctly separated from the overlying moraine, which may have been discordantly deposited, perhaps during a later part of the Gotiglacial subepoch. If the moraine covering is continuous it speaks for this conclusion, but continued varve-measurements may elucidate the whole question, and, if carried down to the bottom-varve, at the same time also fix the rate of the Gotiglacial ice-recession.

Now that it has been possible to obtain accurate teleconnection between Late Quaternary varves in Scotland and the Swedish time-scale, it seems highly desirable by continued varve-measurements to elaborate a closer parallelisation concerning other stages of the last deglaciation in Scotland and other countries.

REFERENCES TO LITERATURE.

ANTEVS, ERNST, 1928. "The Last Glaciation," *Res. Ser. Amer. Geogr. Soc.*, No. 17, New York.

DE GEER, GERARD, 1934. "Geology and Geochronology," *Geogr. Ann.*, vol. xvi, pp. 1–52, Stockholm.

REEDS, CHESTER A., 1926. "The Varved Clays at Little Ferry, New Jersey," *Amer. Mus. Novitates*, No. 209, New York.

SIMPSON, J. B., 1933. "The Late-Glacial Readvance Moraines of the Highland Border West of the River Tay," *Trans. Roy. Soc. Edin.*, vol. lvii, pp. 633–646.

(*Issued separately February 25, 1935.*)

III.—The Invariant Theory of the Correlation.

By **Professor H. W. Turnbull,** F.R.S., University of St Andrews.

(MS. received October 15, 1934. Read November 5, 1934.)

INTRODUCTION.

BEYOND the ternary and quaternary cases considered by Clebsch (1891) and Weitzenböck (1910), there appears to be no writing on the bilinear form in cogredient variables—the correlation—from the point of view of the projective invariant theory. In the present work a complete system of concomitants is found for the single bilinear ground form. Incidentally it is shown that such a system belongs not to the single form but to the simultaneous system of two mutually conjugate forms. Symbolic methods are used, and the processes are analogous to those employed in the cases of two quadratic forms (Turnbull and Williamson, 1929), and also of one bilinear form (Turnbull, 1932) in contragredient variables. In contrast with these previous results it is remarkable that the present system involves no forms of weight greater than two (§ 3, § 6).

Certain general remarks given in Turnbull and Williamson (1929, pp. 9–10) continue to apply: and in particular the Clebsch system has not yet been found. Owing to the length of the proofs here established the subsequent further investigation of the non-symbolic form of the result and the strict conditions for irreducibility have not been undertaken. The present system is virtually the same as that of a quadratic and a linear complex in n variables (a symmetric and a skew symmetric bilinear form).

§ 1. *Symbolic Notation of a Correlation.*—A bilinear form

$$\phi=\sum_{i=1}^{n}\sum_{j=1}^{n}x_i a_{ij} y_j \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

will be called a correlation when the two sets of variables x and y are cogredient. Thus they may be subjected to the same linear transformation, with non-singular matrix $[h_{ij}]$, so that ϕ becomes a bilinear form in new variables ξ_i and η_i. In fact

$$x_i=\sum_{j=1}^{n}h_{ij}\xi_j, \qquad y_i=\sum_{j=1}^{n}h_{ij}\eta_j, \qquad \phi=\sum_{i,j}\xi_i\theta_{ij}\eta_j, \quad . \quad . \qquad (2)$$

where $\theta_{ij}=\sum_{k,l} h_{ki}a_{kl}h_{ij}$. It is convenient to employ further sets of variables u, v, etc., contragredient to x; that is, such that the expression

$$u_x=u_1x_1+u_2x_2+\ .\ .\ .\ +u_nx_n\ . \qquad . \qquad . \qquad . \qquad (3)$$

is an absolute invariant of the transformation. Symbols can now be introduced, denoted by the letters a, b, c, a', b', c', . . ., accented or unaccented, each of which is cogredient with the variable u, such that the actual coefficient a_{ij} is resolved into the following symbolic factors:—

$$a_{ij}=a_ia'_j=b_ib'_j=c_ic'_j=\ .\ .\ ., \qquad . \qquad . \qquad . \qquad . \qquad (4)$$

wherein each suffix i and j takes all the values 1, 2, . . ., n. A gradient of degree δ in the coefficients a_{ij} is expressible unambiguously as a gradient in the symbols a_i, each term containing exactly δ distinct letters a, b, c, . . ., and each letter occurring twice, once with and once without an accent. The correlation assumes the equivalent symbolic forms

$$\phi=a_xa'_y=b_xb'_y=c_xc'_y=\ .\ .\ ., \qquad . \qquad . \qquad . \qquad (5)$$

where a_x is analogous to u_x in (3). According to the usual theory (Turnbull, 1929, p. 201) it is permissible in any symbolic expression to interchange any pair of its letters a, b, provided that the corresponding pair of accented letters is simultaneously interchanged. This leaves the actual expression unaltered in value. But the interchange of accent (of a with a', b with b', . . .) for all the symbols produces the *conjugate* expression.

For example, the *conjugate correlation* of ϕ is

$$\phi'=\sum_{i,j}^{n} x_ia_{ji}y_j=a'_xa_y=b'_xb_y=c'_xc_y=\ .\ .\ ., \qquad . \qquad . \qquad (6)$$

which is in general distinct from ϕ. If in particular $\phi'\equiv\phi$, then $a_{ij}=a_{ji}$ for all i, j, and the correlation is *symmetrical* (geometrically it represents a quadric): while, if $\phi'\equiv-\phi$, then $a_{ij}=-a_{ji}$, for all i, j, and it is *skew symmetrical* (geometrically it represents a *linear complex* or *null system*). Evidently the general unsymmetrical correlation determines a pencil of forms $\lambda\phi+\phi'$, including one symmetrical ($\lambda=1$) and one skew symmetrical ($\lambda=-1$) member.

§ 2. By resolving each variable x into $n-1$ of the contragredient variables u, v, . . . in the usual way the simplest approach to the concomitants of ϕ is obtained. According to the fundamental theorem (Turnbull, 1929, p. 190) every concomitant is expressible as an aggregate of factors of the *outer product* type

$$g=(a\ .\ .\ .\ b'\ .\ .\ .\ u\ .\ .\ .), \qquad (1)$$

containing exactly n letters, with or without accents, representing symbols a or variables u. If no symbols appear g is the absolute invariant, conveniently written

$$U_n=(uU_{n-1})=(uv\ .\ .\ .\ w), \qquad (2)$$

where U_{n-1} denotes a convolution of $(n-1)$ cogredient variables $v, \ldots,$ w. When these are in fact the components into which x has been resolved then $U_{n-1}=x$ and $U_n=u_x$. Similarly U_s will denote a convolution of s variables u.

Next, if within g each symbol is paired with its conjugate; that is, appears once with and once without an accent, then g represents an actual concomitant. Such are $(aa'U_{n-2})$, $(aba'b'U_{n-4})$, $(abca'b'c'U_{n-6})$, etc. Since g is an n-rowed determinant, each letter a, a', u representing a column, there is essentially only one such type of concomitant of unit weight for each value of r the number of different symbols a, b involved. This is written

$$(A_rA'_rU_{n-2r}). \qquad (3)$$

When n is even there is an *invariant* of unit weight, no variables appearing: $(A_rA'_r)$, $n=2r$. When n is odd there is a *contravariant* $(A_rA'_ru)$, $n=2r+1$, involving a single variable u.

More generally, every further concomitant is expressible as a sum of terms P, each of which is a product of factors

$$P=g_1g_2\ .\ .\ .\ g_w, \qquad (4)$$

each of the w factors containing exactly n letters as in g. Allowing for every possibility within a factor g, let it have m pairs of symbols, q further unaccented and r accented unpaired symbols, and s variables: and let these be arranged as follows:—

$$g=(A_mA'_mB_qC'_rU_s), \qquad (5)$$

where

$$2m+q+r+s=n,$$

$m, q, r, s > 0$. The suffix w denotes the *weight* of P; the number $(m+q)$ of equivalent symbols a convolved in g is called the *currency* of g in such symbols; also $(m+r)$ is the currency in conjugate symbols. To each of the q symbols of B_q corresponds a conjugate occurring in one of the other factors of P. Similarly for C'_r.

Now let forms P be considered in ascending weight: and, for equal weights, in ascending degree in the symbols: and, for equal degrees, in descending currency. Therefore in a given P let the factor of highest currency in either type of symbol be placed first. Without loss of generality take this currency to be $(m+q)$ and P to be

$$P=(A_{m+q}A'_mC'_rU_s)\,(B'_{q_1}\ldots)\,(B'_{q_2}\ldots)\ldots,(B'_{q_\mu}\ldots)\ldots, \quad . \qquad (6)$$

where $A_{m+q}=A_mB_q$, the various B'_{q_i} including all the symbols conjugate to B_q, so that

$$q=q_1+q_2+\ \ldots\ +q_\mu.$$

By a process analogous to that in the theory of quadrics (Turnbull, 1929, p. 194) interchange the $(m+q)$ equivalent symbols of A_{m+q} in all possible ways and add the results. Hence

$$(m+q)\,!\,P=m\,!\,q_1\,!\,q_2\,!\ldots q_\mu\,!\,\Omega'(A_{m+q}A'_mC'_rU_s)\,(B'_{q_1}\ldots)\,(B'_{q_2}\ldots)\ldots, \qquad (7)$$

where Ω' denotes the determinantal permutation of the $(\mu+1)$ matrices A'_m, B'_{q_i}. A Sylvester identity (Turnbull, 1929, p. 45) immediately expresses the right-hand side as a sum of terms, in each of which all these $(\mu+1)$ matrices are assembled within any prescribed factor, let us say the second. The first two symbolic factors now assume the form

$$g_1g_2=(A_{m+q}\ldots)\,(A'_{m+q}\ldots)\,. \quad . \quad . \quad . \quad . \qquad (8)$$

The process has therefore removed the m symbols of A'_m from g_1 of (6), replacing them by m new symbols or variables from g_2. If $k(>0)$ new symbols equivalent to a have thereby been introduced into g_1, a repetition of the process yields the same result (8), but with $(m+q+k)$ replacing $(m+q)$ in both factors g_1 and g_2. Briefly the currencies of the conjugate symbols A and A' have been raised.

If, however, $k=0$ there may still be further symbols a', besides A', in g_2; in which case the *conjugate* process will assemble the corresponding symbols a within g_1, with increased currency as before. Let these processes continue until no such further step is possible: the first two factors therefore must take the form

$$g_1g_2=(A_iC'_rU)\,(A'_iD_sV), \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

with $i\geqslant r$, $i\geqslant s$, $r\geqslant 0$, $s\geqslant 0$, the currency of U being $(n-i-r)$ and of V, $(n-i-s)$. If $w=2$ this is a complete concomitant, so that the r symbols conjugate to C'_r must be included in those of D_s, and *vice versa.* Thus $C_r=D_s$. If $w>2$ the conjugates of C'_r may occur distributed among any of the remaining factors $g_2g_3\ \ldots\ g_w$.

The cases when $r=i$ or $s=i$ are reducible. For if $r=i$ the matrix A'_i may be transferred by a Sylvester identity from g_2 to g_1 giving (when $2i\leqslant n$)

$$g_1g_2=(A_iA'_iU)\,(C'_iD_sV)+\sum_{k>0}(A'_iC'_k\ldots)\,(C'_l\ldots D_sV), \quad . \qquad (10)$$

where $C'_k C'_l = C'_i$. The first term on the right contains the concomitant (3) of unit weight, while the currency $(i+k)$ of each subsequent term is increased. Similarly if $s=i$. Also, when $2i > n$, (10) holds with the suppression of this first term.

The requisite forms of weight two, which include all irreducible forms of this weight, can now be written down:

$$P=(A_i B'_r U_{n-i-r})\,(A'_i B_r V_{n-i-r}), \qquad (11)$$

where

$$i > r \geqslant 0, \qquad n \geqslant i+r > 0.$$

Next if $w > 2$ and is even, two more factors $g_3 g_4$ may be paired off according to the greatest surviving currency, by the same methods. Such a process may alter the contents of the earlier factors $g_1 g_2$ but cannot disturb A_i or A'_i of (9). If fresh symbols a are thereby introduced into g_1 the original process (with increased value of i) is repeated. Similarly for a'. Eventually in this way, when $w=2\nu$, every successive pair of factors $g_1 g_2$, $g_3 g_4$, $g_5 g_6$, . . . assumes the form (9), and we can take as representative

$$P_{2\nu}=(A_i\,.\,.\,.)\,(A'_i\,.\,.\,.)\,(A_j\,.\,.\,.)\,(A'_j\,.\,.\,.)\,.\,.\,.\,(A_l\,.\,.\,.)\,(A'_l\,.\,.\,.), \qquad (12)$$

where each odd factor g_1, g_3, . . . contains no further unaccented, and each even factor g_2, g_4, . . . no further accented, symbol. Furthermore these factor pairs may evidently be arranged in descending suffix value, so that

$$i \geqslant j \geqslant \,.\,.\,.\, \geqslant l. \qquad (13)$$

Likewise when $w=2\nu+1$, only that an extra factor (denoted by g_0) will be left over. Thus, let us say,

$$\begin{aligned} P_{2\nu+1} &= (A_i\,.\,.\,.)\,(A'_i\,.\,.\,.)\,.\,.\,.\,(A_l\,.\,.\,.)\,(A'_l\,.\,.\,.)\,(A_m B'_r U) \\ &= g_1 g_2\,.\,.\,.\,g_{2\nu} g_0, \quad l \geqslant m, \quad l \geqslant r. \qquad (14) \end{aligned}$$

In consonance with the order already adopted a form P will be considered prior to P_1, of the same weight and degree, if the suffix set $[i, j, \,.\,.\,.\,, l]$ of P is higher than $[i_1, j_1, \,.\,.\,.\,, l_1]$ of P_1; that is, if the first of the differences $(i-i_1)$, $(j-j_1)$, . . . which is not zero is positive.

The case $w=2$ has been considered: therefore let $P_{2\nu+1}$ be taken next, and finally $P_{2\nu}$. If in (14) $r > m$ the rôles of the conjugate symbols may be interchanged. Suppose then that $m > r$. Now consider the m accented symbols conjugate to A_m. By hypothesis they cannot be situated in the factors $g_2 g_4 \,.\,.\,.\, g_{2\nu}$ but must be somewhere in the residual factors. Let them be convolved explicitly in the last factor g_0 by a Sylvester identity (obtained by interchanging the m pairs of letters in all possible ways) so that

$$m\,!\,P_{2\nu+1}=\sum_{h\geqslant 0}(A_i\,.\,.\,.\,)\,(A'_i\,.\,.\,.\,)\,.\,.\,.\,(A'_l\,.\,.\,.\,)\,(A_{m-h}A'_mB'_kU). \qquad (15)$$

If $h > 0$, then h of the symbols of A_m have been displaced to one (or more) of the factors which already contain A_i, A_j, . . ., A_l, so that one (or more) of the suffixes $i, j, \ldots, l$ has been increased without decrease of others. Thus the term is reducible. If, however, $h=0$, then either $k=0$ or $k > 0$: and if $k=0$ the final factor of (15) is an actual concomitant, while if $k > 0$ the final factor has more accented than unaccented symbols so that the whole term is of type (14) with $r > m$ and with $i, j, \ldots, l$ unchanged. Hence in each case $P_{2\nu+1}$ is reducible.

Finally $P_{2\nu}(\nu > 1)$ must be considered; let us say

$$P_{2\nu}=(A_i\,.\,.\,.\,)\,(A'_i\,.\,.\,.\,)\,(A_j\,.\,.\,.\,)\,.\,.\,.\,(A_l\,.\,.\,.\,)\,(A'_lB_rU). \quad . \qquad (16)$$

If $r=0$, then accented symbols must occur in the last but one factor, else $g_{2\nu+1}g_{2\nu}$ is an actual concomitant. The rôles of conjugate symbols may then be reversed. Hence we assume that $r > 0$. Once more the symbols conjugate to B_r must occur in the factors $g_1g_3\,.\,.\,.\,g_{2\nu-1}$: and they may be convolved explicitly in the factor $g_{2\nu-1}$. This step expresses $P_{2\nu}$ as a sum of terms $\sum R$, where

$$R=(A_i\,.\,.\,.\,)\,(A'_i\,.\,.\,.\,)\,(A_j\,.\,.\,.\,)\,.\,.\,.\,(A_lB'_rC'_sV)\,(A'_lB_rU), \quad . \qquad (17)$$

together with terms wherein some or all of the l symbols of A_l have been displaced leftwards into factors already containing A_i, A_j, . . . Such terms are reducible through increase of a suffix $i, j, \ldots$ prior to l. In R the matrix C'_s denotes such of the accented symbols belonging to $g_{2\nu-1}$ in $P_{2\nu}$ which have not been displaced by the process. If $s=0$, then the final pair of factors form an actual concomitant, while if $s > 0$ the process may be repeated by reversing the rôles of conjugate symbols and with $(r+s)$ replacing r. As before, the process leads to nothing new and ultimately reduces R.

§ 3. *The Complete $C(a, u)$ System of a Correlation.*—By the preceding investigation a complete system has been obtained wherein all the variables that occur are only of type u and are cogredient with the symbols a. Such a system * is denoted by $C(a, u)$. Incidentally the following theorem has been proved.

Every concomitant of a correlation whose weight exceeds two is reducible in terms of the $C(a, u)$ system.

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* *Cf.* Turnbull and Williamson, 1929, p. 9, where the corresponding system for two quadrics is called the U system.

The said system is as follows:—

(i) *Absolute Concomitant,*

$$(U_n) = (uv\ .\ .\ .\ w).$$

(ii) *Weight One,*

$$(A_m A'_m U_{n-2m}) \qquad 0 < 2m \leqslant n.$$

(iii) *Weight Two,*

$$(A_i B'_j U_k)\ (A'_i B_j V_k)$$
$$i + j + k = n, \qquad i > j \geqslant 0, \qquad k \geqslant 0.$$

§ 4. The same methods apply to the complete system of type $C\{a, u, x\}$; that is, one in which any number of cogredient and contragredient variables may occur. It is only necessary to introduce, besides the outer product type of factor already employed, the generalized inner and outer products analogous to those used in the case of quadrics (Turnbull and Williamson, 1929, p. 11).

(i) *Generalized Inner Product Type,*

$$(A_m \mid X_m) = \Omega a_x b_y\ .\ .\ .\ c_z, \qquad . \quad . \quad . \quad . \qquad (1)$$

where $A_m = ab\ .\ .\ .\ c$, $X_m = xy\ .\ .\ .\ z$, and Ω denotes the $m!$ terms obtained by determinantal permutation of $ab\ .\ .\ .\ c$.

(ii) *Generalized Outer Product Type,*

$$(A_i B'_j U_k X_m) = \Omega_B (A_i B'_{j_1} U_k)\ (B'_m \mid X_m), \qquad . \quad . \quad . \qquad (2)$$

where $B'_j = B'_{j_1} B'_m$; that is, $j = j_1 + m$, and Ω_B is the determinantal permutation of this distribution of B' into two factors, and therefore contains $\binom{j}{m}$ terms.

Clearly if $m = 1$ the generalized inner product is merely the ordinary product: and if $m = 0$ the generalized outer product is the ordinary outer product $(A_i B'_j U_k)$. The suffixes satisfy the relations

$$2n > i + j + k - m = n, \qquad . \quad . \quad . \quad . \qquad (3)$$

the inequality being due to the fact that (X_n) is an actual concomitant, so that for our purpose $m < n$.

By the fundamental theorem the typical concomitant consists of terms

$$P = g_1 g_2\ .\ .\ .\ g_w h_1 h_2\ .\ .\ .\ h_\varpi, \qquad w \geqslant 0, \quad \varpi \geqslant 0, \qquad . \qquad (4)$$

containing w ordinary outer products (as before) together with ϖ ordinary inner products a_x or a'_x. The ϖ variables x which thus enter into P need not necessarily be all alike, but they must be cogredient. Variables of type u, contragredient to x, may enter as before into the outer products. The suffix w will be called the weight of P, and ϖ the order.

The only irreducible concomitant of zero weight besides u_x, (X_n), is of type

$$\phi = a_x a'_y. \qquad (5)$$

This type includes the conjugate form $a_y a'_x$, the quadric $a_x a'_x$, and forms derived therefrom by polarization. The linear complex

$$(aa' \mid xy) = a_x a'_y - a_y a'_x \qquad (6)$$

and other such generalized inner products are evidently reducible in terms of type (5), when the variables x, y are no longer convolved.

Next if P is of weight unity, it can be written

$$P = (A_r A'_r B_p C'_q U) h_1 \ldots h_\varpi, \qquad (7)$$

where the symbols conjugate to B and those conjugate to C' must occur in the h factors. By $p\,!$ interchanges among the B, and $q!$ among the C symbols, we obtain

$$p\,!\, q\,!\, P = (A_r A'_r B_p C'_q U)\,(B'_p \mid X_p)\,(C_q \mid Y_q), \qquad (8)$$

where X_p, Y_q denote convolutions of p and of q variables cogredient with x. Obviously $p+q=\varpi$. If $p > q$ we may combine $A_r B_p$ as A_{r+p} and proceed as in § 2 (6) to convolve the conjugate symbols $A'_r B'_p$ explicitly in the first factor g_1. If $q > 0$ this step either raises the currency by introducing more than $(r+p)$ accented symbols into g_1, or displaces at least one symbol of C' into the X factor. By resolution of the compound into simple inner products (1) a concomitant of zero weight, of type $c_x c'_y$, emerges. Hence if $q > 0$ the form reduces. Similarly if $q > p > 0$. There remains the possible type

$$P_1 = (A_{r+p} A'_r U)\,(A'_p \mid X_p), \qquad (9)$$

which is evidently equivalent to a generalized outer product (Turnbull and Williamson, 1929, pp. 12–14). By interchange of equivalent symbols this can be written

$$(r+p)\,!\, P_1 = (A_{r+p} A'_{r+p} U X_p), \qquad (10)$$

and by convolving A'_{r+p} explicitly it is also equivalent to

$$(-)^r (A'_{r+p} A_{r+p} U X_p), \qquad (11)$$

together with reducible terms involving the type u_ϖ, due to the displacement of variables u into the X factor.

For higher weight the argument now proceeds as before, starting with the form P of § 2 (6): only in this case an extra submatrix $(B'_{q_0} \mid X)$ must be included where $q = q_1 + q_2 + \ldots + q_\mu + q_0$, to allow some of the symbols of B'_q to belong originally to h factors. This implies that on reaching the step (§ 2 (8)) the second factor g_2 may be a generalized outer product: and, allowing for the conjugate process, both factors of (9) may be such. Concomitants of weight two are therefore of the form

$$P = (A_i B'_r C'_s UX)\,(A'_i B_r D_t UX)\,(C_s \mid X_s)\,(D'_t \mid X_t) \quad . \quad . \quad (12)$$

together with simpler ones with one or both pairs CC', DD' suppressed —that is, $n > s \geqslant 0$, $n > t \geqslant 0$. Let BC be convolved in g_2, the second factor, since $B'C'$ is already convolved. Resulting terms R, wherein a symbol d of D or u of U has been shifted to replace a c of C_s, are reducible by yielding the types $d_x d'_y$ or u_x as before. The process gives

$$\binom{r+s}{r} P = \sum Q + R,$$

where

$$Q = (A_i B'_{r+s} UX)\,(A'_j B_{r+s} D_t UX)\,(A'_s \mid X_s)\,(D'_t \mid X_t) \quad . \quad . \quad (13)$$

with $B'_{r+s} = B'_r B'_s$, $A_i = A_j A_s$. Similarly $B'D'$ may be convolved in g_1, and in this way P is made equivalent to the form

$$(A_i B'_{r+s+t} U_k X)\,(A_i B_{r+s+t} U_l X). \quad . \quad . \quad . \quad (14)$$

A reduction analogous to that of § 2 (10) reduces this if $i = r+s+t$. Hence $i > r+s+t$.

§ 5. For weights higher than 2 the preceding argument applies on the supposition that each factor g_i of the expressions in § 2 (12) and (14) may be a generalized factor and involve both U and X. In place of (15) the process now yields, let us say,

$$m\,!\,P_{2\nu+1} = \sum_{h \geqslant 0} \{A_i \ldots\}\{A'_i \ldots\} \ldots \{A'_l \ldots\}\,(A_{m-h-p} A'_m B'_k U)\,(A_p \mid X_p), \quad (1)$$

where the brackets { } denote generalized factors, the final one being written out explicitly. As before h denotes the number of symbols of A_m which have been displaced to earlier factors. The original arguments hold if $h > 0$, or if $k > 0$, while if $h = 0$, $k = 0$, a concomitant of weight one can be removed

$$(A_{m-p} A'_m U)\,(A_p \mid X_p) \quad . \quad . \quad . \quad . \quad (2)$$

which is seen to be equivalent to an earlier type (§ 4 (9)).

Similarly there is no material modification in the case $P_{2\nu}$. Hence the complete system as before is entirely confined to forms whose weight cannot exceed two.

Among the covariantive forms involving variables X, but not u, certain further conditions apply. Thus if

$$P = (A_i B'_j X_k)\,(A'_i B_j X_k) \quad . \quad . \quad . \quad . \quad (3)$$

we first have $i+j>n$ and $i+j-n=k$, $i>j$. But if $2k>j$, the form reduces, by containing factors of type $b_x b'_y$ when P is written as

$$\sum (A_i B'_{n-i})\,(B'_k \mid X_k)\,(A'_i B''_{n-i})\,(B''_k \mid X_k), \qquad . \quad . \qquad (4)$$

where $B_j = B''_{n-i} B''_k$. For at least one symbol pair b, b' is common to the two X factors in every term of the summation (Turnbull and Williamson, 1929, p. 20). Hence the irreducible terms are included among those for which $2k \leqslant j$; that is,

$$2i+2j > 2n \geqslant 2i+j, \qquad i>j>1 \qquad . \quad . \quad . \qquad (5)$$

the latter inequality ($j>1$) being implied by what precedes.

For $(A_i B'_j U_k X_p)\,(A'_i B_j U_l X_q)$ in a similar way the number of symbols x must not exceed the smaller of i or j; that is, $p+q \leqslant j$. But

$$p=i+j+k-n, \qquad q=i+j+l-n.$$

Hence

$$\left.\begin{array}{c} 2i+j+k+l \leqslant 2n \\ n < i+j+k, \qquad n < i+j+l, \qquad i>j>1. \end{array}\right\} \qquad . \quad . \qquad (6)$$

Forms for which one of p and q is zero are such as

$$(A_i B'_j U_k)\,(A'_i B_j U_l X_{i+j+l-n}), \qquad . \quad . \quad . \qquad (7)$$

where

$$i+j+k=n < i+j+l < 2n, \qquad 0<j<i. \qquad . \quad . \quad . \qquad (8)$$

Those for which $p=q=0$ have already been considered.

§ 6. *The Complete System involving two Contragredient Sets of Variables.*

(i) *Absolute concomitants,*

$$(uv\,.\,.\,.\,w), \qquad (xy\,.\,.\,.\,z), \qquad u_x.$$

(ii) *Contravariantive forms* (with variables x absent),

$$(A_m A'_m U_{n-2m}) \qquad n \geqslant 2m > 0;$$

$$(A_i B'_j U_k)\,(A'_i B_j V_k) \qquad n \geqslant i > j \geqslant 0.$$

(iii) *Covariantive forms* (with variables u absent),

$$a_x a'_y, \qquad a_y a'_z,$$

$$(A_m A'_m X_{2m-n}) \qquad 2m > n > m;$$

$$(A_i B'_j X_{i+j-n})\,(A'_i B_j Y_{i+j-n}),$$

where

$$2n \geqslant 2i+j, \qquad i+j > n > i > j > 1.$$

(iv) *Mixed forms* (with both sets of variables present),

$$(A_m A'_m U_p X_{2m+p-n}) \qquad 2m+p > n \geqslant m+p, \qquad m>0, \qquad p>0;$$

$$(A_i B'_j U_k)\,(A'_i B_j U_l X_{i+j+l-n}),$$

where

$$2n > i+j+l > n = i+j+k, \qquad i > j > 0;$$
$$(A_i B_j' U_k X_p)\,(A_i' B_j U_l X_q) \qquad i > j > 1,$$

where also

$$2n \geqslant 2i+j+k+l, \qquad i+j+k > n, \qquad i+j+l > n.$$

§ 7. SPECIAL TYPES OF CONCOMITANTS.

I. *Invariants.*—These, which contain neither type of variable, may be picked out at once. They are:

(*a*) Weight one,

$$(A_m A_m'), \qquad 2m = n;$$

(*b*) Weight two,

$$(A_i B_{n-i}')\,(A_i' B_{n-i}), \qquad n \geqslant i > n-i.$$

The former occurs only if n is even. Of the latter there are $\frac{1}{2}n$ if n is even, and $\frac{1}{2}(n+1)$ if n is odd. Those of weight two may be identified as the coefficients in the binary n-ic,

$$|\,\lambda A + \mu A'\,| = \phi_0 \lambda^n + \phi_1 \lambda^{n-1}\mu + \;.\;.\;.\; + \phi_i \lambda^{n-i}\mu^i + \;.\;.\;.\; + \phi_n \mu^n, \qquad (1)$$

where $A = [a_i a_j']$ and $A' = [a_i' a_j]$ are the matrices of the original correlation and its conjugate. The expansion (1) denotes the characteristic determinant of the pencil $\lambda A + \mu A'$. Since a determinant is unaltered by transposition the coefficients are equal in pairs, $\phi_i = \phi_{n-i}$. Indeed it is easy to verify that

$$\phi_0 = \phi_n = \frac{1}{n\,!}\,(A_n)\,(A_n'),$$

$$\phi_i = \phi_{n-i} = \frac{1}{n\,!\,(n-i)\,!}(A_i B_{n-i}')\,(A_i' B_{n-i});$$

which are the invariants of weight two.

II. *The Contravariants.*—If one variable u alone enters the forms are called contravariants. They are—

(*a*) Weight one,

$$(A_m A_m' u), \qquad n = 2m + 1;$$

(*b*) Weight two,

$$(A_i B_{n-i-1}' u)\,(A_i' B_{n-i-1} u), \qquad n > i > n-i-1 \geqslant 0.$$

The former occurs only if n is odd. There are $\frac{1}{2}n$ of the latter if n is even, and $\frac{1}{2}(n-1)$ if n is odd. These forms occur if the reciprocal matrix $(\lambda A + \mu A')^{-1}$ is expanded as a polynomial in $\lambda : \mu$ of order $(n-1)$ divided by the previous n-ic $|\,\lambda A + \mu A'\,|$.

Evidently if $n = 4$ there are two possible values $i = 2$, 3, and no value of m. Hence the quaternary correlation has two contravariants (Weitzenböck, 1910, p. 135).

III. *Covariants.*—If the variable x alone occurs the forms are called covariants. They are:

(*a*) Weight zero,

$$a_x a'_x;$$

(*b*) Weight one,

$$(A_m A'_m x), \qquad n = 2m - 1;$$

(*c*) Weight two,

$$(A_i B'_{n-i+1} x)\,(A'_i B_{n-i+1} x), \qquad n > i > n - i + 1 > 1.$$

The first, (*a*), is the quadric or symmetrical member of the correlation pencil $\lambda A + \mu A'$, $(\lambda = \mu)$. The second occurs only if n is odd. The third, (*c*), type contain $\left(\frac{n}{2} - 1\right)$ members if n is even, and $\frac{n-1}{2}$ if n is odd. They are generated most simply by forming the reciprocal of the reciprocal pencil, $(\lambda A^{-1} + \mu A'^{-1})^{-1}$, and again picking out the coefficients.

If $n = 4$, then i can be 3 only, while m has no value. Hence there are two quaternary covariants, $a_x a'_x$ and $(A_3 B'_2 x)\,(A'_3 B_2 x)$ (Weitzenböck, 1910, p. 135).

IV. *The Line Complexes.*—It is natural to enquire what forms exist which contain the single variable $p = xy$ denoting a straight line joining two points x and y. Such forms must ultimately arise from terms of the X system where only x and y are retained. Let the requisite members of this system be selected from § 6 and denoted as follows:—

(i) $i_x i'_y = (A_i B'_{n-i+1} x)\,(A'_i B_{n-i+1} y), \qquad 2i > n + 1;$
(ii) $r_x = (A_r A'_r x), \qquad n = 2r - 1;$
(iii) $(A_i B'_{n-i+2} p)\,(A'_i B_{n-i+2} p), \qquad 2i > n + 2;$
(iv) $(A_i A'_i p), \qquad n = 2i - 2;$
(v) $(A_2 p)\,(A'_2 p).$

The above represent all types which may occur when only two cogredient variables, x and y, are retained. Forms in the single variable p are therefore compounded of these types, of which (iii), (iv), and (v) already are explicit functions of p. All remaining irreducible forms must therefore be combinations of types (i) and (ii); that is, are products of symbolic factors

$$i_x, \quad i'_x, \quad i_y, \quad i'_y, \quad r_x, \quad r_y, \quad . \quad . \quad . \quad (1)$$

in which as many x's occur as y's, each x being convolved with a y. The problem reduces to one in binary types, by means of the following Lemma:

LEMMA.—*A form containing two equivalent symbols i convolved is reducible.*

Proof.—Write $\theta_{ix}=(A_iB'_jx)$, $\theta'_{ix}=(A'_iB_jx)$, where $i>j=n-i+1$; and let ϕ_i be a symbol equivalent to θ_i, and λ, μ, ρ, σ be symbols or variables cogredient with x. A form with two equivalent symbols i convolved is

$$(\theta_i\phi_i \mid \lambda\mu)\ (\theta'_i \mid \rho)\ (\phi'_i \mid \sigma),$$

which by interchange of equivalent symbols is the same as

$$\tfrac{1}{2}(\theta_i\phi_i \mid \lambda\mu)\ (\theta'_i\phi'_i \mid \rho\sigma).$$

Apart from a non-zero numerical factor this may be written

$$(A'_ib_2\ldots b_j)\ (C'_id_2\ldots d_j)\ (b_1d_1 \mid \lambda\mu)\ (A_ib'_1b'_3\ldots b'_j)\ (C_id'_1d'_3\ldots d'_j)\ (b'_2d'_2 \mid \rho\sigma),$$

where

$$b_1b_2b_3\ldots b_j=B_j \quad\text{and}\quad d_1d_2d_3\ldots d_j=D_j.$$

In this six-factor expression P convolve b_2d_2 in g_1 the first factor, since the conjugate convolution occurs in g_6. Hence

$$P=\frac{j-2}{2}X+\frac{i}{2}Y,$$

where

$$X=(A'_ib_2d_2b_4\ldots b_j)\ (C'_ib_3d_3\ldots d_j)\ (b_1d_1 \mid \lambda\mu)\ldots,$$
$$Y=(d_2A'_{i-1}b_2\ldots b_j)\ (C'_ia'_1d_3\ldots d_j)\ (b_1d_1 \mid \lambda\mu)\ldots$$

In X the symbols $b'_3d'_3\ldots d'_j$ conjugate to those in g_2 may be collected in g_5. This yields reducible terms together with terms with $(i+1)$ unaccented symbols within the factor g_4. In Y there are $(i+1)$ accented symbols in g_2. The currency i of P has thus been raised, so reducing the form.

Now let the variables p be constructed from those of the factors (1). The result is a possible set of factors

$$(ii'p),\quad (ijp),\quad (ij'p),\quad (i'j'p),\quad (irp),\quad (i'rp),\quad (rrp),$$

where $2n>2i>n+1$, $2n>2j>n+1$. By the Lemma the form reduces if, in (ijp), $i=j$. Also $(ii'p)$ is a concomitant, and (rrp) vanishes identically since r_x is a linear covariant. If (ijp) occurs in a form, i' must also occur: let us say

$$(ijp)\ (i'\lambda),$$

where $\lambda=kp$ or rp or $k'p$. Now, by a fundamental identity,

$$\begin{aligned}(ijp)\ (i'kp)&=(i'jp)\ (ikp)+(ii'p)\ (jkp)\\ &\equiv(i'jp)\ (ikp).\end{aligned}$$

Hence the symbols i, i' are virtually interchangeable. The problem of constructing irreducible forms is now isomorphic with that of binary quadratic types (Turnbull, 1927, p. 78). *Hence the only possible irreducible complexes are contained among*

$$(ii'p), \qquad (ijp)\,(i'rp)\,(j'rp), \qquad (ijp)\,(i'j'p), \qquad (ij'p)\,(jk'p)\,(ki'p), \quad . \qquad (2)$$

where i, j, k all differ. This set (2), together with (iii), (iv), (v) above, constitute the system of line complexes. For the general value of n the forms (2) are not all irreducible, but the detailed treatment is very lengthy.

To illustrate the result reached; in the original notation the third form of the set (2) is

$$(ijp)\,(i'j'p) = (A_iB'_k\,.\,C_jD'_l\,.\,p)\,(A'_iB_k\,.\,C'_jD_l\,.\,p),$$

where $i+k=n+1=j+l$. In terms of the original inner and outer products the first factor is

$$(A_iB'_k\,.\,C_jD'_l\,.\,p) = \sum (A_iB'_{k-1})\,(C_jD'_{l-1})\,(b'_xd'_y - b'_yd'_x)$$

summed for the k-termed derangement of B'_k and the l-termed of D'_l. Equivalent to this is the alternative form $(ij'p)\,(i'jp)$.

If n is even the forms of the type involving r in (2) are non-existent.

EXAMPLE.—*To find the complexes of the quaternary correlation.*—Here $n=4$, $8 > 2i > 5$. Hence $i=3$ only, in type (i) above. The set (2) therefore provides the single form

$$(33'p) = (A_3B'_2\,.\,A'_3B_2\,.\,p) \quad . \quad . \quad . \quad . \qquad (3)$$

which is equivalent to

$$(A_3b')\,(A'_3c)\,(bc'p). \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

Here (3) is a sum of four terms such as (4) obtained by deranging b', b and c, c' independently, where $B=bc$. Of types (ii), (iii), (iv), (v) only the two latter supply one each. The system has three forms:—

$$\Omega_1 = \tfrac{1}{2}(aa'p), \qquad \Omega_2 = \tfrac{1}{4}(A_2p)\,(A'_2p), \qquad \Omega_3 = \tfrac{1}{2}(33'p).$$

With these numerical coefficients inserted the notation agrees with the Ω_1, Ω_2, Ω_3 of Weitzenböck (**2**, p. 135). The fourth form Ω_4 of the work just quoted is the same as

$$\tfrac{1}{4}(a'A_3)\,(bA'_3)\,(b'cp)\,(c'B_3)\,(dB'_3)\,(d'ap)$$

which reduces at once when the symbol b is transferred by a Sylvester identity from the second to the fifth factor.

REFERENCES TO LITERATURE.

Clebsch, Lindemann, 1891. *Vorlesungen über Geometrie* (Leipzig), Bd. ii (1), p. 343.

Turnbull, H. W., 1927. "The Invariant Theory of the Quaternary Quadratic Complex," *Proc. Roy. Soc. Edin.*, vol. xlviii, pp. 70–91.

Turnbull, H. W., 1929. *Theory of Determinants, Matrices and Invariants*, Glasgow.

Turnbull, H. W., 1930. "The Complete System of two Quadratic Forms in n Variables," *Proc. London Math. Soc.* (2), vol. xxx, pp. 469–480.

Turnbull, H. W., 1932. "The Invariant Theory of a General Bilinear Form," *ibid.*, vol. xxxiii, pp. 1–21.

Turnbull, H. W., and **Williamson, J.**, 1929. "Further Invariant Theory of two Quadratics in n Variables," *Proc. Roy. Soc. Edin.*, vol. l, pp. 8–25.

Weitzenböck, R., 1910. "Das Formensystem der Korrelation im R_3," *Rend. di Palermo*, vol. xxx, pp. 122–135.

(*Issued separately February* 25, 1935.)

IV.—On Least Squares and Linear Combination of Observations.

By **A. C. Aitken,** M.A., D.Sc., Mathematical Institute, University of Edinburgh.

(MS. received November 28, 1934. Read March 4, 1935.)

1. INTRODUCTORY.

IN a series of papers W. F. Sheppard (1912, 1914) has considered the approximate representation of equidistant, equally weighted, and uncorrelated observations under the following assumptions:—

(i) The data being $u_1, u_2, \ldots, u_n$, the representation is to be given by linear combinations

$$y_i = c_{1i}u_1 + c_{2i}u_2 + \ldots + c_{ni}u_n, \qquad i = 1, 2, \ldots, n.$$

(ii) The linear combinations are to be such as would reproduce any set of values that were already values of a polynomial of degree not higher than the kth.

(iii) The sum of squared coefficients $\sum_j c_{ji}^2$, which measures the mean square error of y_i, is to be a minimum for each value of i.

It was found by Sheppard (see also Lidstone, 1933) that the values y_i were identical with those which would have been obtained by directly fitting a polynomial of degree k to the u_i by the method of Least Squares.

It is proposed in the present paper to treat the problem from a more general standpoint, extending it in the following respects :—

(*a*) The data need not be equidistant.

(*b*) They may be weighted and even correlated. (This case was also considered by Sheppard.)

(*c*) The representation need not be restricted to one based on a set of values of a polynomial, but may be in terms of periodic exponential, or indeed any arbitrary set of sufficiently defined functional values, provided only that they are linearly independent over the n values of x.

In order to attain full generality with adequate compactness we shall employ the notation of matrices and vectors, using A' and x' to denote the matrix, or vector, obtained by transposing A, or x, that is, substituting rows for columns, columns for rows.

2. DERIVATION BY LEAST SQUARES.

The approach from the standpoint of Least Squares will first be considered, some classical results being obtained by way of illustration.

Let the approximate representation of the vector of data

$$u=\{u(x_1)\,u(x_2)\;\ldots\;u(x_n)\}$$

by the vector

$$y=\{y(x_1)\,y(x_2)\;\ldots\;y(x_n)\}$$

be linearly in terms of a set of prescribed functions

$$1,\; p_1(x),\; p_2(x),\; \ldots,\; p_k(x),$$

restricted only by the condition that they must be linearly independent over the n values

$$x=x_1,\; x_2,\; \ldots,\; x_n.$$

Let P denote the matrix of these functional values, the ith row of P being therefore the row vector

$$p_i'=[1\;\; p_1(x_i)\;\; p_2(x_i)\;\ldots\; p_k(x_i)].$$

Then P is of order $n\times(k+1)$, and of rank $k+1$. Let a denote a column vector of $k+1$ coefficients independent of x, namely,

$$a=\{a_0\,a_1\,a_2\;\ldots\;a_k\}.$$

Then the set of values y_i is simply the vector $y=Pa$.

If the elements of u, namely the data, are uncorrelated and of equal weight, the principle of Least Squares dictates that the sum of squared residuals shall be a minimum. Since this is the vector product

$$s^2=(u-Pa)'(u-Pa),$$

the minimal conditions

$$\tfrac{1}{2}\partial s^2/\partial a=0$$

give at once

$$P'Pa=P'u,$$

and these are the normal equations for the coefficients a_i of the representation.

Again, since

$$a=(P'P)^{-1}P'u,$$

the representation of u by $y=Pa$ is a certain linear transformation of the u_i, namely,

$$y=P(P'P)^{-1}P'u=C'u$$

let us say.

Certain properties of the transformation are evident at once. For example, it is symmetric—that is, $C'=C$; for

$$[P(P'P)^{-1}P']'=P(P'P)^{-1}P'.$$

It is also (Lidstone, p. 158) idempotent—that is, $C^2=C$; for

$$\begin{aligned} C^2 &= P(P'P)^{-1}P'P(P'P)^{-1}P' \\ &= P(P'P)^{-1}P' = C. \end{aligned}$$

It follows that C', or C, satisfies the "reduced characteristic equation" ("minimal equation") $\lambda^2-\lambda=0$, and so its latent roots are without exception units and zeros in certain multiplicities.

These multiplicities are easily determined. In fact C' is of rank $k+1$, for it transforms any arbitrary set u into a set of the special type $y=Pa$, depending upon $k+1$ constants a_i. Hence all the y_i for $i>k+1$ —that is, those obtained by the operation on u of rows of C' later than the $(k+1)$th—are linearly dependent on the first $k+1$ elements of y. Hence, u being arbitrary, these rows are themselves linearly dependent on the first $k+1$ rows of C', and so C' is of rank $k+1$. (It cannot be of lower rank, otherwise it would transform arbitrary data u_i into values y_i depending on fewer than $k+1$ constants.) Hence the trace (sum of diagonal elements) of C' is $k+1$, the sum of the latent roots.

A simpler proof of the above result is derivable from the theorem in matrices that the rank of a product of matrices cannot exceed the rank of any of the factors.

Again, the vector of residuals is $u-y$, obtained from u by the transformation $I-C'$, where I is the unit matrix. This again is symmetric and idempotent, for

$$\begin{aligned} (I-C')^2 &= (I-C)^2 \\ &= I^2-2C+C^2 \\ &= I-C', \quad \text{since} \quad C^2=C, \qquad C'=C. \end{aligned}$$

The latent roots of $I-C'$ are therefore again units and zeros; it is evidently of rank $n-k-1$, and so its trace is $n-k-1$.

Now when a set of data u with equal variance of error (which we may take as unity) is transformed according to $y=C'u$, the variances of error of the y_i are the diagonal elements of the product matrix $C'C$, the product variances being the non-diagonal elements. Hence the aggregate error variance of the residuals in the present case is the trace of $(I-C)'(I-C)$—that is, of $I-C'$, namely, $n-k-1$. We thus have a simple proof of a classical result.

3. EXTENSION TO CORRELATED CASE.

The argument may be extended to the case of data having correlated errors.

Let u be a set of data with variances and product variances of error arrayed by the elements of a symmetric matrix V. In such a case we have to make

$$(u - Pa)' V^{-1}(u - Pa)$$

a minimum. Differentiating with respect to a in the manner of § 2, we obtain the more general normal equations

$$P' V^{-1} Pa = P' V^{-1} u,$$

which show that the representation is a linear transformation

$$y = P(P' V^{-1} P)^{-1} P' V^{-1} u.$$

The matrix of transformation here is not symmetric, but is easily shown to be of rank $k + 1$.

4. STANDPOINT OF LINEAR COMBINATION.

Having established these results from the point of view of Least Squares, let us begin *ab initio* from Sheppard's assumptions. The matrix C' which emerges will prove identical, in the sequel, with the C' of §§ 2 and 3 above, but in the meantime we regard it as determined solely by Sheppard's assumptions.

Our demonstration is based on a simple but very useful Lemma, which has a field of application (Henderson, 1932) much wider than the present context.

LEMMA.—*If* C *is a rectangular matrix with more rows than columns, and if the trace of* C'C *is a minimum, subject to the condition* C'A = P, *where* A *and* P *are prescribed matrices, then*

$$C' = P(A'A)^{-1}A'.$$

Proof.—The ith element in the diagonal of $C'C$ is $c_i'c_i$, where c_i' is the ith row of C'. In view of the way in which the elements of c_i' occur in the condition $C'A - P = 0$, we introduce undetermined multipliers λ_{ij}, and consider the expression

$$\begin{aligned}\phi &= \sum_i \{c_i'c_i - 2(c_i'a_1 - p_{1i})\lambda_{1i} - \ . \ . \ . \ - 2(c_i'a_k - p_{ki})\lambda_{ki}\} \\ &= \sum_i (c_i'c_i - 2c_i'A\lambda_i + 2p_i'\lambda_i),\end{aligned}$$

where a_j, p_j, and λ_j denote the jth columns of the respective matrices A, P, and $\Lambda = [\lambda_{ij}]$.

The minimal conditions

$$\tfrac{1}{2}\partial\phi/\partial c_i = 0$$

give vector conditions

$$c_i = A\lambda_i, \qquad i = 1, 2, \ . \ . \ ., n;$$

in brief, the matrix condition

$$C = A\Lambda \quad \text{or} \quad C' = \Lambda' A'.$$

Hence, since $C'A = P$, we have

$$\Lambda' A' A = P, \qquad \Lambda' = P(A'A)^{-1},$$

and so

$$C' = P(A'A)^{-1}A',$$

which establishes the Lemma.

We may remark that the same result is arrived at whether we postulate that the trace of $C'C$ shall be a minimum, or that the diagonal elements shall individually (Sheppard, 1912, p. 369) assume their minimum values

Now in the application to Sheppard's problem the diagonal elements of $C'C$ are to have minimum values subject to the condition that $C'y$ reproduces y when y is already of the form Pa. Accordingly

$$C'Pa = Pa$$

for arbitrary vectors a, and so $C'P = P$. Applying the Lemma, we see that C' is given by

$$C' = P(P'P)^{-1}P',$$

and this (§ 2) is the same matrix as we obtained by the method of Least Squares.

5. LINEAR COMBINATION OF CORRELATED DATA.

Sheppard's explicit purpose was to obtain that particular linear combination which, under the "fidelity condition," should have the least mean square of error. The procedure can therefore be extended with ease to the case of weighted and correlated data.

Let the data u be characterised by variances and product variances of error, arrayed as elements of a symmetric matrix V.

The matrix of error variance of $y = C'u$ is then $C'VC$. Hence the natural extension of Sheppard's problem will be occupied with finding the minimum of the trace of $C'VC$, subject to the condition $C'P = P$.

In this connexion there is a Lemma so similar in statement and justification to that of § 2 that we content ourselves by merely enunciating it.

LEMMA.—*If* C *is a rectangular matrix with more rows than columns, and if the trace of* C'VC *is a minimum, subject to the condition* C'A = P, *then*

$$C' = P'(A'V^{-1}A)^{-1}A'V^{-1}.$$

Applying this Lemma to the case before us, we deduce at once that the desired linear transformation C' is given by

$$C' = P(P'V^{-1}P)^{-1}P'V^{-1},$$

and this again (§ 3) is the same matrix as we obtained by the method of Least Squares, extended to cover the case of correlated data.

6. CONCLUDING REMARKS.

In conclusion, a few casual observations touching the uncorrelated case may be of interest.

(i) If the set of functional values $p_j(x)$ forms an orthogonal set over the range of x, the matrix $P'P$ in the normal equations

$$P'Pa = P'u$$

is purely diagonal. For example, the matrix $T'T$ constructed from appropriate Tchebychef polynomials is diagonal, so that the a's are found at once.

When the set is not only orthogonal but also normalised, we have

$$P'P = I, \qquad a = P'u, \qquad C' = P'P.$$

(ii) If the values of x are spaced symmetrically about a central value (which may or may not appear in the given values, according as n is odd or even), the matrix C' is not merely axisymmetric but also *centrosymmetric*. The reason is that in such a case the data can be reversed by reflection about the central value. The vector y given by $y = C'u$ ought therefore to be in agreement with the vector obtained by reversing the elements of u, applying C', and then reversing the elements of the result. Since this must hold for *arbitrary* vectors u, we see that C' itself must be unchanged by a complete reversal of the order of its rows. Hence, being already symmetric, it must in addition be centrosymmetric.

(iii) A point of interest about the Lemmas of §§ 4, 5 is that, under the conditions stated, not merely the trace (sum of diagonal elements) of $C'C$, or of $C'VC$, but the sum of the principal minors of *every* order r, assumes a minimal value. This is an immediate consequence of the theorem that the product of the rth compounds of two or more matrices is identical with the rth compound of the product of those matrices. In virtue of this we may replace every matrix used in the course of the demonstrations of §§ 4, 5 by its rth compound, arriving finally at the result that the trace of the rth compound of $C'C$, or of $C'VC$, is a minimum. But the trace of the rth compound of a matrix is the sum of the rth principal minors of the matrix itself; and so the stated result follows.

[REFERENCES.

REFERENCES TO LITERATURE.

HENDERSON, R., 1932. "A Postulate for Observations," *Ann. Math. Statistics*, vol. iii, pp. 32–37.

LIDSTONE, G. J., 1933. "Notes on Orthogonal Polynomials," *Journ. Inst. Act.*, vol. lxiv, pp. 153–159.

SHEPPARD, W. F., 1912. "Reduction of Errors by Negligible Differences," *Proc. Fifth Internat. Congr. Math.* (Cambridge), vol. ii, pp. 348–384.

SHEPPARD, W. F., 1914. "Fitting of Polynomials by Method of Least Squares," *Proc. Lond. Math. Soc.* (2), vol. xiii, pp. 97–108.

SHEPPARD, W. F., 1914. "Graduation by Reduction of Mean Square of Error," *Journ. Inst. Act.*, vol. xlviii, pp. 171–185, 390–412; vol. xlix, pp. 148–157.

(*Issued separately March* 6, 1935.)

V.—The Mathematical Representation of the Energy Levels of the Secondary Spectrum of Hydrogen.—II.

By **Ian Sandeman,** Ph.D., Late Carnegie Research Scholar in the University of St Andrews.

(MS. received October 29, 1934. Read January 7, 1935.)

IN a recent paper (Sandeman, 1933) the question of the mathematical representation of the energy levels of the hydrogen molecule by methods based on the old mechanics has been considered. In view of the fact that the wave mechanics has supplanted the older theory, the further application of newer methods to this spectrum is desirable. Richardson and Davidson (1929 *b*, p. 45) worked out the potential energy function $U(\xi)$ (ξ is defined on p. 51) for the state $1s\sigma 3p\pi\,^3\Pi$ on the old quantum mechanics and on the wave mechanics with the following results: $U(\xi)=5{\cdot}50\xi^2(1-1{\cdot}556\xi+1{\cdot}58\xi^2-1{\cdot}56\xi^3)$ volts on the old quantum mechanics and $U(\xi)=5{\cdot}71\xi^2(1-1{\cdot}55\xi+1{\cdot}58\xi^2-1{\cdot}59\xi^3)$ volts on the wave mechanics. They also (*loc. cit.*, pp. 30 and 31) gave the $U(\xi)$'s for the states $1s\sigma 2p\sigma\,^1\Sigma$, $1s\sigma 2s\sigma\,^3\Sigma$, and $1s\sigma 3d\pi\,^1\Pi_b$, and on p. 45 they gave the formulæ, based on Fues's (1926) results, for transcribing the old quantum mechanics into the wave mechanics series. Richardson and Das (1929, pt. ii) gave similar data for the state $1s\sigma 3p\sigma\,^3\Sigma$.

The problem of the rotating vibrator in the wave mechanics has received the attention of a number of workers, and a solution, in a form convenient for spectral analysis and which should be particularly accurate for light molecules like H_2, has recently been given by J. L. Dunham (1932), whose early death is a loss to theoretical spectroscopy. Dunham's work not only shows the difficulties that had to be overcome in the mathematical theory, but also indicates that the solution obtainable is an asymptotic solution which may be expected to hold closely for the heavier elements. The tentative application of this solution to the hydrogen molecule with its small moment of inertia is therefore a matter of some interest.

Validity of Applying Dunham's Solution of the Wave Mechanics Equation to the Hydrogen Molecule.—The equation of Schrödinger, for a linear oscillator of mass *m*, can be written:

$$d^2\psi/dx^2+(8\pi^2 m/h^2)(E-V)\psi=0 \quad . \quad . \quad . \quad (1)$$

To solve this equation by the method of Wentzel (1926), Brillouin (1926), and Kramers (1926), followed by Dunham, we put

$$\psi = e^{(2\pi i/h)\int y\,dx}. \qquad (2)$$

y is then determined by the relation:

$$(h/2\pi i)(dy/dx) = 2m(\mathrm{E} - \mathrm{V}) - y^2. \qquad (3)$$

The energy level condition is:

$$\oint y\,dx = nh, \qquad (4)$$

where n is an integer. This result was first given by Wentzel, although Dunham shows that his derivation of it was not quite satisfactory. To use this condition we expand y in powers of $h/2\pi i$:

$$y = y_0 + (h/2\pi i)y_1 + (h/2\pi i)^2 y_2 + \dots \qquad (5)$$

Substitution in equation (3) and equating of the coefficients of the various powers of h to zero give a set of equations from which the y_n's are determined. Dunham evaluates a few of these quantities, and shows that equation (5) has singular points where $\mathrm{E} = \mathrm{V}$, in the neighbourhood of which it does not behave even approximately like a solution of equation (3).

The conditions under which equation (5) gives a good approximation to y, and hence to ψ, are considered in Dunham's first paper. Such conditions are fulfilled under the following circumstances:—

(1) When the value of $\mathrm{E} - \mathrm{V}$ is large.
(2) When the potential curve approaches some finite value asymptotically, so that its derivatives vanish.
(3) When the quantity $h^2/8\pi^2 m$ is small.
(4) When x is large.

The conditions in the hydrogen molecule, which has an exceptionally large value of $h^2/8\pi^2 m$ and for which the potential curves are known to be steep, are not therefore *a priori* favourable for an application of the method. The essential uncertainty introduced by the use of an asymptotic solution must not be lost sight of.

On applying the energy level condition to equation (5) Dunham obtains the general phase integral in the convenient form:

$$\oint(\mathrm{E} - \mathrm{V})^{\frac{1}{2}}dx - (\mathrm{B}/32)\oint \mathrm{V}'^2(\mathrm{E} - \mathrm{V})^{-\frac{5}{2}}dx + \dots = (n + \tfrac{1}{2})2\pi \mathrm{B}^{\frac{1}{2}}, \qquad (6)$$

where $\mathrm{B} = h^2/8\pi^2 m$. When this result is applied to the molecule, it can be shown from spectral theory considerations that the terms which have been omitted cannot have an appreciable effect. It should be noted that the uncertainty inherent in the use of this result does not arise from the omission of these terms. The validity of the result of this integration depends on whether equation (5) is a sufficiently accurate solution of equation (3) during the whole path of the integration. As Dunham remarks: "The way in which the quality of the approximation depends on the presence of quantum effects is well illustrated by the case of a potential function with a sharp kink in it. This kink will give rise to characteristic quantum diffraction effects which would not be present if the kink were smoothed over. Such a kink often arises from the presence of a singularity in the potential function near the real axis. If, in carrying the integral . . . around Q and R (the points on the real axis for which E = V. As potential functions with a single minimum are presumed, we shall have two such points in dealing with a discrete spectrum), we are forced to go near such a singularity, there will be a considerable part of the path for which equation (5) does not give a good approximation, and so the energy level formula will be relatively inaccurate."

Dunham's Treatment of the Molecular Rotating Vibrator.—Equation (6) takes the general solution of the problem as far as it can be taken without a specific knowledge of the form of the potential function V. For the molecule in his second paper Dunham uses Kratzer's co-ordinate $\xi = (r - r_e)/r_e$, where r_e is the equilibrium nuclear separation, and expands V, or its equivalent in wave-number units U, in a power series round the point $\xi = 0$:

$$U = V/hc = a_0\xi^2(1 + a_1\xi + a_2\xi^2 + a_3\xi^3 + \dots), \qquad (7)$$

where $a_0 = \omega_e^2/4B_e$, ω_e being the classical frequency of a small oscillation and B_e being equal to $h/8\pi^2 r_e^2 m$. $a_1, a_2, \dots$ are constants depending on the molecular configuration.

Schrödinger's equation applied to the molecule becomes

$$d^2\psi/d\xi^2 + (8\pi^2 r_e^2 m/h^2)\left(E - V - \frac{h^2K(K+1)}{8\pi^2 r_e^2 m(1+\xi)^2}\right)\psi = 0. \qquad (8)$$

Dunham puts this equation in a form suitable for the application of the energy level condition by an ingenious device. He takes the two terms, V and $\frac{h^2K(K+1)}{8\pi^2 r_e^2 m(1+\xi)^2}$, together, and expands the united expression in a power series similar to (7) in a new variable such that the power series has a minimum value for zero value of the new variable. This process is made possible because of the large value of the constant a_0 in comparison with $h/8\pi^2 r_e^2 m$, on account of which an approximation to the desired degree of accuracy can be obtained. (The condition that the series has a minimum for zero value of the variable causes the linear term in the new series to vanish.) Approximations are required in the process of integration and again in the final step when the result of the phase integration is inverted in order to obtain the energy E (or its equivalent in wave-number units F) as a power series in the integration constant—the quantum number $v+\frac{1}{2}$. The type of approximation is, however, familiar from the treatment of the similar problem on the old mechanics first carried out by Kratzer.

In the final energy equation the ratio of the coefficients of successive powers of $v+\frac{1}{2}$ is of the order of B_e/ω_e—in general a very small quantity. It is accordingly generally possible to calculate the first few coefficients numerically from the experimental data with an accuracy comparable with the accuracy of the data. For hydrogen, however, this is scarcely the case. The ratio B_e/ω_e assumes a relatively large value, about 0·014. The accuracy attainable in analysis will therefore not be quite comparable with the accuracy of the data.

Dunham writes the energy level expression in the following form:—

$$F_{vK} = \sum_{lj} Y_{lj}(v+\tfrac{1}{2})^l K^j (K+1)^j, \quad . \quad . \quad . \quad . \quad (9)$$

and has carried his approximations far enough to evaluate all the Y_{lj} in terms of the molecular constants, ω_e, B_e, a_1, a_2, a_3, a_4, a_5, and a_6, taking into account all the Y_{lj} for which the sum $l+j$ does not exceed 4.

Just as Richardson and Davidson found in their first paper on the "Bands Analogous to the Parhelium Line Spectrum" (1929 *a*), a fourth-power formula in the vibration quantum number is not capable of representing the observational data accurately. The following appears to be the best mode of procedure: we may calculate experimental values of the Y_{lj}, taking into account as many as appear to be necessary. These experimental values will be nearly accurate for low values of l and j. We can then proceed to compare these low values with the theoretical values of the first few Y_{lj} given by Dunham, and to deduce the molecular constants.

From what has been said it will be seen that it is not possible to calculate the constants of the hydrogen molecule with high numerical accuracy. The writer believes that his results give a sufficiently consistent and interconnected scheme to indicate that, at least for the $1s2s\,^3\Sigma$ and $1s2p\,^1\Sigma$ states, which are the final states of the strongest lines in the visible spectrum, the application of the energy level condition of Wentzel is justified. It is believed that the analysis of the two states here given carries the problem substantially as far as the accuracy of the experimental data allows, and provides a basis for the next advance which will be possible when spectral measurements have been improved in accuracy and scope beyond the already high standard attained by those of Gale, Monk, and Lee (1928).

An Analysis of the $1s2s\,{}^3\Sigma$ *State of Hydrogen.*—A study of this state (final state of the Fulcher Bands) made it clear that, owing to slight irregularities in the experimental data, it was useless to attempt to find accurate values of the small coefficients Y_{03}, Y_{13}, and Y_{04} from the data. The following procedure was accordingly adopted: a rough analysis was first carried out from which approximate values of ω_e, B_e, a_1, a_2, and a_3 were deduced. Y_{03}, Y_{13}, and Y_{04} were calculated from these approximate values by means of Dunham's relations.

Y_{02}, Y_{12}, and Y_{22} were next found from the experimental rotational term differences, and Y_{01}, Y_{11}, Y_{21}, Y_{31}, and Y_{41} from both rotational and vibrational data. (Only the rotational data give Y_{01}. The whole set of Y_{l1}'s was calculated to give the best fit for both data.) Finally, Y_{10}, Y_{20}, Y_{30}, Y_{40}, Y_{50}, and Y_{60} were found from the vibrational term differences. The method of least squares was used in each determination.

The values found for the various Y_{lj} are shown in Table I. To preserve consistency in the results these quantities are given to seven figures, just as they were determined in the analysis, although not all the figures are significant for theoretical application. From Table I it appears that the Y_{lj} form an alternately positive and negative series both horizontally and vertically.

TABLE I.—CALCULATED VALUES OF THE Y_{lj} ($1s2s\,{}^3\Sigma$ STATE).

	$j=0.$	1.	2.	3.	4.
$l=0$	..	34·22051	−0·02250640	0·000019931	−0·00000002153
1	2665·9252	−1·694884	0·0008207384	−0·00000014357	
2	−72·79390	0·03454932	−0·000004053144		
3	1·429492	−0·002034446			
4	−0·1473890	0·00000733776			
5	0·01246012				
6	−0·0005853230				

Table II shows the observed and calculated rotational term differences. In this table the observed values are taken as the means of the term differences yielded by the red and green Fulcher Bands (Richardson and Das, 1929, pt. i, and Sandeman, 1929). Residuals are shown in italics. Only in two cases, which are starred, does the residual come out greater than would be expected. In both cases the term difference is derived from only one band, and one of the two band lines concerned in each case may be affected by interference with a line of another system.

TABLE II.—ROTATIONAL TERM DIFFERENCES ($1s2s\,^3\Sigma$ STATE).

ΔK.		$v=0$.	1.	2.	3.	4.
2–0	Observed	199·51	189·75	180·26	171·05	162·0
	Calculated	199·50	189·73	180·27	171·05	162·0
	Residual	*0·01*	*0·02*	*−0·01*	*0·00*	*0·0*
3–1		330·76	314·54	298·83	283·57	268·5
		330·75	314·54	298·84	283·53	268·5
		0·01	*0·00*	*−0·01*	*0·04*	*0·0*
4–2		459·44	436·91	415·05	393·73	372·5*
		459·45	436·89	415·04	393·74	372·8
		−0·01	*0·02*	*0·01*	*−0·01*	*−0·3*
5–3		584·64	555·85	528·02	500·9	
		584·65	555·87	528·01	500·9	
		−0·01	*−0·02*	*0·01*	*0·0*	
6–4		705·57	670·91*	636·93		
		705·50	670·69	636·99		
		0·07	*0·22*	*−0·06*		
7–5		821·24	780·62	741·29		
		821·27	780·62	741·28		
		−0·03	*0·00*	*0·01*		

Table III similarly shows the fit for the vibrational term differences. The experimental values here given are those of Richardson and Das (1929, pt. i). Four values for which the residuals are greater than would be expected are starred. The single value in the lowest horizontal row is inserted for the sake of completeness. As this term difference is a little doubtful, it was not used in making the calculation. As the successive Y_{ij} do not decrease in magnitude sufficiently rapidly to compensate for the increase in successive powers of $v+\frac{1}{2}$, the formula cannot be expected to extrapolate very satisfactorily. By the addition of further Y_{ij} the formula could be made to extrapolate in any desired manner.

In Table III the term differences corresponding to the same change in the quantum number v are shown in the same horizontal row. The second and third horizontal rows appear in the mean to show deviations of $+0{\cdot}02$ cm.$^{-1}$ and $-0{\cdot}02$ cm.$^{-1}$ respectively from the calculated values, and the fifth and sixth rows similar deviations of $+0{\cdot}06$ cm.$^{-1}$ and $-0{\cdot}06$ cm.$^{-1}$ This seems to indicate a perturbation of $+0{\cdot}02$ cm.$^{-1}$ in the vibrational level for which $v=2$, and a perturbation of $+0{\cdot}06$ cm.$^{-1}$ in the level for which $v=5$. It is, however, also possible that the deviations are to be attributed to the fact that insufficient Y_{ij} have been used in the calculation.

TABLE III.—VIBRATIONAL TERM DIFFERENCES (1s2s$^3\Sigma$ STATE).

Δv.		K = 0.	1.	2.	3.	4.	5.
1–0	Observed	2524·32	2521·07	2514·55	2504·85	2492·00	2476·13
	Calculated	2524·33	2521·07	2514·57	2504·86	2492·01	2476·09
	Residual	*−0·01*	*0·00*	*−0·02*	*−0·01*	*−0·01*	*0·04*
2–1		2388·26	2385·12	2378·80	2369·41	2357·06*	2341·63*
		2388·24	2385·08	2378·78	2369·38	2356·93	2341·52
		0·02	*0·04*	*0·02*	*0·03*	*0·13*	*0·11*
3–2		2256·10	2253·05	2246·91	2237·77	2225·60	2210·39*
		2256·15	2253·07	2246·93	2237·76	2225·62	2210·60
		−0·05	*−0·02*	*−0·02*	*0·01*	*−0·02*	*−0·21*
4–3		2126·86	2123·85	2117·78	2108·83	2096·57*	
		2126·89	2123·86	2117·83	2108·83	2096·91	
		−0·03	*−0·01*	*−0·05*	*0·00*	*−0·34*	
5–4			1996·57	1990·58	1981·75		
			1996·51	1990·54	1981·64		
			0·06	*0·04*	*0·11*		
6–5			1869·80	1863·89	1854·96		
			1869·85	1863·90	1855·02		
			−0·05	*−0·01*	*−0·06*		
7–6			1742·15				
			1742·14				
			0·01				
8–7			1612·5				
			1609·6				
			2·9				

If we assume only four Y_{i0}'s, as contemplated by Dunham in his analysis, we obtain by an application of the method of least squares the following values for these Y_{i0}'s:—

$$Y_{10} = 2665{\cdot}4861,$$
$$Y_{20} = -72{\cdot}18137,$$
$$Y_{30} = 1{\cdot}068784,$$
$$Y_{40} = -0{\cdot}04863957.$$

Consecutive rows would then in the mean show the following deviations from the calculated values:—

1st Row	−0·03,
2nd ,, ...	+0·07,
3rd ,, ...	+0·01,
4th ,, ...	−0·09,
5th ,, ...	+0·04,
6th ,, ...	+0·11,
7th ,, ...	−0·10,
8th ,, ...	−3·6.

Here, as before, the single term difference in the eighth row has not been used in carrying out the calculation.

It seems clear that four Y_{l0}'s are not sufficient to represent the data accurately. On the other hand, as we add more and more Y_{l0}'s, these merely contort themselves to fit small inaccuracies in the experimental data. The difficulty is inherent in any application of an approximate solution of the phase integral to hydrogen. The best plan appears to be to take the values of the Y_{l0} up to Y_{60}, which gave a fairly good fit, and to use the earlier members to gain information regarding the structural constants.

With the help of the first three Y_{l0}'s and first three Y_{l1}'s, using Dunham's relations and neglecting his correction terms, we find:

$$\begin{aligned} \omega_e &= 2665{\cdot}9252, \\ B_e &= 34{\cdot}22051, \\ a_1 &= -1{\cdot}6438, \\ a_2 &= 1{\cdot}9588, \\ a_3 &= -2{\cdot}0945, \\ a_4 &= 2{\cdot}2775. \end{aligned}$$

We should therefore expect the potential expression (7) to approximate to the following series:—

$$U = a_0\xi^2(1 - 1{\cdot}6438\xi + 1{\cdot}9588\xi^2 - 2{\cdot}0945\xi^3 + 2{\cdot}2775\xi^4 - \ldots) \quad . \quad (10)$$

The coefficients of the next two powers of ξ, a_5, and a_6, are deducible from Y_{31} and Y_{40}. In view of the fact that the accuracy of these two Y's is a little doubtful, it seems best to omit them.

The equation for the $U(\xi)$ of $1s\sigma 2s\sigma\,{}^3\Sigma$ found by Richardson and Davidson (1929 *b*, equation (23), p. 31) on the old quantum mechanics was, in volts,

$$U(\xi) = 6{\cdot}24\xi^2(1 - 1{\cdot}637_5\xi + 1{\cdot}943\xi^2 - 2{\cdot}15\xi^3 + 2{\cdot}5\xi^4 \ldots) \quad . \quad (10.1)$$

As will be seen, the coefficients in (10.1) are not very different from those in (10).

We may compare expression (10) directly with the potential function of Kratzer (1922), which approximates to the form:

$$U = \text{const. } \xi^2(1 - 2\xi + 3\xi^2 - 4\xi^3 + 5\xi^4 - \ldots) \quad . \quad . \quad (11)$$

It is not surprising that Kratzer's potential energy expression has not been found to yield a very good representation of the experimental data.

In order to compare the experimental function (10) with that of Morse (1929), we throw expression (10) into the form:

$$U = a_0\xi^2(1 - k\xi + 0{\cdot}725k^2\xi^2 - 0{\cdot}472k^3\xi^3 + 0{\cdot}312k^4\xi^4 - \ldots), \quad . \quad (12)$$

where $k=1{\cdot}6438$. Morse's expression may be written $U=D(1-e^{-a\xi})^2$, and this, when expanded as a power series in ξ, becomes

$$U=\text{const. }\xi^2(1-a\xi+0{\cdot}583a^2\xi^2-0{\cdot}250a^3\xi^3+0{\cdot}086a^4\xi^4-\ldots)\quad . \qquad (13)$$

Morse's function, although providing a better fit than Kratzer's function, is not at all a good approximation—a result which could have been predicted from the fact that the coefficients Y_{30}, Y_{40}, . . ., which should vanish for a potential function of Morse's form, do not even approximate to zero.

If we proceed to take into account Dunham's second order terms and to correct B_e and ω_e by the method indicated by him, we find

$$\begin{aligned} Y_{01}&=B_e(1-0{\cdot}64889(B_e/\omega_e)^2),\\ Y_{10}&=\omega_e(1-37{\cdot}46391(B_e/\omega_e)^2), \end{aligned}$$

which give

$$\begin{aligned} B_e&=34{\cdot}22417,\\ \omega_e&=2682{\cdot}465. \end{aligned}$$

The correction in the case of ω_e is large, and indicates that further orders of correction terms beyond the range of Dunham's approximations, and probably beyond the range of experimental determination, would be necessary to determine ω_e accurately to the seventh figure. It seems also probable that the value found for a_1 itself requires correction. We cannot accordingly claim very high accuracy for the determinations of B_e and ω_e just given. On applying Dunham's correction term to the coefficient Y_{02} we obtain

$$Y_{02}=D_e(1-11{\cdot}41981(B_e/\omega_e)^2).$$

This gives $D_e=-0{\cdot}02255$ agreeing within the limits of obtainable accuracy with the value of $-4(B_e^3/\omega_e^2)$, viz. $-0{\cdot}02228$, obtained by using the corrected values of B_e and ω_e. As Dunham points out, the test for the presence of *l*-uncoupling is to see whether the observed and calculated values of Y_{02} agree. There is accordingly no evidence of *l*-uncoupling in this case.

An Analysis of the $1s2p\,^1\Sigma$ State of Hydrogen.—The results of an analysis of the $1s2p\,^1\Sigma$ state are shown in Tables IV, V, and VI. The

TABLE IV.—CALCULATED VALUES OF THE Y_{lj} ($1s2p\,^1\Sigma$ STATE).

	$j=0$.	1.	2.	3.
$l=0$	..	20·03318	−0·01766104	0·00002555
1	1359·516	−1·216331	0·003095295	−0·000003616
2	−22·54405	0·1483786	−0·0004584659	
3	1·594593	−0·02103160	0·00003230402	
4	−0·3019905	0·001801606		
5	0·03868993	−0·00006698531		
6	−0·002853472			
7	0·0000906746			

TABLE V.—ROTATIONAL TERM DIFFERENCES (1s2p $^1\Sigma$ STATE).

ΔK.		v=0.	1.	2.	3.	4.	5.	6.	7.
2–0	Obs.	116·15	110·38	105·50	101·17	97·21	93·58	90·09	(86·74)
	Calc.	116·18	110·38	105·49	101·17	97·22	93·54	90·06	86·73
	Resid.	*−0·03*	*0·00*	*0·01*	*0·00*	*−0·01*	*0·04*	*0·03*	*0·01*
3–1		192·37	182·88	174·86	167·73	161·27	155·14	149·46	
		192·37	182·88	174·86	167·75	161·24	155·17	149·47	
		0·00	*0·00*	*0·00*	*−0·02*	*0·03*	*−0·03*	*−0·01*	
4–2		266·72	253·85	242·85	233·03	224·09	215·76	207·87	
		266·71	253·80	242·83	233·05	224·09	215·74	207·92	
		0·01	*0·05*	*0·02*	*−0·02*	*0·00*	*0·02*	*−0·05*	
5–3		338·58	322·61	308·85	296·64	285·38	(274·78)	265·04	
		338·61	322·61	308·92	296·65	285·35	274·83	265·04	
		−0·03	*0·00*	*−0·07*	*−0·01*	*0·03*	*−0·05*	*0·00*	
6–4		(407·57)	(388·83)	(372·86)	358·12	344·67		(320·46)	
		407·55	388·88	372·75	358·16	344·66		320·48	
		0·02	*−0·05*	*0·11*	*−0·04*	*0·01*		*−0·02*	

Note.—In each case where only one experimental value is available that value has been placed in brackets.

TABLE VI.—VIBRATIONAL TERM DIFFERENCES (1s2p $^1\Sigma$ STATE).

Δv.		K=0.	1.	2.	3.	4.	5.	6.	7.
1–0	Obs.	1318·35	1316·41	1312·55	1306·94	1299·68	1290·99	1280·94	1269·84
	Calc.	1318·36	1316·41	1312·57	1306·93	1299·66	1290·94	1280·99	1270·04
	Resid.	*−0·01*	*0·00*	*−0·02*	*0·01*	*0·02*	*0·05*	*−0·05*	*−0·20*
2–1		1281·48	1279·84	1276·60	1271·82	1265·59	1258·04	1249·32	1239·81
		1281·48	1279·84	1276·59	1271·81	1265·61	1258·12	1249·48	1239·83
		0·00	*0·00*	*0·01*	*0·01*	*−0·02*	*−0·08*	*−0·16*	*−0·02*
3–2		1246·72	1245·25	1242·38	1238·13	1232·61	1225·89	1218·02	1209·88
		1246·71	1245·25	1242·38	1238·14	1232·61	1225·88	1218·03	1209·15
		0·01	*0·00*	*0·00*	*−0·01*	*0·00*	*0·01*	*−0·01*	*0·73*
4–3		1213·02	1211·76	1209·06	1205·17	1200·10	1193·86	1186·75	
		1213·01	1211·69	1209·06	1205·18	1200·10	1193·88	1186·59	
		0·01	*0·07*	*0·00*	*−0·01*	*0·00*	*−0·02*	*0·16*	
5–4		1179·95	1178·77	1176·33	1172·74	1168·03	1162·25	1155·5	
		1180·02	1178·78	1176·34	1172·72	1167·99	1162·20	1155·4	
		−0·07	*−0·01*	*−0·01*	*0·02*	*0·04*	*0·05*	*0·1*	
6–5		1147·50	1146·32	1144·00	1140·64	1136·15	1130·77	1124·8	
		1147·49	1146·33	1144·02	1140·62	1136·20	1130·51	1124·6	
		0·01	*−0·01*	*−0·02*	*0·02*	*−0·05*	*0·26*	*0·2*	
7–6		1115·5	1114·32	1112·04	1108·7	1104·3			
		1115·4	1114·29	1112·07	1108·8	1104·7			
		0·1	*0·03*	*−0·03*	*−0·1*	*−0·4*			
8–7			1081·29						
			1081·04						
			0·25						

data for this state are so numerous that it was possible to determine all the Y's experimentally. The rotational term differences given in Table V were obtained by taking averages of all the data from the bands ending on this state given by Richardson and Davidson (1929*a*). The vibrational term differences quoted in Table VI are those given by Richardson and Davidson in their first paper (1929*a*). All the Y's except the Y_{l0}'s were calculated from the rotational term differences. As will be seen, the calculated term differences give a very good fit. In Table VI some of the edge values show rather large deviations. As the edge values are, however, derived from faint and rather doubtful band lines, it is difficult to use them to correct the results. There remains some evidence of systematic deviation in Table VI. In view, however, of the fact that all the Y's except the Y_{l0}'s were found from the rotational term differences, the fit is surprisingly good.

For this state the potential expression corresponding to equation (10) comes out as

$$U = a_0\xi^2(1 - 1{\cdot}6867\xi + 2{\cdot}8061\xi^2 - 4{\cdot}1813\xi^3 + 5{\cdot}3836\xi^4 - \ldots) \quad . \quad (14)$$

Here we are dealing with successive a_n which increase much more rapidly than in the $2s^3\Sigma$ state. a_2 is very nearly equal to the square of a_1. This potential expression (14) is quite incompatible either with that of Kratzer or with that of Morse. It bears, however, a close resemblance to the $U(\xi)$ of $1s\sigma 2p\sigma {}^1\Sigma$ got by Richardson and Davidson (1929*b*, equation (21), p. 30) on the old quantum mechanics. They found, in volts,

$$U(\xi) = 2{\cdot}84\xi^2(1 - 1{\cdot}63\xi + 2{\cdot}63\xi^3 - 4{\cdot}1\xi^4 + 6\xi^6 \ldots) \quad . \quad . \quad (14.1)$$

As before, on taking into account Dunham's second order terms, we find

$$\begin{aligned} Y_{01} &= B_e(1 - 4{\cdot}12075(B_e/\omega_e)^2), \\ Y_{10} &= \omega_e(1 - 46{\cdot}02352(B_e/\omega_e)^2), \\ Y_{02} &= D_e(1 + 8{\cdot}19586(B_e/\omega_e)^2), \end{aligned}$$

which gives $B_e = 20{\cdot}05118$ and $\omega_e = 1373{\cdot}239$. For reasons which have been given before, we cannot claim very high accuracy for these values. The value of D_e comes out as $-0{\cdot}01763$ in sufficiently good agreement with the calculated value, $-0{\cdot}01710$, obtained by substituting the corrected values of B_e and ω_e in $-4(B_e^3/\omega_e^2)$. Here again there is very little evidence of *l*-uncoupling.

A curious point which arises from this investigation is the difference in type between the potential expressions of the two hydrogen states considered. Kratzer (1922) anticipated that the same or a very similar type of expression would hold for all diatomic molecules. This has been shown not to be the case for the two hydrogen states. Although the first constant in the potential expansion, a_1, has very nearly the same value, $-1{\cdot}66$, for both states, the second constant differs considerably. This could have been predicted from the fact that, apart from the difference in sign, for the $2p\,^1\Sigma$ state Y_{20} is nearly equal to Y_{01}, whereas for the $2s\,^3\Sigma$ state it is nearly twice as large as Y_{01}.

It would be very advantageous if we were able to substitute for the potential expansion a single mathematical function which would lend itself to integration and so dispense with the need for approximations, the cumulative effect of which make it impossible to obtain numerically accurate results from an application of the phase integral to hydrogen with its high value of B_e/ω_e. The potential expansions here given do not, however, look as if they could be summed to give a single convenient function. The most interesting results obtained from this study are contained in equations (10) and (14), as in the wave mechanics potential functions are of primary importance.

SUMMARY.

The validity of applying the results of the wave mechanics to the secondary spectrum of hydrogen and the practical difficulties involved in the analysis of the band states are discussed. Empirical representations of the two states, $1s2s\,^3\Sigma$ and $1s2p\,^1\Sigma$, of the form

$$F_{vK} = \sum_{ij} Y_{ij}(v+\tfrac{1}{2})^i K^j (K+1)^j$$

are given. As far as the nature of the experimental problem allows, values for the structural constants are calculated on the basis of J. L. Dunham's solution of the wave mechanics equation for the diatomic molecule. The consistency of the results indicates very little evidence of *l*-uncoupling for these states. Potential functions are obtained which indicate that the potential function of Morse is not applicable to the two states.

I am indebted to Professor H. Stanley Allen for helpful criticisms, and to Professor R. T. Birge for directing my attention to the work of J. L. Dunham.

REFERENCES TO LITERATURE.

BRILLOUIN, L., 1926. "La méchanique ondulatoire de Schrödinger; une méthode générale de résolution par approximations successives," *C.R. Acad. Sci. Paris*, T. clxxxiii, p. 24.

DUNHAM, J. L., 1932. "The Wentzel-Brillouin-Kramers Method of Solving the Wave Equation," *Phys. Rev.*, vol. xli, p. 713; "The Energy Levels of a Rotating Vibrator," *ibid.*, p. 721.

FUES, E., 1926. "Das Eigenschwingungsspektrum zweiatomiger Moleküle in der Undulationsmechanik," *Ann. Phys.*, Leipzig, Bd. lxxx, p. 367.

GALE, H. G., MONK, G. S., and LEE, K. O., 1928. "Wave-lengths in the Secondary Spectrum of Hydrogen," *Astrophys. Journ.*, vol. lxvii, p. 89.

KRAMERS, H. A., 1926. "Wellenmechanik und halbzahlige Quantisierung," *Zeits. Phys.*, Bd. xxxix, p. 828.

KRATZER, K., 1922. "Die Gesetzmässigkeit der Bandensysteme," *Ann. Phys.*, Leipzig, Bd. lxvii, p. 127.

MORSE, P. M., 1929. "Diatomic Molecules according to the Wave Mechanics. II. Vibrational Levels," *Phys. Rev.*, vol. xxxiv, p. 57.

RICHARDSON, O. W., and DAS, K., 1929. "The Spectrum of H_2: The Bands Analogous to the Ortho-helium Line Spectrum," pt. i, *Proc. Roy. Soc.*, A, vol. cxxii, p. 688; pt. ii, *ibid.*, vol. cxxv, p. 312.

RICHARDSON, O. W., and DAVIDSON, P. M., 1929*a*. "The Spectrum of H_2: The Bands Analogous to the Parhelium Line Spectrum," pt. i, *Proc. Roy. Soc.*, A, vol. cxxiii, p. 54; pt. ii, *ibid.*, vol. cxxiii, p. 466; pts. iii and iv, *ibid.*, vol. cxxiv, p. 50.

RICHARDSON, O. W., and DAVIDSON, P. M., 1929*b*. "The Energy Functions of the H_2 Molecules," *Proc. Roy. Soc.*, A, vol. cxxv, p. 23.

SANDEMAN, I., 1929. "The Fulcher Bands of Hydrogen," *Proc. Roy. Soc. Edin.*, vol. xlix, p. 48.

SANDEMAN, I., 1933. "The Mathematical Representation of the Energy Levels of the Secondary Spectrum of Hydrogen," *Proc. Roy. Soc. Edin.*, vol. liii, p. 347.

WENTZEL, G., 1926. "Eine Verallgemeinerung der Quantenbedingungen für die Zwecke der Wellenmechanik," *Zeits. Phys.*, Bd. xxxviii, p. 518.

(*Issued separately May* 30, 1935.)

VI.—The New Crystallography. By **W. L. Bragg,** F.R.S., Langworthy Professor of Physics, University of Manchester. (Bruce-Preller Lecture, delivered on February 4, 1935.) (With Four Figures.)

(MS. received February 23, 1935. Read February 4, 1935.)

1. INTRODUCTION.

THE mapping of atomic arrangement by X-ray analysis, first applied only to crystalline solids, but more recently extended to amorphous solids, liquids, and gases, has now been pursued for more than twenty years. In the present paper an attempt is made to review the position which has been reached and to forecast the probable lines of development in the future. Examples are chosen from inorganic chemistry, organic chemistry, mineralogy, metallurgy, and biochemistry, which are as far as possible representative of the problems to which X-ray analysis can be effectively applied. It is perhaps not generally realised how extensive these applications are and how much they are influencing other branches of science. A knowledge of the actual atomic arrangement introduces an important precision, simplicity, and novelty of outlook, and a plea might well be made for a wider use of the new knowledge.

May I just remind you that this X-ray investigation is a branch of physical optics. It is based on ordinary optical principles, but they acquire rather a new aspect because the wave-lengths employed are so short. Briefly, X-rays are scattered when they are allowed to fall on a substance, and it is possible, with ease in some cases and with greater difficulty in others, to draw conclusions about the arrangement of the atoms from the pattern of the scattered or "diffracted" rays recorded by a photographic plate or some other means.

2. INORGANIC CHEMISTRY.

As an example in this field we may take the structures of the heteropoly acids and their salts. The exact formulæ of these compounds have long been a subject of speculation. The acid radicle is complex, containing many oxygen atoms together with a number of tungsten or molybdenum atoms, and a single atom of P, Si, As, B, etc., which appears to be in a key position. Numerous suggestions as to the composition of the radicle have been made which differ widely from each other.

The difficulty arises as follows: The best-known examples of this class have twelve atoms of W or Mo associated with each P, Si, etc. The acids are very stable and crystallise in the cubic system from aqueous solution with much water of crystallisation, a great part of which is rapidly lost when the crystals are exposed to the air. It is difficult to tell whether a given specimen represents a single compound or a mixture of components, and to decide how much of the H_2O content is water of crystallisation and how much an essential part of the acid radicle. Further, the salts are isomorphous with the acids, and it is difficult to ascertain how many kations combine with each acid radicle.

An X-ray analysis of the twelve-acids has been made by Keggin (*Proc. Roy. Soc.*, A, vol. cxliv, 1934, p. 75), and of their salts by Santos and Illingworth (in press). The factors contributing to the success of these analyses are as follows:—

(*a*) It is possible to tell by X-rays whether the substance under examination is a pure specimen or is a mixture of constituents. In the former case the powder method of X-ray examination gives a sharply defined set of lines referable to a single crystal type, in the latter a jumble of lines, generally diffuse, representing several crystal types. Keggin found, for instance, that freshly crystallised phosphotungstic acid is a definite compound (later defined as $H_3PW_{12}O_{40} . 29H_2O$) and the same compound when dehydrated over P_2O_5 is also definite ($H_3PW_{12}O_{40} . 5H_2O$). Intermediate stages of dehydration all represent mixtures.

(*b*) The W or Mo atoms are so heavy that their positions in the cubic structure can be accurately fixed from the powder photographs.

(*c*) The W and Mo atoms being fixed, the positions of the oxygen atoms in the acid radicle are uniquely determined by considerations of packing. The acid radicle proves to be $(PW_{12}O_{40})^{3-}$, confirming one of the formulæ previously suggested on chemical grounds.

(*d*) The number of molecules of water of crystallisation can be counted. This is a very ingenious solution, which shows the possibilities of the new angle of attack. Chemical analysis proves the number in the higher hydrate to be approximately 28–30, and it is known that the arrangement of the water molecules must conform to the tetrahedral symmetry of their site in the crystal. This requires one H_2O at the centre of the site, four surrounding it tetrahedrally, and still further out groups of twelve. Clearly the exact number must be 29.

$$1 + 4 + 12 + 12 = 29.$$

The acid radicle $(PW_{12}O_{40})^{3-}$ is an interesting entity, a "two-decker" radicle. P lies at the centre, surrounded by 4O. This in turn is surrounded by 12W or 12Mo, and the outer layer consists of a further 36O. The 29 hydrate has its immense amount of water of crystallisation (232 molecules per unit cell) in the form of *ice of crystallisation*, stabilised at the high temperature by the surrounding acid radicles.

The heteropoly acids are used in organic analysis, as precipitating agents in biochemistry, and commercially to form stable lakes with fugitive dyes. The number so far examined represents a very small fraction of the possible types, but at any rate a nucleus of definitely determined compounds has been created which will form the basis of further examination (see Table I).

TABLE I.—FORMULÆ OF HETEROPOLY ACIDS AND THEIR SALTS.

	(*a*) As indicated by X-ray analysis.	(*b*) Suggested on chemical grounds.
	Acids.	
	$H_3PW_{12}O_{40} . 5H_2O$.	. . . .
	$H_4GeW_{12}O_{40} . 5H_2O$.	. . . .
	$H_3PW_{12}O_{40} . 29H_2O$.	$H_7[P(W_2O_7)_6] . 28H_2O$.
	$H_4SiW_{12}O_{40} . 29H_2O$.	$H_8[Si(W_2O_7)_6] . 28H_2O$.
	$H_5BW_{12}O_{40} . 29H_2O$.	$H_9[B(W_2O_7)_6] . 28H_2O$.
	$H_6H_2W_{12}O_{40} . 29H_2O$.	$H_2W_4O_{13} . 9H_2O$.
	$H_3PMo_{12}O_{40} . 29H_2O$.	$H_7[P(Mo_2O_7)_6] . 28H_2O$.
	$H_4SiMo_{12}O_{40} . 29H_2O$.	$H_8[Si(Mo_2O_7)_6] . 28H_2O$.
	Salts.	
	$X_3AB_{12}O_{40}$,	$(NH_4)_3H_4[P(W_2O_7)_6] . nH_2O$.
where		$2(NH_{42}O) . SiO_2 . 12MoO_3 . nH_2O$.
	$X = NH_4$, K, Rb, Cs, Tl.	$2K_2O . P_2O_5 . 24MoO_3 . 4H_2O$.
	$A = P$, As, HSi, HMn	$4Rb_2O . SiO_2 . 12WO_3 . nH_2O$.
	HZr, HTi, H_2B, H_3H_2.	$Cs_8[Si(W_2O_7)_6 . nH_2O$.
	$B = W$, Mo.	$2TlOH . SiO_2 . 12WO_3 . 26H_2O$, etc.

3. ORGANIC CHEMISTRY.

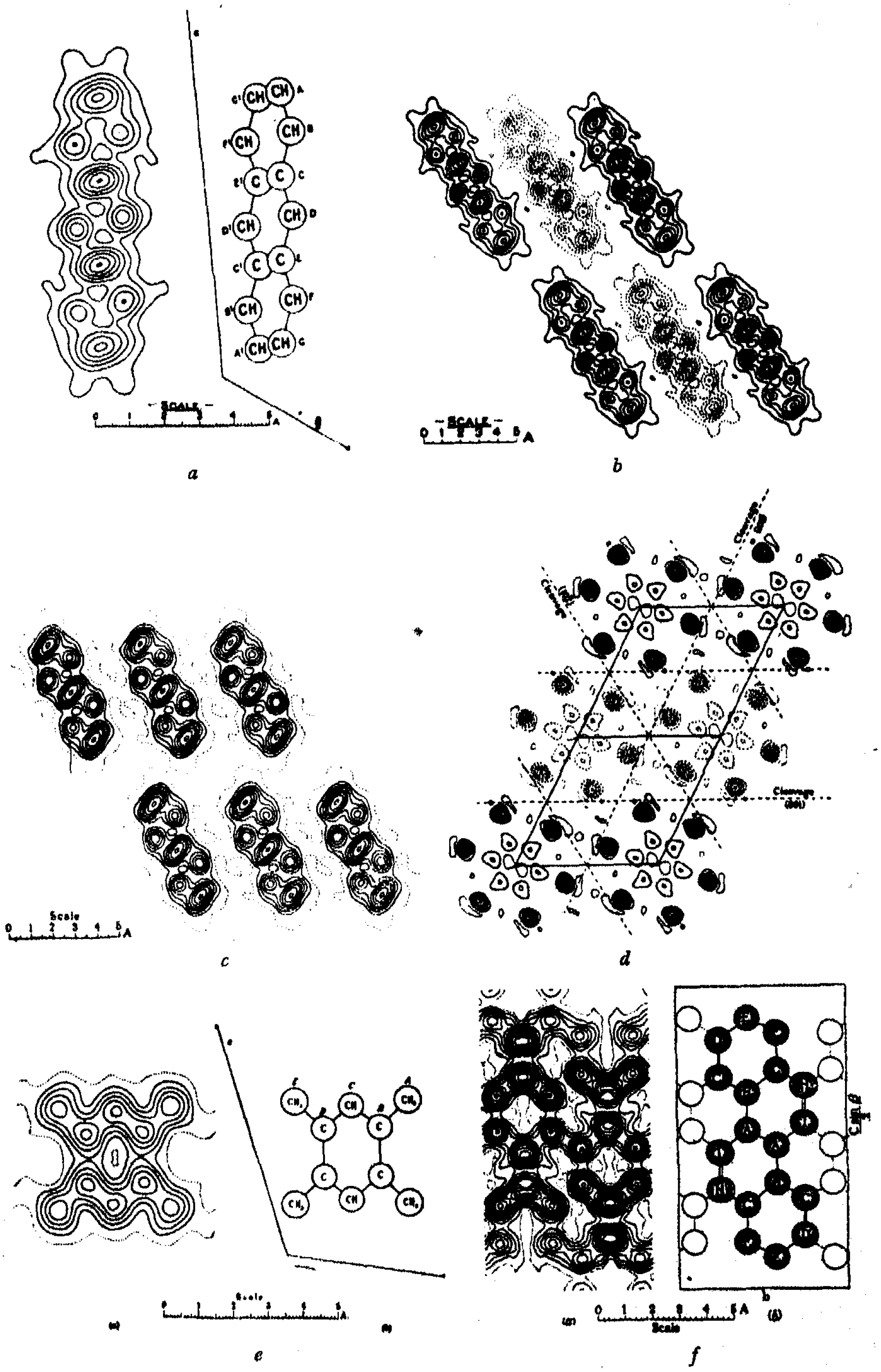
The relative positions of the carbon, hydrogen, oxygen, and nitrogen atoms in organic compounds has been sketched by the labours of the organic chemist. The nature of an organic compound depends essentially upon their relative positions. The first task of X-ray analysis has been to give precision to the stereochemical formulæ. We wish to know, for instance, the precise distance between the carbon atoms in a benzene ring, whether the ring is flat, how groups such as CH_3 or NO_2 are joined on to it, the form of the COOH radicle, and many similar details. Once the rules for the distances between atoms and the angles between bonds are established, it is possible to build a three-dimensional model of any organic compound true to scale.

These details are being ascertained in a series of investigations in which the researchers at the Davy Faraday laboratory are taking the lead. The work is laborious in its initial stages, the solution of any crystal only being obtained after numerous precise measurements and calculations extending over many months. As the available data accumulate, however, it is possible to proceed more boldly and rapidly.

Let us suppose that we were able actually to *see* the pattern in a crystal structure. Our eyes would have to be adapted to X-rays, and their focussing power and delicacy of construction would have to be so marvellous that we could resolve these minute details of structure which are on a scale of an Angstrom unit (10^{-8} cm.). If endowed with this power, we would not see the atoms as objects with a definite outline and shape. We would, on the other hand, see a continuous cloud of scattering matter, very tenuous in most parts but becoming much denser around certain points where atomic nuclei are situated. Only the electrons are capable of scattering X-rays, and the cloud represents the distribution of electron density in the crystal, which rises to high values just around the nuclei of the atoms. The pattern of the crystal would be seen as a regular repetition of these denser parts.

Though direct vision is impossible, it is one of the most elegant features of X-ray diffraction measurements that they can be combined in such a way as to form a picture of what would be seen if the crystal were viewed in any given direction. This property was first pointed out by W. H. Bragg (*Bakerian Lecture*, Roy. Soc., 1915), and has been extensively used to represent the results of analysis. The X-ray measurements of scattered amplitude are substituted as coefficients in a double Fourier series, which when summed gives a map of the electron density. A series of such maps for various organic crystals analysed in the Davy Faraday laboratory is shown in fig. 1.

Investigations of this kind do not merely confirm and give precision to formulæ which have already been determined; they lead on to the determination of structural formulæ for far more complex molecules, whose constitution has not been fully established. Chemical investigation breaks up such large molecules into recognisable fragments and gives hints as to how these fragments were pieced together in the original substance, but there are always cases where the complete solution has not yet been obtained. X-ray analysis holds out promise of a very important application in this field. As an instance, one may quote Bernal's observations on the Sterols (*Nature*, vol. cxxix, 1932, p. 277; *Chem. and Ind.*, vol. li, 1932, pp. 259 and 466). An X-ray examination of the unit cell and symmetry readily gives an estimate of the space occupied



a

b

c

d

e

f

FIG. 1.—*a*, Single molecule of anthracene. *b*, Anthracene molecules in crystal. *c*, Naphthalene. *d*, Hexachlorbenzene. *e*, Durene. *f*, Chrysene.

by each molecule. This is sufficient to decide that the suggested structure (*a*) is incompatible with the observations, whereas (*b*) or (*c*), proposed by Rosenheim and King, are in good accord. The first formula represents a molecule which is both too broad and too short to fit into the cell determined by X-rays, whereas the second and third represent a molecule of suitable dimensions (fig. 2). The analysis is here of direct help, without the necessity of a laborious determination of atomic positions. We decide as to the nature of the molecule by seeing what sort of box it fits into, just as children try to guess what present they are going to get for Christmas from the shape of the parcel before they are allowed to open it.

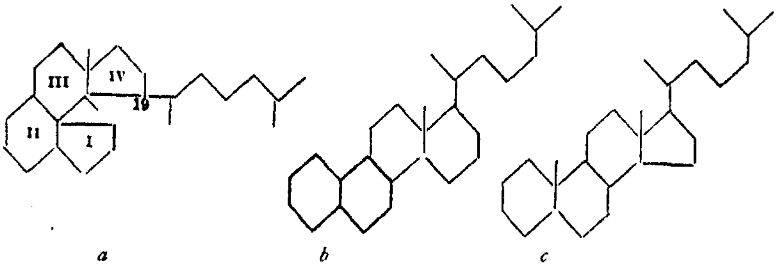


FIG. 2.

4. MINERALOGY.

Minerals were amongst the first substances to which the new analysis was applied. A ready supply of beautiful crystals is available, and they have always afforded tempting material for trying out new methods. In consequence a great deal of attention has been paid to them, and at the present time the structures of nearly all the important minerals are known. A large amount of work remains to be done in filling in the details, and some of this will be of fundamental importance. Nevertheless, the general scheme has been very fully worked out.

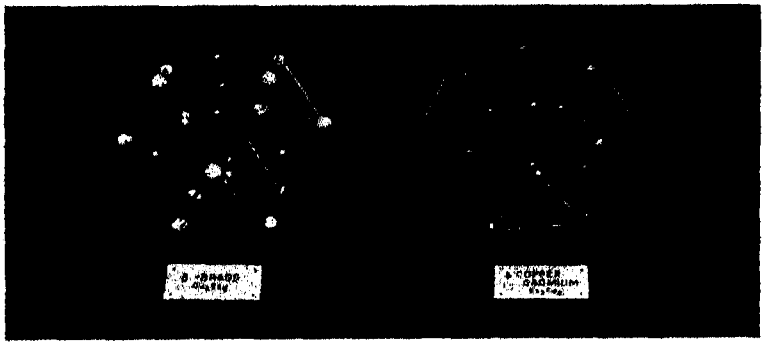
It is the object of the present paper to show how X-ray analysis may reveal a quite new aspect of structure, hitherto unsuspected, which when once realised immensely simplifies all our conceptions. The examination of the silicates affords a striking example. The silicates constitute the greater part of the earth's crust. In any one mineral family such as the micas or amphiboles there is so much isomorphous replacement of one element by another that their formulæ have been the subject of much speculation. Hypothetical "silicic acids" of many kinds have been postulated. The solution is simple. The framework of silicates in general consists of linked tetrahedral groups, every silicon atom being surrounded by four oxygen atoms, each of which may be common to two neighbouring groups. The silicon-oxygen content of the silicate is not represented by isolated acid radicles, but by these firmly bound and continuously extended frameworks. This readily explains the silicon-oxygen proportions of characteristic structures, replacing the complex scheme of "silicic acids" which had previously been thought necessary. Aluminium may proxy for silicon in the framework, and may alternatively act as a kation combined with the framework. This separation of the two functions of aluminium casts a flood of light on previously enigmatic chemical compositions. The structures of the most prevalent minerals, such as olivine, pyroxene, amphibole, mica, felspar, and quartz are seen to be the most simple ways of linking up tetrahedra.

5. METALLURGY

Here again, in this very brief survey, I wish to draw attention to the revolution in ideas which the new knowledge may bring about. Just as typical metals constitute a class of solid bodies with characteristic properties, so the alloys formed by two or more metals differ in character from ordinary chemical compounds. The fundamental feature of a chemical compound, on which Dalton's atomic hypothesis was founded, is the definite integral ratio of its atomic constitution. In alloys there is in general no such definiteness. Two metals such as silver and gold may be alloyed in all proportions, and form a homogeneous substance. If all metallic alloys were of this type it would be natural to say that one was dealing with a case of isomorphous substitution of one metal atom for another of similar character, such as is common in many inorganic crystals. However, in most alloy systems one runs through a series of "phases," or alloys of distinctive types, as the proportion of metal A to metal B is varied. Each phase has a certain range of composition. In extreme cases, these phases may have ranges which are so narrow, and correspond so nearly to simple integral atomic ratios, that they merge into well-defined chemical compounds of the usual type.

One of the central problems of metal chemistry is therefore the following: What imposes such definite characteristics on an alloy phase, while yet permitting its composition to vary over a range which is often considerable?

I will confine myself to a single example which illustrates the hints to be gained from an analysis of atomic arrangement. Fig. 3 shows the structures of two corresponding alloys of copper with zinc and with cadmium respectively. The compositions are approximately represented by Cu_5Zn_8 and Cu_5Cd_8. Since zinc and cadmium are so similar in their properties, it is not surprising to find that these alloy phases are very similar in their physical properties, being characterised for example by great brittleness and hardness. X-ray analysis shows that this similarity extends to the crystal patterns. Leaving out of account for the moment any distinction between copper, zinc, or cadmium, it is found that metal atoms in both alloys are arranged in an almost identical way.



Cu_5Zn_8 Cu_5Cd_8.

FIG. 3.

However, a detailed comparison of these structures by Bradley and Gregory (*Phil. Mag.*, vol. xii, 1931, p. 143) revealed the significant contrast shown by fig. 3. Although the sites occupied by metal atoms are identical in both, the distribution of the copper atoms amongst these sites is quite different in Cu_5Zn_8 as compared with Cu_5Cd_8. (The copper atoms are the darkest, and marked with white discs.) It is a rigorous condition of isomorphous substitution that similar atoms (*e.g.* Cl and Br in a mixed crystal of KCl and KBr) should be in similar positions. Here there is nothing of the kind.

One is forced to conclude that the peculiar and complex structure common to these two alloy phases *is not a result of direct links between atoms*, for if it were the copper atoms would be in the same positions in both.

A body of similar evidence has led to the formulation of a promising theory of alloy structure. Westgren has given examples of alloys with a definite arrangement of sites for atoms, but quite irregular distribution of atoms amongst the sites. Hume Rothery's rule states that the nature of an alloy structure depends upon the ratio of free electrons to metallic atoms. Finally H. Jones (*Proc. Roy. Soc.*, A, vol. cxliv, 1934, p. 225) has put forward a theory based on the wave mechanics, which, though it has as yet many tentative and approximate features, appears to be a brilliant and successful solution. The alloy phase results from an interaction between the atoms of all kinds on the one hand and the "free" electrons on the other hand. The arrangement of atomic sites is such as to allow the electrons to be in states of low potential energy. In highly metallic alloys the interaction between an atom and its neighbours is of secondary importance. As we pass to less typically metallic alloys, this latter interaction increases in importance until finally it is dominant and the substance becomes a typical chemical compound.

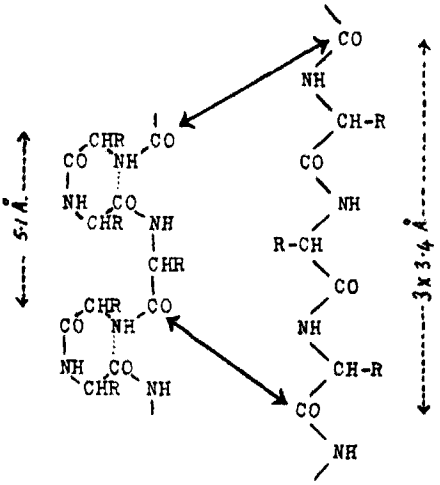
6. BIOCHEMISTRY

The X-ray investigation of structures produced by living matter has a very recent history, but it is perhaps the most interesting field of all. These structures are not crystalline, but they show a degree of order in their atomic arrangement which may be quite high. As an example I will take the pioneer work carried out by W. T. Astbury at Leeds on hair, wool, and related substances (see W. T. Astbury and H. J. Woods, *Phil. Trans. Roy. Soc.*, A, vols. ccxxx, 1931, p. 75, and ccxxxii, 1933, p. 333).

Again, in so brief a survey, we must concentrate on a single point, a new conception due to the results of X-ray analysis. Astbury started work with what appeared to be most unpromising material. When diffraction effects are obtained with truly crystalline matter, they are very sharp and perfect. The spots in "rotation" photographs, for example, are numerous and sharp, and exact measurements may be made on them. In the case of such a substance as hair, only a few diffuse spots are to be seen, because the arrangement of the atoms is not perfect as in a crystal. The existence even of diffuse spots shows, however, that a certain amount of orderly arrangement exists.

The constituent of hair, wool, feathers, quills, whalebone, nails, etc. is keratin. Astbury based his work on the discovery that all these substances give one characteristic X-ray diffraction pattern when unstretched, and an alternative characteristic pattern when stretched. The patterns are of the type called "fibre" photographs, indicating a regular repetition of units in the direction of the hair fibre. The stretched photograph is very similar to that of a much simpler substance, fibroin, which constitutes silk. Astbury concluded that the keratin fibre is based on a type of endless linking —CO—CH—NH— (see fig. 4), the polypeptide chain, which is believed to be the basis of fibroin.

The new conception is as follows: Unstretched α-keratin and stretched β-keratin both give fibre patterns, so clearly the difference between them is not due to an orienting of the molecules by the tension. The alternative explanation is that mechanical tension, which can extend hair to nearly double its original length, brings about an *extension of the molecular chain itself.* The α-keratin chain in its crumpled form is shown on the left-hand side in fig. 4, and β-keratin to which it is transformed on stretching is shown on the right. The chain is supposed to be held in its crumpled form by the weak link NH . . . CO (dotted) which is broken on extension.



α-Keratin β-Keratin

FIG. 4.

The conception of an elastic type of molecular aggregate, whose whole form alters reversibly on stretching and subsequent contraction, is startlingly novel. Its occurrence in a substance formed by living matter is highly significant. There is no space here to mention the evidence with which Astbury supports his conclusions or the elaborations of his hypothesis, but this brief account may serve to indicate the vistas which are being opened up.

(*Issued separately May* 31, 1935.)

VII.—The Mathematical Representation of the Energy Levels of the Secondary Spectrum of Hydrogen.—III. By **Ian Sandeman,** Ph.D., Late Carnegie Research Scholar in the University of St Andrews.

(MS. received February 16, 1935. Read May 13, 1935.)

IN a recent communication (Sandeman, 1935) some problems arising out of the attempt to obtain accurate values of the molecular constants in applying the results of the wave mechanics to the secondary spectrum of hydrogen have been discussed, and an analysis of the two molecular states $1s2s\,^3\Sigma$ and $1s2p\,^1\Sigma$ on the basis of Dunham's (1932) solution of the Schrödinger equation for the molecular rotating vibrator has been carried out and compared with the previous calculations of Richardson and Davidson and Richardson and Das. In the present paper an analysis of the ground state $(1s1s\,^1\Sigma)$ of the hydrogen molecule—made possible by recent advances in the measurement and allocation of the band lines—has been attempted. For comparison a similar analysis of the ground state of the molecular ion $H_2^+(1s\,^2\Sigma)$ is included. Although the spectrum of the molecular ion is not known, the fact that the variables in the Schrödinger equation are separable enables an analysis to be built up from theoretical considerations.

In view of the interest attaching to the ground states of the neutral and ionised molecules, an analytical treatment of these on the basis of Dunham's theoretical work is due. The potential energy expression deducible for the molecular ion is of particular interest, as, in the neighbourhood of the equilibrium position of nuclear separation, it assumes a very simple mathematical form.

The Ground State of the Neutral Molecule $(1s1s\,^1\Sigma)$.—The hydrogen bands, discovered by Schumann and first measured by Lyman in 1906, which end on this state, have been the subject of numerous investigations, notably those of Werner, Hori, Dieke and Hopfield, Schaafsma and Dieke, and Hyman, an account of which is given by Richardson (1934). Spectral measurements in the extreme ultra-violet present some difficulties, in particular the absence of suitable standards, and until recently a few problems in connection with the bands have remained unsolved—notably an apparent discrepancy between the constants of the lowest vibrational state yielded by the Raman Effect measured by Rasetti (1929) and those of the higher vibrational states found from the spectral measurements. Recent measurements of the bands by Jeppesen (1933) have removed this discrepancy, and have added a number of new bands to the list of those already measured, thus enabling an analysis to be carried out with greater precision.

In spite of the progress that has been made no very accurate numerical reckoning of the molecular constants is yet possible owing to slight irregularities in the observational data. The tentative analysis here given must be subject to scrutiny and revision as further experimental data become available.

Jeppesen gives the successive differences between vibrational levels (ΔG) based on his own and Hyman's (1930) investigations as:

v.	ΔG.	Δ.
$\frac{1}{2}$	4161·70	238·80
$1\frac{1}{2}$	3922·90	228·37
$2\frac{1}{2}$	3694·53	226·01
$3\frac{1}{2}$	3468·52	228·30
$4\frac{1}{2}$	3240·22	225·54
$5\frac{1}{2}$	3014·68	231·93
$6\frac{1}{2}$	2782·75	238·62
$7\frac{1}{2}$	2544·13	250·77
$8\frac{1}{2}$	2293·36	266·20
$9\frac{1}{2}$	2027·16	288·65
$10\frac{1}{2}$	1738·51	320·47
$11\frac{1}{2}$	1418·04	

If we assume the vibrational term to be capable of expression by a six-term formula of the form

$$F(v) = \sum_{l=1}^{6} Y_{l0}(v+\tfrac{1}{2})^l,$$

and calculate values of the constants Y_{l0} from the experimental term-differences, a least-squares determination gives the following values in cm.$^{-1}$ for the various Y_{l0}:—

$$\begin{aligned} Y_{10} &= 4412{\cdot}982, \\ Y_{20} &= -130{\cdot}7476, \\ Y_{30} &= 3{\cdot}671688, \\ Y_{40} &= -0{\cdot}3894256, \\ Y_{50} &= 0{\cdot}0220899, \\ Y_{60} &= -0{\cdot}006966374. \end{aligned}$$

This formula fits Jeppesen's ΔG with the following residuals: +0·07, −0·18, −0·87, −0·95, −1·84, +0·12, +0·13, +0·69, −0·03, −0·26,

−0·17, +0·14. This fit is not altogether satisfactory, as it is actually better for the ΔG of high quantum number which are less accurately known than those of low quantum number. If, however, we calculate Y_{70} and Y_{80} by the method of least squares, the sequence of coefficients comes out very irregular both in sign and magnitude and the fit of the lowest members is scarcely improved. Although it is possible that the ΔG are not capable of representation by a formula of the type suggested, reasons have been given in Part II of this paper for thinking that they should be nearly so. Moreover, an examination of the first differences between the ΔG in Jeppesen's table suggests that some of the irregularity present in these may be fortuitous. The best plan seems to be to accept the six-constant formula given above, the first few constants of which are likely to be nearly correct, and to use these first few constants to gain information regarding the molecule.

In his rotational analysis Jeppesen unfortunately uses the quantum number $(K+\frac{1}{2})^2$ instead of $K(K+1)$, the form naturally arising from the wave mechanics and used by Dunham in his theoretical work. If we indicate the rotational band constants used by Jeppesen by a bar, we should write his rotational term in the form:

$$\bar{F}(K)=\bar{B}(K+\tfrac{1}{2})^2+\bar{D}(K+\tfrac{1}{2})^4+\bar{F}(K+\tfrac{1}{2})^6.$$

The form of this expression suited to Dunham's analysis would be:

$$F(K)=BK(K+1)+DK^2(K+1)^2+FK^3(K+1)^3.$$

By equating the coefficients of $K(K+1)$ in the two expressions we see that the relation between the barred and unbarred B's is:

$$B=\bar{B}+\tfrac{1}{2}\bar{D}+\tfrac{3}{16}F.$$

Jeppesen has made a careful determination of the values of $\bar{B}$ for the various vibrational states. His values together with the corresponding values of B are given below:

v.	$\bar{B}$.	B.
0	59·396	59·373
1	56·413	56·391
2	53·528	53·506
3	50·697	50·676
4	47·870	47·850
5	44·973	44·953
6	42·094	42·075
7	39·134	39·116
8	36·086	36·068
9	32·712	32·695

On the assumption that the coefficients of $K(K+1)$ are expressible by a six-term formula

$$\sum_{l=0}^{5} Y_{l1}(v+\tfrac{1}{2})^l,$$

a least-squares determination of the values of the various Y_{l1} yielded the following result:—

$$\begin{aligned}
Y_{01} &= 60{\cdot}98352,\\
Y_{11} &= -3{\cdot}318288,\\
Y_{21} &= 0{\cdot}2334901,\\
Y_{31} &= -0{\cdot}05315730,\\
Y_{41} &= 0{\cdot}0057322667,\\
Y_{51} &= -0{\cdot}0002449350.
\end{aligned}$$

The resulting formula, when applied to the B's, gave the following residuals: −0·004, +0·012, −0·011, −0·006, +0·016, −0·012, +0·012, −0·015, +0·008, −0·002.

The values of the various Y's here given naturally do not differ very greatly from those found previously by Hyman and Hori. (See Richardson, 1934.) The analysis given by Jeppesen which is designed to fit the lowest quantum members may also be compared. The formula for the ΔG given by Jeppesen is equivalent to a four-constant representation of the vibrational term with the following values for the Y_{l0}:—

$$Y_{10}=4416{\cdot}411, \qquad Y_{20}=-131{\cdot}22, \qquad Y_{30}=3{\cdot}115397, \qquad Y_{40}=-0{\cdot}19.$$

With the help of Dunham's theory we can use the lowest Y_{l0}'s and Y_{l1}'s to calculate the constants of the state. We find the variable part of the potential energy to be of the form:

$$E_v = a_0\xi^2(1-1{\cdot}6563\xi+1{\cdot}9996\xi^2-1{\cdot}7301\xi^3+0{\cdot}0225\xi^4+\ldots), \quad . \qquad (1)$$

where a_0 is a constant and ξ is equal to $(r-r_e)/r_e$, r being the nuclear separation and r_e its value at the position of equilibrium. The last coefficient within the bracket in equation (1) is a little doubtful owing to the uncertainty attaching to Y_{30}, the constant from which it is calculated. The other three cannot be very far from correct.

On taking Dunham's correction terms into account, we find:

$$\begin{aligned}
Y_{01} &= B_e(1+\ 2{\cdot}5682(B_e/\omega_e)^2),\\
Y_{10} &= \omega_e(1-43{\cdot}5253(B_e/\omega_e)^2).
\end{aligned}$$

We accordingly obtain the following values for the constants of the state:—

$$\begin{aligned}
B_e &= 60{\cdot}95363 \text{ cm.}^{-1},\\
\omega_e &= 4449{\cdot}970 \text{ cm.}^{-1},\\
a_0 &= 81218{\cdot}4 \text{ cm.}^{-1},\\
r_e &= 0{\cdot}740 \text{ A.U.}
\end{aligned}$$

ω_e is the classical frequency of a small oscillation, B_e is equal to $h/8\pi^2\mu r_e^2 c$, where μ is the reduced mass of the molecule, and a_0 is equal to $\omega_e^2/4B_e$. For reasons detailed in a previous paper (Sandeman, 1935) very high numerical accuracy cannot be attributed to values of the molecular constants derived in this way. In the present case we have also to reckon with a certain amount of uncertainty in the experimental data from which the Y_{ij} are found. This question will be referred to again. It is believed that the analysis here given carries the problem substantially as far as the experimental data allow.

The Ground State of the Hydrogen Molecular Ion ($1s^2\Sigma$).—Numerical values for the energy of the ground state were first calculated by Burrau in 1927. The molecular ion has been the subject of numerous communications in recent years. The problem has been approached from the mathematical side by Alexandrow and by Wilson, and from the physical side by Morse and Stueckelberg, by Teller, and by many others. A bibliography of the papers dealing with this question is given by Jaffé (1934). The method followed in the present paper is to calculate numerical values for the energy for a number of nuclear separations close to the equilibrium value, and then to search for the simplest mathematical formula expressing the energy in terms of the nuclear separation.

If, following the mathematical treatment of Wilson (1928), we insert elliptical co-ordinates, ξ, η, ϕ, in the wave equation, we are able to separate the variables and obtain the differential equation for ξ in the following form:—

$$\frac{d}{d\xi}\left\{(1-\xi^2)\frac{dX}{d\xi}\right\}+\left(\lambda^2\xi^2-x\xi-\frac{n_3^{\,2}}{1-\xi^2}+\mu'\right)X=0. \qquad (2)$$

Here n_3 is the quantum number belonging to the co-ordinate ϕ, μ' is a constant of separation, and λ^2 and x have the following values:—

$$\lambda^2=-\frac{2\pi^2mr^2E'}{h^2}, \qquad x=\frac{8\pi^2mre^2}{h^2}.$$

Here m is the mass of the electron, r is the nuclear separation, and E' is the electronic part of the energy, the total energy E being given by

$$E=E'-\frac{e^2}{r}.$$

$x/2$ measures the nuclear separation in terms of the radius of the hydrogen atom (a), and $(4\lambda^2/x)^2$ gives the electronic part of the energy in terms of the Rydberg Constant (E_H). We have in fact:

$$\begin{aligned} r/a &= x/2, \\ -E'/E_H &= (4\lambda^2/x)^2, \qquad (3) \\ -E/E_H &= (4\lambda^2/x)^2-4/x. \end{aligned}$$

The differential equation for η is the same as (2) except that the term in x is missing.

Equation (2) has been treated by Jaffé. It leads to a relation between x, λ, and μ' in the form of a continued fraction called by Jaffé T_{ξ}. This continued fraction is very convergent and presents no difficulty for numerical treatment.

The corresponding differential equation for η leads to a relation between λ^2 and μ' also in the form of a continued fraction called by Jaffé T_{η}. This continued fraction is not very convergent and presents a little difficulty for numerical treatment. Wilson showed that by means of it μ' could be expressed as a power series in λ^2, and gave the power series for the root of the continued fraction corresponding to the ground state up to the coefficient of λ^8. His value for the last coefficient, however, needs modification. Hylleraas (1931) has given approximate values for the coefficients of λ^8 and λ^{10}. If, however, we desire to express μ' correctly to six places of decimals for any given value of λ^2 in the range under consideration, five powers of λ^2 are not sufficient.

The writer has calculated the first seven coefficients to be:

$$-\mu' = \frac{\lambda^2}{3} + \frac{2\lambda^4}{3^3 \cdot 5} + \frac{4\lambda^6}{3^5 \cdot 5 \cdot 7} - \frac{26\lambda^8}{3^7 \cdot 5^3 \cdot 7} - \frac{92\lambda^{10}}{3^9 \cdot 5^2 \cdot 7 \cdot 11} \\ - \frac{513988\lambda^{12}}{3^{10} \cdot 5^5 \cdot 7^3 \cdot 11 \cdot 13} + \frac{2894282\lambda^{14}}{3^{12} \cdot 5^6 \cdot 7^3 \cdot 11 \cdot 13}. \qquad (4)$$

Even then the term in λ^{16} was found to be affecting the sixth decimal place of μ'. As this coefficient is troublesome to calculate, an approximate value was found by trial, and T_{η} was finally taken as:

$$-\mu' = \frac{\lambda^2}{3} + 0{\cdot}0148148 1\lambda^4 + 0{\cdot}000470312\lambda^6 - 0{\cdot}000013587\lambda^8 \\ - 0{\cdot}000002428\lambda^{10} - 0{\cdot}0000000568\lambda^{12} \\ + 0{\cdot}0000000071\lambda^{14} + 0{\cdot}0000000020\lambda^{16}. \qquad (5)$$

A few trials showed that in the region near the equilibrium position ($x \doteqdot 4$) this formula gave μ' accurately to 1 in the sixth decimal place.

The procedure followed in finding the energy values was that described by Jaffé. The fact that the writer's value for the energy at $x=4$ differs slightly from those of Hylleraas and Jaffé appears to be due to the use by them of an approximate value of μ'.

The question arose as to what values of the nuclear separation x are to be taken into account in order to arrive at an estimate of the energy of vibration. It is not possible to derive from the differential equations any information regarding the amplitude of vibration. From considerations based on the old mechanics we should, however, suppose the ratio of the semi-amplitude of vibration to the internuclear distance to be something of the order of $\sqrt{2B_e/\omega_e}$. From the estimated values of B_e and ω_e for the ion it was considered sufficient to work between the limits $x = 3{\cdot}3$ and $x = 4{\cdot}7$. Fifteen values of the energy calculated between these limits are given in Table I.

TABLE I.

x.	λ^2.	$-E'/E_H = (4\lambda^2/x)^2$.	$4/x$.	$-E/E_H = (4\lambda^2/x)^2 - 4/x$.
3·3	1·633589	2·400131	1·212121	1·188010
3·4	1·712057	2·369629	1·176471	1·193158
3·5	1·791614	2·340067	1·142857	1·197210
3·6	1·872240	2·311407	1·111111	1·200296
3·7	1·953918	2·283615	1·081081	1·202534
3·8	2·036630	2·256654	1·052632	1·204022
3·9	2·120360	2·230490	1·025641	1·204849
4·0	2·205098	2·205098	1·0	1·205098
4·1	2·290827	2·180442	0·975610	1·204832
4·2	2·377538	2·156497	0·952381	1·204116
4·3	2·465220	2·133235	0·930233	1·203002
4·4	2·553865	2·110632	0·909091	1·201541
4·5	2·643463	2·088662	0·888889	1·199773
4·6	2·734008	2·067303	0·869565	1·197738
4·7	2·825494	2·046533	0·851064	1·195469

For the limited region about the position of equilibrium nuclear separation considered λ^2 can be expressed to a close approximation by the following four-term formula in x:—

$$\lambda^2 = 0{\cdot}04191111x^2 + 0{\cdot}54797767x - 0{\cdot}7810752 + 0{\cdot}49475222x^{-1}. \qquad (6)$$

Equation (6) fits the 15 values of λ^2 in Table I with the following residuals:—

+0·000004
+0·000004
+0·000002
0·0
−0·000001
−0·000001
−0·000001
+0·000001
+0·000001
+0·000001
0·0
0·0
−0·000002
−0·000004
−0·000004

On substituting the value of λ^2 given by equation (6) in the relation

$$-E/E_H = (4\lambda^2/x)^2 - 4/x,$$

and equating its differential coefficient to zero, we find the value of x corresponding to the energy minimum to be $x = 3{\cdot}996$. As x represents twice the internuclear distance in atomic units, the equilibrium internuclear distance cannot differ very greatly from 2 atomic units. As the equation on which the turning-point value of x is based is an approximation, high accuracy cannot be claimed for the value $x = 3{\cdot}996$ found here. The minimum of the energy curve is so flat, that the matter of trying to locate the turning-point value of x accurately from the continued fractions T_3 and T_η would be laborious. It seems probable that the equilibrium value of x is not exactly 2 atomic units, but is a little short of that value.

On taking the equilibrium nuclear separation as

$$x_e = 2r_e/a = 3{\cdot}996,$$

and substituting the new variable $\xi = (r - r_e)/r_e$, we finally obtain the following expression for the total energy:—

$$-\mathrm{E}/\mathrm{E_H} = 0{\cdot}67058 + \frac{1{\cdot}1931}{1+\xi} - \frac{0{\cdot}78264}{(1+\xi)^2} + \frac{0{\cdot}12406}{(1+\xi)^3} \quad . \quad . \qquad (7)$$

Or, omitting the constant part of the total energy, we obtain the variable part of the potential energy, E_v, as:

$$\mathrm{E}_v = 0{\cdot}41046\mathrm{E_H}\xi^2(1 - 1{\cdot}6978\xi + 2{\cdot}0933\xi^2 - 2{\cdot}1865\xi^3 + 1{\cdot}9775\xi^4 - 1{\cdot}4663\xi^5 + 0{\cdot}6528\xi^6 + \ldots) \quad . \qquad (8)$$

The general similarity between this expression and the potential energy expression for the $1s2s\,^3\Sigma$ state of the neutral molecule previously given (Sandeman, 1935) is striking. The latter expression may be written:

$$\mathrm{E}_v = 0{\cdot}47924\mathrm{E_H}\xi^2(1 - 1{\cdot}6438\xi + 1{\cdot}9588\xi^2 - 2{\cdot}0945\xi^3 + 2{\cdot}2775\xi^4 + \ldots) \quad . \qquad (9)$$

For the ground state of H_2 the first two terms within the brackets resemble those of equations (8) and (9), but some factor comes into play which causes subsequent terms to become numerically much smaller. Compare equation (1) which may be written:

$$\mathrm{E}_v = 0{\cdot}74051\mathrm{E_H}\xi^2(1 - 1{\cdot}6563\xi + 1{\cdot}9996\xi^2 - 1{\cdot}7301\xi^3 + 0{\cdot}0225\xi^4 + \ldots) \quad . \qquad (10)$$

From the various constants of equation (8) it is possible with the help of Dunham's analysis to forecast all the band constants for the ground state of H_2^+. We have in fact for the first few:

$$\begin{aligned}
r_e &= 1{\cdot}055 && \text{A.U.,}\\
\mathrm{B}_e &= 29{\cdot}93 && \text{cm.}^{-1},\\
a_0 &= 45018{\cdot}3 && \text{cm.}^{-1},\\
\omega_e &= 2321{\cdot}5 && \text{cm.}^{-1},\\
\mathrm{Y}_{10} &= 2321{\cdot}1 && \text{cm.}^{-1},\\
\mathrm{Y}_{20} &= -67{\cdot}77 && \text{cm.}^{-1},\\
\mathrm{Y}_{01} &= 29{\cdot}93 && \text{cm.}^{-1},\\
\mathrm{Y}_{11} &= -1{\cdot}617 && \text{cm.}^{-1},\\
\mathrm{Y}_{02} &= -0{\cdot}01991 && \text{cm.}^{-1} \quad . \quad . \quad . \quad . \quad . \qquad (11)
\end{aligned}$$

The value of ω_e here given is intermediate between those previously found by Richardson and Davidson from the curvature of Burrau's energy graph and from an extrapolation of the singlet states, respectively. (See Richardson, 1934, p. 250.)

After deducting zero-point vibrational energy ($0{\cdot}14_1$ volts), we obtain from equation (7) the heat of dissociation of the ion as $16{\cdot}16_5$ volts in essential agreement with Richardson's value.

Equation (7) shows that for the ground state of H_2^+ the wave equation for the molecular rotating vibrator could be written in a very simple form involving only the first three inverse powers of $1+\xi$. Unfortunately this simplification is not capable of extension to the states of the neutral molecule. The potential expansions for the $1s1s\,^1\Sigma$ and $1s2s\,^3\Sigma$ states of H_2, while bearing considerable resemblance to the corresponding expansion for the $1s\,^2\Sigma$ state of H_2^+, do not appear to be capable of reduction to any simple mathematical form.

A few figures deduced from the present analysis may be of interest: the ratio B_e/ω_e comes out as 0·01370 and 0·01289 respectively for the ground states of H_2 and H_2^+. The moments of inertia come out as 4·538 and $9{\cdot}241 \times 10^{-41}$ gm. cm.^{2} respectively in essential agreement with previous determinations. Just as for all the states of the neutral molecule which the writer has analysed, the first constant of the potential expansion ("a_1" in Dunham's symbolism) of the ground state of the ionised molecule comes out about $-1{\cdot}7$, while the second constant (a_2) comes out about $+2{\cdot}0$, as it does for the $1s1s\,^1\Sigma$ and $1s2s\,^3\Sigma$ states of the neutral molecule.

Accuracy of Observational Data in the Extreme Ultra Violet and the Values of the Constants of the $1s2p\,^1\Pi$ *State.*—An important question raised by the present work is that of how much reliance can be placed on the experimental data available from the recent measurements of Jeppesen, when it is desired to use these to obtain accurate numerical values of the molecular constants. If we use Jeppesen's wave numbers of the band lines to deduce the rotational term differences in the manner described by him, we find that the term differences corresponding to each vibrational state are in good agreement with one another, which means that in each spectral region where a band occurs the lines are in their correct positions relative to one another, at least well within the experimental limit of error of 2 cm.$^{-1}$ given by Jeppesen. If, on the other hand, we take the vibrational term differences, got by subtracting any band line from the corresponding line of the band with vibration quantum number higher by one—say the line for which the quantum numbers are (v', v'', K$'$, K$''$) from the line for which they are ($v'+1$, v'', K$'$, K$''$)—we do not obtain so good a measure of agreement. This indicates that there have been small, but quite appreciable, shifts between the regions where the bands occur.

This point will be made clear by a brief consideration of the $1s2p\,{}^1\Pi$ state. As no analysis of this state on the basis of Dunham's work has yet been given, the tentative values of the constants here given will also serve the purpose of comparison with the other states. From the table of rotational term differences of the $1s2p\,{}^1\Pi_c$ states (upper states of the Q series) given by Jeppesen the writer has deduced the following values for the constants Y_{ij}:—

$Y_{01} = 31{\cdot}3853$, $Y_{11} = -1{\cdot}5516$, $Y_{21} = 0{\cdot}00195$; $Y_{02} = -0{\cdot}020717$,
$Y_{12} = 0{\cdot}000614$; $Y_{03} = 0{\cdot}000189$ (Jeppesen's value).

The above values fit all the term differences except two within 1 cm.$^{-1}$. Probably more constants would be required to obtain a better fit, but the accuracy of the data scarcely justifies the calculation of any more.

We can now use the "reciprocal-action coefficients," *i.e.* the coefficients of terms in both v and K (Y_{11}, Y_{21}, and Y_{12}), to eliminate the effect of rotation from the vibration term differences and obtain pure vibration term differences, *i.e.* the part of the term difference involving only the Y_{i0}. Table II shows all the vibration term differences corresponding to the transition 2 ⟶ 1 of the initial vibration quantum number together with the bands from which they are deduced. The figures in brackets underneath are the corresponding pure vibration term differences.

TABLE II.—$1s2p\,{}^1\Pi_c$ STATE. VIBRATION TERM DIFFERENCES CORRESPONDING TO A TRANSITION 2 ⟶ 1 OF THE INITIAL VIBRATION QUANTUM NUMBER v'.

Bands involved.	K = 1.	2.	3.	4.	5.
$(2 \longrightarrow 3) - (1 \longrightarrow 3)$	2167·5 (2170·6)	2162·4 (2171·6)	2153·5 (2171·9)	2140·8 (2171·4)	*2131·5* (*2177·3*)
$(2 \longrightarrow 4) - (1 \longrightarrow 4)$	2175·5 (2178·6)	2167·6 (2176·8)	2158·5 (2176·9)	*2150·5* (*2181·1*)	*2134·1* (*2179·9*)
$(2 \longrightarrow 5) - (1 \longrightarrow 5)$	2171·4 (2174·1)	2164·8 (2174·0)	2155·2 (2173·6)	2143·7 (2174·3)	

It is evident that, apart from the three values in italics, the figures for the pure vibration term differences yielded by each horizontal row are consistent within the limits of error along the row, but the rows do not agree very well with each other. This indicates that slight shifts have taken place in Jeppesen's measurements in passing from one region to another.

In view of this it is believed that no very high accuracy can be claimed for the ΔG of any of the states, although the values of these given by Jeppesen cannot be far from correct, and that therefore, in the case of $1s1s\,{}^1\Sigma$ where a large number of ΔG are available, an approximate fit by means of a six-constant formula in the vibration quantum number is more likely to yield good values of the constants than a four-constant formula designed to fit the estimated values of the ΔG for the lowest quantum states with great numerical fidelity. This question also shows how desirable it is to push the limits of observational accuracy beyond the already high standard attained.

As far as the $1s2p\,{}^1\Pi_c$ state is concerned, as only four vibrational term differences are available, we cannot estimate the coefficients Y_{i0} very accurately. A weighted least-squares determination of the first three, on the assumption that a three-constant formula is adequate, gave $Y_{10}=2443{\cdot}1$, $Y_{20}=-68{\cdot}018$, and $Y_{30}=0{\cdot}252$. With the above values of the Y_{ij}, assuming Jeppesen's conclusion that the value of B_e for $1s2p\,{}^1\Pi_c$ is the true value for the $1s2p\,{}^1\Pi$ state, we obtain the first two constants of the potential expansion as $-1{\cdot}64$ and $1{\cdot}92$. This indicates that the potential expansion for the state conforms to the same general model as those of $1s1s\,{}^1\Sigma$, $1s2s\,{}^3\Sigma$, and $1s\,{}^2\Sigma$.

Morse's Rule and Douglas Clark's Modification of it.—Morse (1929) has suggested that the quantity $\omega_e r_e^3$ is, approximately at least, a constant for all molecules, and this rule has frequently been applied to forecast r_e for molecular states for which ω_e is known but not r_e. In a recent communication Douglas Clark (1934) has suggested that Morse's rule should be modified to read:

$$\omega_e r_e^3\sqrt{n_c}=k-k',$$

where n_c is the "group number" or number of electrons shared between the two atoms forming the diatomic molecule excluding those forming completed rings in either atom, k is a constant for any particular period (all the molecules consisting of one atom with n completed rings and the other atom with m completed rings for the same values of m and n are classed together as belonging to the same period), and k' is a correction for ionised molecules. While Douglas Clark's rule is essentially empirical, it appears to hold fairly closely for diatomic molecules of non-hydride type.

The group number n_c should be 2 for the neutral hydrogen molecule and 1 for the ionised molecule. The result of an application of Douglas Clark's rule to the hydrogen states is shown in the following table:—

		$\omega_e r_e^3\times10^{21}$ cm.^{3}.	$\omega_e r_e^3\sqrt{2}\times10^{21}$ cm.^{3}.
H_2^+	$1s\,{}^2\Sigma$	2·66	
H_2	$1s1s\,{}^1\Sigma$	1·80	2·55
	$1s2s\,{}^3\Sigma$	2·57	3·64
	$1s2p\,{}^1\Pi$	2·67	3·77
	$1s2p\,{}^1\Sigma$	2·94	4·16

The table suggests that the rule holds closely only for the ground states. For these the ionisation correction k' comes out as $-0{\cdot}11\times10^{-21}$ cm.^{3}, which may be compared with the value $+0{\cdot}50\times10^{-21}$ cm.^{3} found by

Douglas Clark for the molecules of the KK period. The state $1s2p\,^1\Sigma$ which departs most widely from the rule is peculiar in having a different type of potential energy expression from that of the other states.

Morse's rule and Douglas Clark's modification of it seem to be successive approximations towards some law the ultimate form of which is yet to be discovered.

SUMMARY.

With use of the observational data of C. R. Jeppesen, a determination of the molecular constants of the ground state ($1s1s\,^1\Sigma$) of H_2 is carried out on the basis of J. L. Dunham's solution of the wave equation for the molecular rotating vibrator. For comparison a similar analysis of the ground state ($1s\,^2\Sigma$) of the molecular ion H_2^+ is carried out, the method being to calculate the energy for a number of nuclear separations close to the equilibrium position by the method of G. Jaffé and to search for the simplest mathematical formula expressing the energy as a function of the nuclear separation. For the region close to the position of equilibrium the energy can be expressed in terms of the nuclear separation by a very simple mathematical formula. A forecast of the band constants for the ground state of H_2^+ is given. The potential energy expansions of the $1s1s\,^1\Sigma$ and $1s2s\,^3\Sigma$ states of the neutral molecule closely resemble the expansion for the ground state of H_2^+, but, unlike the latter, do not appear to be capable of representation by any compact mathematical expression. The application to the hydrogen states of C. H. D. Clark's modification of Morse's Rule for diatomic molecules is discussed.

I am indebted to Professor H. Stanley Allen and to Professor O. W. Richardson for helpful criticisms and suggestions and to Dr A. C. Aitken of Edinburgh for suggestions as to the method of carrying out the least-squares determinations undertaken in this paper.

REFERENCES TO LITERATURE.

CLARK, C. H. D., 1934. "The Relation between Vibration Frequency and Nuclear Separation for some simple Non-Hydride Diatomic Molecules," *Phil. Mag.*, vol. xviii, p. 459.

DUNHAM, J. L., 1932. "The Wentzel-Brillouin-Kramers Method of Solving the Wave Equation," *Phys. Rev.*, vol. xli, p. 713; "The Energy Levels of a Rotating Vibrator," *ibid.*, p. 721.

HYLLERAAS, E. A., 1931. "Über die Elektronenterme des Wasserstoffmoleküls," *Zeits. Phys.*, Bd. lxxi, p. 739.

JAFFÉ, G., 1934. "Zur Theorie des Wasserstoffmolekülions," *Zeits. Phys.*, Bd. lxxxvii, p. 535.

JEPPESEN, C. R., 1933. "The Emission Spectrum of Molecular Hydrogen in the Extreme Ultraviolet," *Phys. Rev.*, vol. xliv, p. 165.

MORSE, P. M., 1929. "Diatomic Molecules according to the Wave Mechanics. II. Vibrational Levels," *Phys. Rev.*, vol. xxxiv, p. 57.

RASETTI, F., 1929. "Incoherent Scattered Radiation in Diatomic Molecules," *Phys. Rev.*, vol. xxxiv, p. 367.

RICHARDSON, O. W., 1934. *Molecular Hydrogen and its Spectrum*, Yale University Press.

SANDEMAN, I., 1935. "The Mathematical Representation of the Energy Levels of the Secondary Spectrum of Hydrogen.—II," *Proc. Roy. Soc. Edin.*, vol. lv, 1935, pp. 49–61.

WILSON, A. H., 1928. "A Generalised Spheroidal Wave Equation," *Proc. Roy. Soc.*, A, vol. cxviii, p. 617; "The Ionised Hydrogen Molecule," *ibid.*, p. 637.

(*Issued separately August* 1, 1935.)

VIII.—Some Series and Integrals involving Associated Legendre Functions, regarded as Functions of their Degrees. By **Professor T. M. MacRobert,** M.A., D.Sc.

(MS. received April 6, 1935. Read July 1, 1935.)

§ 1. *Introductory.*—In a former paper (*Proc. Roy. Soc. Edin.*, vol. liv, 1934, pp. 135–144) the author discussed the evaluation of a number of integrals of Associated Legendre Functions, regarded as functions of their degrees. The methods employed depended mainly on contour integration, and most of the integrals were evaluated in terms of infinite series of Associated Legendre Functions. In the present paper the methods employed are of a more elementary character, depending mainly on the use of Dirichlet Integrals; the results obtained are more general; and the integrals and the corresponding series are evaluated in simpler forms. The same notation is employed as in the previous paper. The Mehler-Dirichlet Integral

$$(\sin\theta)^{\mu}\mathrm{T}_{\lambda-\frac{1}{2}}^{-\mu}(\cos\theta)=\frac{2}{\Gamma(\mu+\frac{1}{2})\sqrt{(2\pi)}}\int_0^{\theta}\cos(\lambda\phi)(\cos\phi-\cos\theta)^{\mu-\frac{1}{2}}d\phi, \qquad \text{(i)}$$

where $0 < \theta < \pi$, $\mu > -\frac{1}{2}$, is used throughout.

§ 2. *Integrals involving One Associated Legendre Function.*—From (i) it follows that

$$\begin{aligned}(\sin\theta)^{\mu}&\int_0^{\lambda}\cos(\lambda\rho)\mathrm{T}_{\lambda-\frac{1}{2}}^{-\mu}(\cos\theta)d\lambda\\ &=\frac{2}{\Gamma(\mu+\frac{1}{2})\sqrt{(2\pi)}}\int_0^{\theta}(\cos\phi-\cos\theta)^{\mu-\frac{1}{2}}d\phi\int_0^{\lambda}\cos(\lambda\rho)\cos(\lambda\phi)d\lambda\\ &=\frac{1}{\Gamma(\mu+\frac{1}{2})\sqrt{(2\pi)}}\int_0^{\theta}(\cos\phi-\cos\theta)^{\mu-\frac{1}{2}}\left\{\frac{\sin\lambda(\phi-\rho)}{\phi-\rho}+\frac{\sin\lambda(\phi+\rho)}{\phi+\rho}\right\}d\phi.\end{aligned}$$

Hence, when $\lambda\rightarrow\infty$,

$$(\sin\theta)^{\mu}\int_0^{\infty}\cos(\lambda\rho)\mathrm{T}_{\lambda-\frac{1}{2}}^{-\mu}(\cos\theta)d\lambda=\frac{\sqrt{(2\pi)}}{2\Gamma(\mu+\frac{1}{2})}(\cos\rho-\cos\theta)^{\mu-\frac{1}{2}}, \quad . \qquad \text{(ii)}$$

where $-\pi < -\theta < \rho < \theta < \pi$, $\mu > -\frac{1}{2}$.

If $|\rho| > \theta$ the integral vanishes.

If $\rho = \pm\theta$, $0 < \theta < \pi$,

$$(\sin\theta)^{\mu}\int_0^{\infty}\cos(\lambda\theta)\mathrm{T}_{\lambda-\frac{1}{2}}^{-\mu}(\cos\theta)d\lambda=\begin{cases}0, \ \mu > \frac{1}{2},\\ \frac{1}{2}\sqrt{(2\pi)}, \ \mu=\frac{1}{2},\\ \infty, \ -\frac{1}{2} < \mu < \frac{1}{2}.\end{cases} \quad . \quad . \qquad \text{(iii)}$$

These results can be deduced directly from (i) by means of Fourier's Integral Theorem. The particular case in which $\mu = 0$ was given by C. Fox (*Proc. Lond. Math. Soc.*, ser. 2, vol. xxvi, p. 76). In the same paper and on the same page Mr Fox gave also some formulæ, particular cases of which are particular cases of formula (iv) below.

§ 3. *Series of Associated Legendre Functions.*—If $0 < \theta < \pi$, $\mu > -\frac{1}{2}$, and m is a positive integer, it follows from (i) that

$$(\sin\theta)^{\mu}\sum_{n=-m}^{m}\cos\{(n+\beta)\rho\}\mathrm{T}_{n+\alpha-\frac{1}{2}}^{-\mu}(\cos\theta)$$

$$=\frac{1}{\Gamma(\mu+\frac{1}{2})\sqrt{(2\pi)}}\int_0^{\theta}(\cos\phi-\cos\theta)^{\mu-\frac{1}{2}}\sum_{n=-m}^{m}\begin{bmatrix}\cos\{(n+\beta)\rho+(n+\alpha)\phi\}\\ +\cos\{(n+\beta)\rho-(n+\alpha)\phi\}\end{bmatrix}d\phi$$

$$=\frac{1}{\Gamma(\mu+\frac{1}{2})\sqrt{(2\pi)}}$$

$$\times\int_0^{\theta}(\cos\phi-\cos\theta)^{\mu-\frac{1}{2}}\begin{bmatrix}\cos(\beta\rho+\alpha\phi)\dfrac{\sin\{(m+\frac{1}{2})(\rho+\phi)\}}{\sin\frac{1}{2}(\rho+\phi)}\\ +\cos(\beta\rho-\alpha\phi)\dfrac{\sin\{(m+\frac{1}{2})(\rho-\phi)\}}{\sin\frac{1}{2}(\rho-\phi)}\end{bmatrix}d\phi.$$

When $m \to \infty$ this gives

$$(\sin\theta)^{\mu}\sum_{n=-\infty}^{\infty}\cos\{(n+\beta)\rho\}\mathrm{T}_{n+\alpha-\frac{1}{2}}^{-\mu}(\cos\theta)$$

$$=\frac{\sqrt{(2\pi)}}{\Gamma(\mu+\frac{1}{2})}(\cos\rho-\cos\theta)^{\mu-\frac{1}{2}}\cos\{(\alpha-\beta)\rho\}, \qquad \text{(iv)}$$

provided that $-\pi < -\theta < \rho < \theta < \pi$, $\mu > -\frac{1}{2}$.

If $|\rho| > \theta$ the sum of the series is zero.

If $\rho = \pm\theta$, $0 < \theta < \pi$,

$$(\sin\theta)^{\mu}\sum_{n=-\infty}^{\infty}\cos\{(n+\beta)\theta\}\mathrm{T}_{n+\alpha-\frac{1}{2}}^{-\mu}(\cos\theta)=\begin{cases}0, & \mu>\frac{1}{2},\\ \frac{1}{2}\sqrt{(2\pi)}\cos\{(\alpha-\beta)\theta\}, & \mu=\frac{1}{2},\\ \infty, & -\frac{1}{2}<\mu<\frac{1}{2}.\end{cases} \qquad \text{(v)}$$

Again, if $0 < \theta < \pi$, $\mu > -\frac{1}{2}$, and m is a positive integer,

$$(\sin\theta)^{\mu}\sum_{n=0}^{m}\cos\{(n+\beta+\tfrac{1}{2})\rho\}\mathrm{T}_{n+\alpha}^{-\mu}(\cos\theta)$$

$$=\frac{2}{\Gamma(\mu+\frac{1}{2})\sqrt{(2\pi)}}\int_0^{\theta}(\cos\phi-\cos\theta)^{\mu-\frac{1}{2}}\sum_{n=0}^{m}\cos\{(n+\beta+\tfrac{1}{2})\rho\}\cos\{(n+\alpha+\tfrac{1}{2})\phi\}d\phi$$

$$=\frac{1}{\Gamma(\mu+\frac{1}{2})\sqrt{(2\pi)}}$$

$$\times\int_0^{\theta}(\cos\phi-\cos\theta)^{\mu-\frac{1}{2}}\begin{bmatrix}\cos\{\beta\rho+\alpha\phi+\frac{1}{2}(m+1)(\rho+\phi)\}\dfrac{\sin\{\frac{1}{2}(m+1)(\rho+\phi)\}}{\sin\{\frac{1}{2}(\rho+\phi)\}}\\ +\cos\{\beta\rho-\alpha\phi+\frac{1}{2}(m+1)(\rho-\phi)\}\dfrac{\sin\{\frac{1}{2}(m+1)(\rho-\phi)\}}{\sin\{\frac{1}{2}(\rho-\phi)\}}\end{bmatrix}d\phi.$$

This gives, when $m \to \infty$,

$$(\sin\theta)^{\mu}\sum_{n=0}^{\infty}\cos\{(n+\beta+\tfrac{1}{2})\rho\}\mathrm{T}_{n+\alpha}^{-\mu}(\cos\theta)=\frac{\sqrt{(2\pi)}}{2\Gamma(\mu+\frac{1}{2})}(\cos\rho-\cos\theta)^{\mu-\frac{1}{2}}\cos\{(\alpha-\beta)\rho\}$$

$$-\frac{1}{2\Gamma(\mu+\frac{1}{2})\sqrt{(2\pi)}}\int_0^{\theta}(\cos\phi-\cos\theta)^{\mu-\frac{1}{2}}\left[\begin{array}{c}\dfrac{\sin(\beta\rho+\alpha\phi)}{\sin\{\frac{1}{2}(\rho+\phi)\}}\\+\dfrac{\sin(\beta\rho-\alpha\phi)}{\sin\{\frac{1}{2}(\rho-\phi)\}}\end{array}\right]d\phi, \quad . \quad \text{(vi)}$$

provided that $-\pi < -\theta < \rho < \theta < \pi$, $\mu > -\frac{1}{2}$.

If $|\rho| > \theta$ the first term on the right-hand side of (vi) is replaced by zero.

If $\rho = \pm\theta$, $0 < \theta < \pi$, the first term on the right-hand side of (vi) is replaced by zero if $\mu > \frac{1}{2}$, and is halved if $\mu = \frac{1}{2}$; while if $-\frac{1}{2} < \mu < \frac{1}{2}$ the series is divergent.

§ 4. *Integrals involving Two Associated Legendre Functions.*—If $0 < \theta \leq \phi < \pi$, $\kappa > -\frac{1}{2}$, $\mu > -\frac{1}{2}$, it follows from (i) that

$$(\sin\theta)^{\kappa}(\sin\phi)^{\mu}\mathrm{T}_{\lambda-\frac{1}{2}}^{-\kappa}(\cos\theta)\mathrm{T}_{\lambda-\frac{1}{2}}^{-\mu}(\cos\phi)=\frac{1}{\pi\Gamma(\kappa+\frac{1}{2})\Gamma(\mu+\frac{1}{2})}\int_0^{\theta}(\cos\psi-\cos\theta)^{\kappa-\frac{1}{2}}d\psi$$

$$\times\int_0^{\phi}[\cos\{\lambda(\psi-\chi)\}+\cos\{\lambda(\psi+\chi)\}](\cos\chi-\cos\phi)^{\mu-\frac{1}{2}}d\chi.$$

Hence, on multiplying by $\cos(\lambda\rho)$, integrating with respect to λ, and writing $u = \chi+\psi$, $v = \chi-\psi$, we find that

$$(\sin\theta)^{\kappa}(\sin\phi)^{\mu}\int_0^{\lambda}\cos(\lambda\rho)\mathrm{T}_{\lambda-\frac{1}{2}}^{-\kappa}(\cos\theta)\mathrm{T}_{\lambda-\frac{1}{2}}^{-\mu}(\cos\phi)d\lambda=\frac{1}{4\pi\Gamma(\kappa+\frac{1}{2})\Gamma(\mu+\frac{1}{2})}$$

$$\times\left\{\begin{array}{l}\displaystyle\int_0^{\theta}\mathrm{S}(\lambda,u,\rho)\mathrm{I}(-u,u)du+\int_{\theta}^{\phi}\mathrm{S}(\lambda,u,\rho)\mathrm{I}(u-2\theta,u)du\\ \displaystyle+\int_{\phi}^{\theta+\phi}\mathrm{S}(\lambda,u,\rho)\mathrm{I}(u-2\theta,2\phi-u)du+\int_{-\theta}^{0}\mathrm{S}(\lambda,v,\rho)\mathrm{J}(-v,2\theta+v)dv\\ \displaystyle+\int_0^{\phi-\theta}\mathrm{S}(\lambda,v,\rho)\mathrm{J}(v,2\theta+v)dv+\int_{\phi-\theta}^{\phi}\mathrm{S}(\lambda,v,\rho)\mathrm{J}(v,2\phi-v)dv\end{array}\right\}, \text{(vii)}$$

where

$$\mathrm{S}(\lambda,u,\rho)=\frac{\sin\lambda(u+\rho)}{u+\rho}+\frac{\sin\lambda(u-\rho)}{u-\rho},$$

$$\mathrm{I}(p,q)=\int_p^q\mathrm{F}(u,v)dv,\quad \mathrm{J}(p,q)=\int_p^q\mathrm{F}(u,v)du,$$

and

$$\mathrm{F}(u,v)=\{\cos\tfrac{1}{2}(u-v)-\cos\theta\}^{\kappa-\frac{1}{2}}\{\cos\tfrac{1}{2}(u+v)-\cos\phi\}^{\mu-\frac{1}{2}}.$$

If now $\lambda\to\infty$ the values of the integral

$$(\sin\theta)^{\kappa}(\sin\phi)^{\mu}\int_0^{\infty}\cos(\lambda\rho)\mathrm{T}_{\lambda-\frac{1}{2}}^{-\kappa}(\cos\theta)\mathrm{T}_{\lambda-\frac{1}{2}}^{-\mu}(\cos\phi)d\lambda,$$

which correspond to different values of ρ, can be derived from (vii), provided that in each case the integral is convergent.

For example, if $\rho=0$, $\kappa > -\frac{1}{2}$, $\kappa+\mu > 0$, $0 < \theta \leqq \phi < \pi$,

$$(\sin\theta)^{\kappa}(\sin\phi)^{\mu}\int_0^{\infty} \mathrm{T}_{\lambda-\frac{1}{2}}^{-\kappa}(\cos\theta)\mathrm{T}_{\lambda-\frac{1}{2}}^{-\mu}(\cos\phi)d\lambda$$

$$=\frac{1}{2\Gamma(\kappa+\frac{1}{2})\Gamma(\mu+\frac{1}{2})}\int_0^{2\theta}(\cos\tfrac{1}{2}u-\cos\theta)^{\kappa-\frac{1}{2}}(\cos\tfrac{1}{2}u-\cos\phi)^{\mu-\frac{1}{2}}du$$

$$=\frac{1}{\Gamma(\kappa+\frac{1}{2})\Gamma(\mu+\frac{1}{2})}\int_0^{\theta}(\cos\omega-\cos\theta)^{\kappa-\frac{1}{2}}(\cos\omega-\cos\phi)^{\mu-\frac{1}{2}}d\omega, \quad . \quad \text{(viii)}$$

where $\omega=\frac{1}{2}u$. The result also holds for $\kappa > -\frac{1}{2}$, $-1 < \kappa+\mu \leqq 0$, provided that $0 < \theta < \phi < \pi$.

The expression on the right of (viii) can be expanded in double series in several ways.

Put

$$\sin\tfrac{1}{2}\omega=\sin\tfrac{1}{2}\theta\,.\,\zeta^{\frac{1}{2}},$$

so that

$$\cos\tfrac{1}{2}\omega\,.\,d\omega=\sin\tfrac{1}{2}\theta\,.\,\zeta^{-\frac{1}{2}}d\zeta.$$

Then the right-hand side of (viii) is equal to

$$\frac{2^{\kappa+\mu-1}}{\Gamma(\kappa+\frac{1}{2})\Gamma(\mu+\frac{1}{2})}\int_0^{\theta}(\sin^2\tfrac{1}{2}\theta-\sin^2\tfrac{1}{2}\omega)^{\kappa-\frac{1}{2}}(\sin^2\tfrac{1}{2}\phi-\sin^2\tfrac{1}{2}\omega)^{\mu-\frac{1}{2}}d\omega$$

$$=\frac{\sqrt{2}\,.\,(1-\cos\theta)^{\kappa}(1-\cos\phi)^{\mu-\frac{1}{2}}}{2\Gamma(\kappa+\frac{1}{2})\Gamma(\mu+\frac{1}{2})}$$

$$\times\int_0^1\zeta^{-\frac{1}{2}}(1-\zeta)^{\kappa-\frac{1}{2}}\left(1-\frac{\sin^2\frac{1}{2}\theta}{\sin^2\frac{1}{2}\phi}\zeta\right)^{\mu-\frac{1}{2}}(1-\sin^2\tfrac{1}{2}\theta\,.\,\zeta)^{-\frac{1}{2}}d\zeta.$$

On expanding the last factor in the integrand by the binomial theorem, and thereafter integrating term by term, it is found that

$$(\sin\theta)^{\kappa}(\sin\phi)^{\mu}\int_0^{\infty}\mathrm{T}_{\lambda-\frac{1}{2}}^{-\kappa}(\cos\theta)\mathrm{T}_{\lambda-\frac{1}{2}}^{-\mu}(\cos\phi)d\lambda=\frac{\sqrt{2}\,.\,(1-\cos\theta)^{\kappa}(1-\cos\phi)^{\mu-\frac{1}{2}}}{2\Gamma(\mu+\frac{1}{2})}$$

$$\times\sum_{r=0}^{\infty}\frac{\{\Gamma(r+\frac{1}{2})\}^2}{\Gamma(\frac{1}{2})\Gamma(\kappa+r+1)\,.\,r!}(\sin\tfrac{1}{2}\theta)^{2r}\mathrm{F}\left(\tfrac{1}{2}+r,\ \tfrac{1}{2}-\mu;\ \kappa+r+1;\ \frac{\sin^2\frac{1}{2}\theta}{\sin^2\frac{1}{2}\phi}\right), \quad \text{(ix)}$$

$$=\frac{\sqrt{2}\,.\,(1-\cos\theta)^{\kappa}(1-\cos\phi)^{\mu-\frac{1}{2}}}{2\pi}\,.\cos(\mu\pi)$$

$$\times\sum_{r=0}^{\infty}\frac{\Gamma(r+\frac{1}{2})\Gamma(\frac{1}{2}-\mu+r)}{\Gamma(\kappa+r+1)\,.\,r!}\left(\frac{\sin\frac{1}{2}\theta}{\sin\frac{1}{2}\phi}\right)^{2r}\mathrm{F}(\tfrac{1}{2},\ -r;\ \mu-r+\tfrac{1}{2};\ \sin^2\tfrac{1}{2}\phi), \quad . \quad . \quad \text{(x)}$$

the series in (x) being deduced from the series in (ix) by changing the order of summation.

By a further change in the order of summation, or by expanding the second last instead of the last factor in the integrand by the binomial theorem and thereafter integrating term by term, it can be shown that

$$(\sin\theta)^{\kappa}\,(\sin\phi)^{\mu}\int_0^{\infty} \mathrm{T}_{\lambda-\frac{1}{2}}^{-\kappa}\,(\cos\theta)\mathrm{T}_{\lambda-\frac{1}{2}}^{-\mu}\,(\cos\phi)d\lambda$$

$$=\frac{\sqrt{2}\,.\,(1-\cos\theta)^{\kappa}(1-\cos\phi)^{\mu-\frac{1}{2}}}{2\pi}\,.\cos\mu\pi$$

$$\times\sum_{r=0}^{\infty}\frac{\Gamma(\frac{1}{2}-\mu+r)\Gamma(\frac{1}{2}+r)}{\Gamma(\kappa+r+1)\,.\,r!}\left(\frac{\sin\frac{1}{2}\theta}{\sin\frac{1}{2}\phi}\right)^{2r}\mathrm{F}(\tfrac{1}{2},\ \tfrac{1}{2}+r;\ \kappa+r+1;\ \sin^2\tfrac{1}{2}\theta), \qquad \text{(xi)}$$

§ 5. *Integrals involving Two Associated Legendre Functions* (*continued*).—Again, in (vii) let $\rho=\pi$; then, if $\kappa>-\frac{1}{2}$, $\mu>-\frac{1}{2}$, $0<\theta\leqq\phi<\pi$,

$$(\sin\theta)^{\kappa}\,(\sin\phi)^{\mu}\int_0^{\infty}\cos\,(\lambda\pi)\mathrm{T}_{\lambda-\frac{1}{2}}^{-\kappa}(\cos\theta)\mathrm{T}_{\lambda-\frac{1}{2}}^{-\mu}\,(\cos\phi)d\lambda$$

$$=\frac{1}{4\Gamma(\kappa+\frac{1}{2})\Gamma(\mu+\frac{1}{2})}\begin{cases}0, & \theta+\phi<\pi,\\ \frac{1}{2}\mathrm{K}, & \theta+\phi=\pi,\\ \mathrm{K}, & \theta+\phi>\pi,\end{cases} \qquad \text{(xii)}$$

where

$$\mathrm{K}=\int_{\pi-2\theta}^{2\phi-\pi}\{\cos\tfrac{1}{2}(\pi-v)-\cos\theta\}^{\kappa-\frac{1}{2}}\{\cos\tfrac{1}{2}(\pi+v)-\cos\phi\}^{\mu-\frac{1}{2}}dv,$$

or, if $v=2\omega-\pi$,

$$\mathrm{K}=2\int_{\pi-\theta}^{\phi}\{\cos\,(\pi-\omega)-\cos\theta\}^{\kappa-\frac{1}{2}}\{\cos\omega-\cos\phi\}^{\mu-\frac{1}{2}}d\omega, \qquad . \quad \text{(xiii)}$$

Here let

$$\cos\omega=\cos\phi\,\sin^2\alpha-\cos\theta\,\cos^2\alpha;$$

then

$$\mathrm{K}=2(-\cos\phi-\cos\theta)^{\kappa+\mu}$$

$$\times\int_0^{\frac{\pi}{2}}\frac{2(\sin\alpha)^{2\kappa}(\cos\alpha)^{2\mu}d\alpha}{\sqrt{[\{1-\cos\phi\,\sin^2\alpha+\cos\theta\,\cos^2\alpha\}\{1+\cos\phi\,\sin^2\alpha-\cos\theta\,\cos^2\alpha\}]}}, \qquad \text{(xiv)}$$

From this form of K it is clear that, if $\kappa+\mu>0$, $\mathrm{K}\to0$ when $\theta+\phi\to\pi$; while, if $\kappa+\mu<0$, $\mathrm{K}\to\infty$ when $\theta+\phi\to\pi$. If $\kappa+\mu=0$ and $\phi=\pi-\theta$,

$$\mathrm{K}=\frac{2}{\sin\theta}\mathrm{B}(\tfrac{1}{2}-\mu,\ \tfrac{1}{2}+\mu);$$

and therefore, from (xii),

$$\int_0^{\infty}\cos\,(\lambda\pi)\mathrm{T}_{\lambda-\frac{1}{2}}^{\mu}(\cos\theta)\mathrm{T}_{\lambda-\frac{1}{2}}^{-\mu}(-\cos\theta)d\lambda=\frac{1}{4\sin\theta},$$

where $0<\theta<\pi$. This is formula (xxxiv) of the paper referred to in § 1.

§ 6. *Series involving Two Associated Legendre Functions.*—If $\kappa > -\frac{1}{2}$, $\mu > -\frac{1}{2}$, $0 < \theta \leqq \phi < \pi$,

$$(\sin\theta)^{\kappa}(\sin\phi)^{\mu}\sum_{n=-m}^{m}\cos\{(n+\gamma+\tfrac{1}{2})\rho\}\mathrm{T}_{n+\alpha}^{-\kappa}(\cos\theta)\mathrm{T}_{n+\beta}^{-\mu}(\cos\phi)$$

$$=\frac{1}{2\pi\Gamma(\kappa+\frac{1}{2})\Gamma(\mu+\frac{1}{2})}\int_0^{\theta}(\cos\psi-\cos\theta)^{\kappa-\frac{1}{2}}d\psi\int_0^{\phi}(\cos\chi-\cos\phi)^{\mu-\frac{1}{2}}$$

$$\times\left[\begin{array}{r}\cos\{(\alpha+\frac{1}{2})\psi+(\beta+\frac{1}{2})\chi+(\gamma+\frac{1}{2})\rho\}\dfrac{\sin\{(m+\frac{1}{2})(\rho+\psi+\chi)\}}{\sin\frac{1}{2}(\rho+\psi+\chi)}\\ +\cos\{(\alpha+\frac{1}{2})\psi-(\beta+\frac{1}{2})\chi+(\gamma+\frac{1}{2})\rho\}\dfrac{\sin\{(m+\frac{1}{2})(\rho+\psi-\chi)\}}{\sin\frac{1}{2}(\rho+\psi-\chi)}\\ +\cos\{-(\alpha+\frac{1}{2})\psi+(\beta+\frac{1}{2})\chi+(\gamma+\frac{1}{2})\rho\}\dfrac{\sin\{(m+\frac{1}{2})(\rho-\psi+\chi)\}}{\sin\frac{1}{2}(\rho-\psi+\chi)}\\ +\cos\{(\alpha+\frac{1}{2})\psi+(\beta+\frac{1}{2})\chi-(\gamma+\frac{1}{2})\rho\}\dfrac{\sin\{(m+\frac{1}{2})(\rho-\psi-\chi)\}}{\sin\frac{1}{2}(\rho-\psi-\chi)}\end{array}\right]d\chi. \quad \text{(A)}$$

In particular, if $\rho=0$ we see that when m tends to infinity

$$(\sin\theta)^{\kappa}(\sin\phi)^{\mu}\sum_{n=-\infty}^{\infty}\mathrm{T}_{n+\alpha}^{-\kappa}(\cos\theta)\mathrm{T}_{n+\beta}^{-\mu}(\cos\phi)$$

$$=\frac{2}{\Gamma(\kappa+\frac{1}{2})\Gamma(\mu+\frac{1}{2})}\int_0^{\theta}(\cos\omega-\cos\theta)^{\kappa-\frac{1}{2}}(\cos\omega-\cos\phi)^{\mu-\frac{1}{2}}\cos\{(\alpha-\beta)\omega\}d\omega, \quad \text{(xv)}$$

provided that $\kappa > -\frac{1}{2}$, $\kappa+\mu > 0$, $0 < \theta \leqq \phi < \pi$. The result also holds for $\kappa > -\frac{1}{2}$, $-1 < \kappa+\mu \leqq 0$, provided that $0 < \theta < \phi < \pi$.

Again, if $\rho=\pi$, it is clear that if $\theta+\phi<\pi$ the function in (A) tends to zero when m tends to infinity. If $\theta+\phi \geqq \pi$ the first and last terms in (A) lead to non-zero values. Thus, in the case when $\gamma=-\frac{1}{2}$ we have

$$(\sin\theta)^{\kappa}(\sin\phi)^{\mu}\sum_{n=-\infty}^{\infty}(-1)^n\mathrm{T}_{n+\alpha}^{-\kappa}(\cos\theta)\mathrm{T}_{n+\beta}^{-\mu}(\cos\phi)$$

$$=\frac{1}{\Gamma(\kappa+\frac{1}{2})\Gamma(\mu+\frac{1}{2})}\begin{cases}0, & \theta+\phi<\pi,\\ \frac{1}{2}\mathrm{K}', & \theta+\phi=\pi,\\ \mathrm{K}', & \theta+\phi>\pi,\end{cases} \quad \text{(xvi)}$$

where

$$\mathrm{K}'=2\int_{\pi-\theta}^{\phi}\{\cos(\pi-\omega)-\cos\theta\}^{\kappa-\frac{1}{2}}\{\cos\omega-\cos\phi\}^{\mu-\frac{1}{2}}$$

$$\times\cos\{(\alpha+\tfrac{1}{2})\pi+(\beta-\alpha)\omega\}d\omega, \quad \text{(xvii)}$$

It may be noted that if $\beta=\alpha$,

$$\mathrm{K}'=\cos\{(\alpha+\tfrac{1}{2})\pi\}.\mathrm{K}.$$

Most of the formulæ given in the previous paper referred to in § 1 can be easily derived from the formulæ contained in this paper.

(*Issued separately August* 1, 1935.)

IX.—The Lunar Atmospheric Pressure Inequalities at Glasgow.

By **R. A. Robb,** M.A., M.Sc., and **T. R. Tannahill,** M.A., B.Sc., University Observatory, Glasgow.

(MS. received April 18, 1935. Revised MS. received June 21, 1935. Read June 3, 1935.)

IN several papers by Chapman (1918, p. 271; 1919, p. 113, etc.) the effect of the moon on the atmospheric pressure has been analysed; the chief inequality observed is semi-diurnal, being, for example, 0·0120 sin $(2\theta+114^\circ)$ millibar at Greenwich, 0·083 sin $(2\theta+68^\circ)$ millibar at Batavia, and 0·060 sin $(2\theta+60^\circ)$ millibar at Hongkong; θ being measured from upper lunar transit.

A rough examination of the barometric records of Glasgow Observatory (latitude 55° 53′ N., longitude $17^{\mathrm{m}}\ 11^{\mathrm{s}}$ W.), extending from 1868 to 1912, showed, however, that a strong diurnal lunar inequality is present; and the present investigation has therefore been carried out in order to establish the possible significance of such a variation. These records consist of hourly readings of atmospheric pressure photographically registered according to the method described by Becker (1925, p. 57). Control readings of a standard barometer were taken five times daily—at 10 a.m., noon, 2 p.m., 6 p.m., and 10 p.m. Greenwich Mean Time.

The procedure adopted was in many ways similar to that given by Chapman (1918, p. 271) for the Greenwich data. In order to reduce accidental variations, which are larger for Glasgow than for Greenwich, the lunar upper transit for every day was marked on the sheets containing the hourly barograph readings, and then only those days were chosen whose variation of atmospheric pressure between two successive upper transits did not exceed 0·10 inch of mercury. The readings for all these days were then retabulated on new sheets, the first entry in any one row being the first occurring after lunar upper transit, and followed by the next 24 entries, thus making up the necessary 25 hours for a lunar day. In this way the break in the lunar day, introduced by Chapman in discussing the Greenwich data, was obviated. The retabulated figures were given to the nearest 0·001 inch as in the original barograph records. Here again the work could have been simplified if the data had been retabulated correct to the nearest 0·01 inch, but this procedure would have introduced accidental errors due to "forcing" (Whittaker and Robinson, p. 187). As an example of this error, the period 1883–1897, containing 1492 days, was examined, and it was found that "forcing" would have introduced the following errors to the means of the columns; the first column, which contains all entries immediately following lunar upper transit, being denoted by 0½ hours, and so on. The unit is 0·0001 millibar:

Hour	0½	1½	2½	3½	4½	5½	6½	7½	8½
Error	−33	+22	+5	+19	+18	+2	+5	+29	+10
Hour	9½	10½	11½	12½	13½	14½	15½	16½	17½
Error	−36	−22	−37	0	−19	+11	−3	+19	+67
Hour	18½	19½	20½	21½	22½	23½	24½		
Error	−18	−33	−55	+24	+20	+12	−9		

The theoretical standard deviation liable to occur in the mean of 1492 numbers, each abbreviated with forcing, is $\sigma = \cdot 002531$ millibar, and thus all the errors lie within $\pm 3\sigma$. The range of 122 units, from +67 to −55, would, however, to some extent have affected the semi-diurnal component, whose amplitude for this period is 132 units (Table III).

The Glasgow readings, covering a period of 45 years, from 1868 to 1912, were divided into nine groups, viz. 1868–1882, 1883–1897, 1898–1912, Winter months (November, December, January, February), Equinoctial months (March, April, September, October), Summer months (May, June, July, August), and days for which, at the initial lunar upper transit, the lunar semi-diameter was (*a*) $<$ 14′·99, (*b*) 15′·00 to 15′·99, (*c*) $>$ 16′·00. The totals obtained for the columns were checked by cross-addition of the rows. The results for the nine sub-groups and for the total data are given in Table I. The figures represent the totals divided by the number of days in the group. The totals were then corrected for any known variations with solar time that may, owing to the selection of data, have been only partially eliminated. The two variations that seemed likely to cause trouble were the solar variation and the vapour-pressure variation. These effects were eliminated by the use of Becker's Tables 2, 25 (pp. 87, 104), the corrections applied being those derived from the monthly curves (45-year normals). Regarding Table 2 (Becker, 1925, p. 87), this has not been corrected for "non-cyclic change"; but the investigation of the Glasgow data for this non-cyclic element showed that there was little rise or fall in pressure in the course of a solar day over the groups considered. For the seasonal groups, in which the non-cyclic element should appear, it was found that the greatest change, amounting to only 0·001 inch or 0·034 millibar per solar day, occurred during the Winter period. Hence, when the data is arranged according to lunar days, this will have extremely little effect on the lunar inequalities and can be neglected.

TABLE I.—UNCORRECTED HOURLY INEQUALITIES.

Unit ·0001 millibar.

Grouping.	1868–1882.	1883–1897.	1898–1912.	Winter.	Equinoxes.	Summer.	S.D. 14.	S.D. 15.	S.D. 16.	Total.
No. of days.	1347.	1492.	1451.	921.	1370.	1999.	1106.	1850.	1334.	4290.
Hour.										
0½	−1129	−498	−882	−1550	−846	−479	−683	−1011	−689	−826
1½	−842	−418	−660	−1290	−634	−330	−383	−704	−742	−633
2½	−682	−301	−431	−963	−448	−247	−232	−386	−766	−465
3½	−416	−258	−219	−709	−306	−95	+6	−128	−774	−294
4½	−200	−186	−24	−388	−129	−24	+113	+81	−641	−135
5½	−30	−156	+86	−168	−42	+31	+198	+143	−474	−35
6½	+130	−54	+150	−27	+143	+71	+244	+184	−223	+74
7½	+302	+75	+227	+23	+370	+162	+268	+227	+98	+198
8½	+481	+205	+270	+157	+569	+211	+313	+178	+502	+314
9½	+657	+266	+325	+151	+684	+338	+371	+132	+824	+409
10½	+803	+423	+436	+427	+813	+420	+416	+246	+1072	+547
11½	+878	+530	+490	+487	+934	+478	+449	+332	+1179	+626
12½	+856	+568	+599	+602	+925	+523	+436	+454	+1159	+669
13½	+711	+453	+558	+693	+755	+385	+332	+524	+829	+569
14½	+565	+427	+506	+685	+594	+347	+259	+612	+534	+497
15½	+481	+326	+488	+836	+339	+304	+213	+688	+250	+430
16½	+429	+249	+411	+816	+165	+284	+197	+622	+134	+360
17½	+307	+195	+205	+745	−1	+159	+127	+464	+2	+234
18½	+187	+120	+45	+696	−148	+28	−13	+253	+31	+115
19½	+72	−56	−50	+453	−171	−122	−170	+77	−10	−14
20½	−135	−127	−152	+326	−308	−236	−252	−149	−29	−138
21½	−432	−234	−321	−21	−488	−355	−267	−479	−162	−326
22½	−631	−352	−490	−275	−677	−453	−323	−678	−355	−486
23½	−1022	−492	−670	−654	−922	−610	−659	−819	−630	−719
24½	−1344	−700	−898	−1052	−1172	−792	−959	−865	−1122	−969

It may be added that there is a large probable error associated with the non-cyclic change which renders it of no significance whatever.

The hourly inequalities, corrected in the above manner for the effect of the solar variation and the vapour-pressure variation, are given in Table II.

In order to test the significance of the variation obtained for the different hours for any one group, the group 1883–1897, comprising 1492 days, was analysed by the variance method (Fisher, 1934, section 42). Table III gives the principal results. No grouping was employed to obtain the necessary sums of squares. Fisher's Table VI (Fisher, 1934, pp. 232–233) shows that the variation for the different hours is significant. In Table III it may be noted that the variance due to the days, *i.e.* to the mean daily pressure varying from day to day, is first eliminated from the total variance, and accidental variations unaccounted for have a variance

TABLE II.—CORRECTED HOURLY INEQUALITIES.

Unit ·0001 millibar.

Grouping.	1868–1882.	1883–1897.	1898–1912.	Winter.	Equinoxes.	Summer.	S.D. 14.	S.D. 15.	S.D. 16.	Total.
No. of days.	1347.	1492.	1451.	921.	1370.	1999.	1106.	1850.	1334.	4290.
Hour.										
0½	−1087	−578	−928	−1531	−984	−458	−674	−861	−1000	−856
1½	−760	−464	−713	−1223	−704	−330	−339	−686	−830	−641
2½	−569	−307	−497	−849	−461	−267	−183	−488	−631	−454
3½	−314	−228	−256	−606	−243	−122	+42	−289	−485	−264
4½	−111	−162	−59	−309	−48	−64	+140	−80	−337	−111
5½	−2	−144	+88	−156	+88	−34	+217	−1	−248	−21
6½	+113	−71	+198	−59	+288	−4	+256	+122	−133	+78
7½	+242	+22	+323	−19	+488	+89	+257	+278	+21	+193
8½	+376	+108	+398	+141	+625	+130	+289	+322	+248	+290
9½	+535	+134	+438	+148	+648	+266	+308	+337	+443	+363
10½	+659	+256	+503	+429	+692	+328	+285	+449	+640	+466
11½	+740	+378	+503	+522	+734	+402	+276	+491	+807	+534
12½	+741	+479	+563	+686	+722	+455	+266	+519	+956	+590
13½	+632	+433	+485	+771	+614	+325	+165	+461	+873	+513
14½	+536	+508	+429	+756	+576	+311	+149	+433	+851	+490
15½	+512	+487	+405	+842	+452	+306	+150	+436	+773	+467
16½	+500	+440	+365	+772	+374	+318	+198	+337	+763	+433
17½	+408	+385	+200	+659	+246	+234	+220	+201	+573	+330
18½	+279	+294	+74	+606	+64	+137	+135	+76	+474	+215
19½	+126	+69	−2	+354	−62	+13	−15	+24	+181	+63
20½	−108	−40	−95	+250	−297	−83	−81	−56	−112	−80
21½	−460	−227	−289	−100	−586	−242	−163	−293	−491	−321
22½	−654	−409	−487	−354	−830	−368	−288	−429	−816	−512
23½	−1030	−582	−698	−700	−1084	−570	−663	−586	−1088	−762
24½	−1302	−780	−944	−1034	−1310	−771	−951	−715	−1434	−999

as shown in the row "Remainder." It is this latter variance which is compared with that obtained from the hours in order that significance may or may not be established.

The significance of the lunar atmospheric inequality at Glasgow has thus been established for one sub-group. The results of the harmonic

TABLE III.

	Degrees of Freedom.	Sum of Squares.	Mean Square.	Standard Deviation. Unit 1 inch Hg.
Hours . .	24	·0400259	·0016677	·040838
Days . .	1491	3449·039	2·31324	
Remainder .	35784	37·2409	·0010407	·032260
Total . .	37299	3486·3201		

analysis of this variation are contained in Table IV; it is assumed that the inequalities can be represented by the series

$$\sum_{r=1}(a_r \cos r\theta + b_r \sin r\theta)$$

or by

$$\sum_{r=1} c_r \,.\, \sin (r\theta + \alpha_r).$$

The first three harmonics are given. It is to be noted that a correction $-7^\circ\cdot5r$ has been applied to each phase angle α_r to allow for the first entry on the lunar sheets corresponding to 0½ hour lunar time, and not to lunar transit.

TABLE IV.—CONSTANTS OF HARMONIC ANALYSIS.

Unit ·0001 millibar.

	Periods.			Seasons.			Semi-diameter.			Total.
	1868–1882.	1883–1897.	1898–1912.	Winter.	Equinoxes.	Summer.	<14′·99.	15′·00–15′·99.	>16′·00.	1868–1912.
a_1	−810	−500	−621	−849	−804	−427	−365	−572	−953	−638
b_1	+86	−62	+114	−255	+254	+38	+143	+64	−47	+44
c_1	814	504	631	887	843	429	393	576	955	640
α_1	269°	256°	273°	246°	280°	268°	284°	269°	260°	267°
a_2	−180	−67	−163	−249	−166	−59	−201	−124	−96	−136
b_2	+81	+113	+27	+73	+87	+77	+83	+20	+149	+76
c_2	198	132	165	259	188	97	218	126	177	156
α_2	280°	315°	265°	242°	283°	308°	278°	265°	313°	285°
a_3	−57	−36	−80	−159	−41	−39	−74	−68	−54	−64
b_3	+115	+51	+17	−17	+84	+79	+74	+1	+118	+62
c_3	128	62	82	160	94	88	105	68	130	89
α_3	312°	303°	260°	244°	312°	312°	293°	249°	314°	292°

Estimates of the probable errors of the harmonic coefficients, for the sub-group 1883–1897 and for the entire data 1868–1912, are given in Table V. The standard deviation found for the sub-group (Table III) has been divided by $\sqrt{3}$ in order to get the standard deviation for the total data.

TABLE V.—PROBABLE ERRORS OF HARMONIC COEFFICIENTS.

Unit ·0001 millibar.

	Period, 1883–1897.	Total, 1868–1912.
c_1, c_2, c_3	±55	±32
α_1	± 6°	± 3°
α_2	±24°	±12°
α_3	±51°	±21°

The striking feature of Table IV is the large first harmonic or diurnal component, with the maximum occurring at lower transit of the moon. This diurnal component cannot be considered as accidental. The second harmonic or semi-diurnal component for the complete period 1868–1912 is

$$0{\cdot}0156 \sin (2\theta + 285^\circ) \text{ millibar}$$

and is quite consistent throughout the various sub-groups. The tidal effect predicted by theory has, of course, a phase angle of 90°; and it would appear that this tidal effect, if present, has been obscured by some other effect. Professor Chapman has kindly pointed out that Bartel's "convex variation" effect for barometrically quiet days would probably explain the observed inequalities. It was found, however, that when this effect was eliminated by the method adopted for the Buenos Ayres data (Chapman and Austin, 1934, p. 23), the results obtained again deviated considerably from those predicted by theory.

SUMMARY.

The Glasgow barograph records have been retabulated according to lunar time, and the mean lunar diurnal inequalities for various groups of years and seasons analysed harmonically. The analysis shows that for the total period of 45 years the inequality can be represented by

$$0{\cdot}0640 \sin (\theta + 267^\circ) + 0{\cdot}0156 \sin (2\theta + 285^\circ) + 0{\cdot}0089 \sin (3\theta + 292^\circ),$$

the amplitude being measured in millibars and θ being reckoned from upper lunar transit.

REFERENCES TO LITERATURE.

BECKER, L., 1925. "The Climatology of Glasgow," *M.O. Geophys. Mem.*, No. 23.

CHAPMAN, S., 1918. "The Lunar Atmospheric Tide at Greenwich, 1854–1917," *Quart. Journ. Roy. Met. Soc.*, vol. xliv, pp. 271–279.

CHAPMAN, S., 1919. "The Lunar Tide in the Earth's Atmosphere," *ibid.*, vol. xlv, pp. 113–137.

CHAPMAN, S., and AUSTIN, M., 1934. "The Lunar Atmospheric Tide at Buenos Ayres, 1891–1910," *ibid.*, vol. lx, pp. 23–28.

FISHER, R. A., 1934. *Statistical Methods for Research Workers*, Edinburgh and London.

WHITTAKER, E. T., and ROBINSON, G., 1924. *The Calculus of Observations*, London.

(*Issued separately August* 1, 1935.)

X.—Charles Darwin as a Student in Edinburgh, 1825–1827. (An Address delivered on October 28, 1935.) By **Professor J. H. Ashworth,** F.R.S., Department of Zoology, University of Edinburgh. (With Three Plates.)

(MS. received October 28, 1935.)

CHARLES DARWIN in his *Autobiography*, written in 1876, gives in half a dozen pages (*Life and Letters*, vol. i, pp. 36–42) an account of his two academic sessions as a medical student in Edinburgh. This account includes brief references to his naturalist friends and acquaintances, a statement that he collected specimens in the tidal pools on the shore of the Firth of Forth and by going out with the "trawlers," and that he made new observations on the "so-called ova of Flustra," which "were in fact larvæ," and on the egg-cases of *Pontobdella muricata*.

It is now possible to form a more adequate conception of his early progress as a naturalist with the help of the minute-book of the Plinian Society of the University of Edinburgh and of a note-book begun by Darwin in Edinburgh in March 1827, the latter of which I have been permitted to see by the kindness of Professor C. G. Darwin and Mr Bernard Darwin. I wish, therefore, to bring before the Royal Society of Edinburgh, of which Darwin was elected an Honorary Fellow just seventy years ago, a brief account of his two academic sessions in Edinburgh with particular reference to the development of his early taste for natural history, more especially in his second session.

Charles Robert Darwin came to Edinburgh at the age of sixteen years and eight months, and with his elder brother Erasmus, who had been a medical student at the University in the previous year, signed the matriculation book, "Charles Darwin—Shropshire," on October 22, 1825. His first classes were on Wednesday, October 26—in Materia Medica at 8 a.m., in Chemistry at 10, and in Anatomy at 1 o'clock. His class-cards (Pl. I) for the academic year 1825–26 were presented to the University of Edinburgh by the late Sir Francis Darwin in 1909, together with the wrapper, inscribed by Darwin, in which they were found among his papers. The cards, with the names of the respective teachers, are:

For the University Library.

Materia Medica, Dietetics and Pharmacy	Andrew Duncan, jun
Chemistry and Pharmacy	Thos. Chas. Hope.
Anatomy, Physiology and Pathology	Alexander Monro.
Clinical Lectures	Dr Graham and Dr Alison.
Principles and Practice of Surgery	Alexr. Monro.

"Perpetual Ticket" for the Royal Infirmary.

In his *Autobiography* Darwin writes (p. 36) that the lectures "were intolerably dull, with the exception of those on chemistry by Hope. . . . Dr Duncan's lectures on Materia Medica at 8 o'clock on a winter's morning are something fearful to remember. Dr —— made his lectures on human anatomy as dull as he was himself." The "Dr" referred to was Alexander Monro *tertius* who did not sustain the great reputation made by his grandfather and his father, who had preceded him in the Chair of Anatomy. Darwin attended the Clinical Lectures and he records that he also "attended regularly the clinical wards in the hospital." He enrolled in the class of Principles and Practice of Surgery, but all that is known about his attendance in this subject is his own note (p. 37) that on two occasions he was present at "very bad operations,"—these were before the days of chloroform—and that he "rushed away before they were completed."

In his second year at the University[*] Darwin, who signed the matriculation-book on November 10, 1826, enrolled in the classes of Practice of Physic and Midwifery, as is shown by the list of members of these classes in the University records, and also in the class of Natural History, but the list of members of this class is wanting. His class-cards for this year were not preserved.

Darwin states his opinion (p. 36) that "there are no advantages and many disadvantages in lectures compared with reading." He excludes from his general condemnation for the dullness of their lectures only one of his teachers in Edinburgh, Dr Thomas Charles Hope, the Professor of Chemistry. Hope's lectures were noted for "uncommon clearness of exposition, and unexampled splendour and success in experimental demonstration" (Christison, *Life*, vol. i, p. 57), and as there were 503 students in this class in Darwin's year, the experiments, to be visible to the class, required to be performed on a large scale which called for great manipulative skill.

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[*] In the academic year 1825–26 there were in the University 2013 students in Medicine, Arts and Law, of whom 902 were in Medicine, and 250 of these came from England. In the year 1926–27 there were 1905 students, of whom 858 were in Medicine, and 215 of these came from England.

It is interesting to compare Darwin's strongly adverse opinion of two of his Professors in Edinburgh with the judgment of other gifted students of about the same period. Robert, afterwards Sir Robert, Christison, who was the distinguished occupant successively of the Chairs of Forensic Medicine and Materia Medica in the University, records (*Life*, vol. i, pp. 68, 69) that Alexander Monro *tertius*, whose class he attended in 1815–16, "was far from being a popular lecturer. In all he did and said his manner betrayed an unimpassioned indifference, as if it were all one to him whether his teaching was acceptable and accepted or not. . . . Nevertheless, Monro gave a very clear, precise, complete course of lectures on anatomy . . . and certainly I learned anatomy well under him." He "studied anatomy another winter under Dr John Barclay [the extra-academical Lecturer], as was then the universal student-fashion."

Other students were much less satisfied with Dr Monro's course. Robert Knox in 1811 had found Monro's teaching "of poor service" and in the following year joined Dr Barclay's course (see Lonsdale, p. 6). John Goodsir (*Anat. Mem.*, pp. 24, 25) "owed a great deal to Dr Knox [who had succeeded Dr Barclay] and always spoke of him as his anatomical teacher and friend." After studying for two years under Knox he enrolled, in 1832–33, in Monro's class. Goodsir's great friend Edward Forbes (*Memoir*, p. 138) also learnt anatomy from Knox, whose class he joined in 1831; he did not attend Monro's course. These and other records support Darwin's opinion of Monro *tertius* as an unsuccessful teacher.

Robert Jameson, whose course Darwin attended in his second year of study (1826–27), was at that time fifty-two years of age and had been for twenty-two years Professor of Natural History, which then included zoology and geology. He devoted himself chiefly to mineralogy, but his published work shows his interest also in marine zoology and in birds, and as Editor of the *Edinburgh Philosophical Journal* and of the *New Philosophical Journal* he maintained a wide outlook on science in general. He formed an extensive and important Natural History Museum in the University of Edinburgh which was notable for the excellent state of preservation of its specimens and their scientific arrangement and for its large collection of birds. The entire museum collection, said to have been in this country "second only to that of the British Museum," was handed over a year after Professor Jameson's death to the new Government Museum of Science and Art, now the Royal Scottish Museum.

Darwin states (p. 41) that he found Professor Jameson's lectures "incredibly dull"; "the sole effect they produced on me was the determination never as long as I lived to read a book on Geology, or in any way to study the science." Fortunately, he did not adhere to this decision; within ten years he had made the observations for his three geological memoirs, one of which, on coral reefs, has become a classic.

The detailed syllabus of Professor Jameson's lectures, as drawn up by him in 1826, is preserved, for it was placed before the Commissioners for the Universities of Scotland in October 1826 (Appendix to the *Evidence*, pp. 115–118). It shows the range of his teaching, which included not only zoology and geology, but also instruction in meteorology and hydrography and some references to botany in its relation to "the animal and mineral kingdoms." The course in zoology began with a consideration of the natural history of man, was followed by an account of the chief classes of vertebrates and invertebrates, and concluded with lectures on the philosophy of zoology in which the first subject was "Origin of the Species of Animals."

This would be the course as given in Darwin's second academic year, and the lectures, which began on November 8, 1826, were on five days per week and were stated to extend over five months, hence there would be about one hundred lectures, in addition to which there were "conversations" with the Professor in the Museum and excursions.

Christison, who attended the shorter course on Natural History in the summer of 1816, states (*Life*, vol. i, p. 90) that Professor Jameson's "lectures were numerously attended in spite of a dry manner, and although attendance on Natural History was not enforced for any University honour or for any profession. [Attendance on lectures in this subject was not required from medical students until 1833.] The popularity of his subject, his earnestness as a lecturer, his enthusiasm as an investigator, and the great museum he had collected for illustrating his teaching, were together the causes of his success."

Edward Forbes took Professor Jameson's course in the summer of 1832 and the fine Museum had great attractions for him. When he succeeded, in May 1854, to the Chair of Natural History he found "Jameson's collection wonderful, even palæontologically," and the amount of material for illustrating the course "very great" (*Memoir*, p. 556). He gratefully acknowledged (p. 554) the encouragement he had received from Jameson and spoke of his Professor's "enthusiastic zeal, his wonderful acquaintance with scientific literature," and he went on to say: "The value of professorial worth should chiefly be estimated by the number and excellence of disciples. A large share of the best naturalists of the day received their first instruction in the science . . . from Professor Jameson. . . . And where else in the British Empire except here, has there been for the last half century a school of Natural History?"

While Professor Jameson's teaching of some parts of geology was unacceptable owing to his adherence to extreme and discredited Wernerian views (see, for instance, Darwin, pp. 41, 42), the recorded judgments of Christison and of Forbes on his teaching in general, and on his enthusiasm for his subject and for the Museum under his charge, show that they would not have assented to Darwin's condemnation of Professor Jameson's course.

I have been unable to find any opinion recorded by a student of the work and teaching of the third Professor to whom Darwin refers, Dr Andrew Duncan *secundus*, who was appointed Professor of Materia Medica in 1821. Christison had graduated two years previously and Forbes, who during his first year of study (1831–32) attended Professor Duncan's class, left no comment upon it. The biographer of Forbes points out (*Memoir*, p. 139) that "he could not possibly profit at that stage of his studies" from such a course. As Darwin attended the class also in his first year he probably experienced a similar handicap. Principal Sir Alexander Grant (*Story*, vol. ii, pp. 424–425) refers to Dr Duncan as "a man of most versatile genius," whose success in teaching Materia Medica was "conspicuous," and states that "an early work of his, *The Edinburgh Dispensatory*, was for many years a standard authority in every medical school in Europe." This estimate of Dr Duncan and his work should be weighed against Darwin's unfavourable comment (p. 98).

Although Darwin apparently did not receive from Professor Jameson's lectures much instruction in zoology, he was able to acquire in other ways while in Edinburgh a considerable acquaintance with this subject.

Soon after coming to Edinburgh Darwin became aware that his father would leave him "property enough to subsist on with some comfort," and this was, as he states (p. 36), "sufficient to check any strenuous effort to learn medicine." Much of the time available after his attendance at classes would probably be devoted to his pursuit of natural history, especially during his second year when his brother was no longer with him in Edinburgh. His rooms were in a top flat in 21 Lothian Street (in which, about twelve years later, lived John and Harry Goodsir and Edward Forbes), less than two hundred yards from the entrance to the University. The Natural History Museum occupied the western block of the University, and consequently such time as was not employed in the three classes in which Darwin had enrolled could immediately be put to good use in the Museum, where he was likely to find at work two experienced naturalists, Dr Robert Grant and William Macgillivray, whom he came to know well. Further, his membership of the Plinian Natural History Society brought him into association with other enthusiastic naturalists in Edinburgh at that time.

The Plinian Society was founded in 1823 and the two minute-books from February 1826 to the end of the Society's existence in 1841 are preserved in the University Library. The Society, which met every Tuesday evening in the College or University of Edinburgh, had about one hundred and fifty members, but the number recorded as being present at the meetings does not usually exceed twenty-five. The Secretary in February 1826 was Dr Robert Grant and the record includes the names of those present and of those who took part in the discussions.

Darwin was elected a member of the Society on November 28, 1826, and at the meeting a week later, at which the election of Officers and Council took place, he was chosen as one of the five members of the Council, from which it may be concluded that he was very favourably known for his interest in natural history. There were five Presidents—Messrs Ainsworth, Coldstream, Kay, Browne, and Fife—three of whom, Browne, Coldstream, and Fife, had proposed Darwin for membership of the Society. He states in his *Autobiography* that he found the meetings of the Society stimulating and that he "used regularly to attend"; the record shows that he was present at all but one of the nineteen meetings held from the date of his election to April 3, 1827.

The minute-book is of interest as showing the subjects discussed in the students' natural history society in Edinburgh one hundred years ago, and the following notes on the proceedings indicate the range of the communications presented during the period of Darwin's membership. At the last five meetings in 1826 the papers included: the alleged oviposition of the cuckoo in the nests of other birds, extra-uterine gestation, the results of the analysis of the Cheltenham waters, oceanic and atmospheric currents, the anatomy of expression, the various purposes to which the formation of a vacuum is applied in the animal kingdom, instinct, and the natural history of the cuckoo. At the meeting on December 26, the members of the Society blackballed Cuvier and Blumenbach, two of the foremost zoologists of that time, whose nomination as Honorary Members had been "signed by all the office-bearers." In January 1827 the subjects before the Society included the mode of obtaining bromine from soap-boilers' waste and on its nature and properties, "the sap-vessels of the Solanum tuberosum, founded on the commencement of a series of experiments in this department of physiological botany" (by Allen Thomson, afterwards Professor of Anatomy in the University of Glasgow), on the principles of natural classification which included some consideration of specific characters (by William F. Ainsworth), on the sea-serpent and the kraken, and by Henry Cheyne on the capture of whales on the coast of the Shetlands, which he had witnessed; 199 whales were taken, the largest of which was 24 ft. 8 in. long. In February the papers were on a peculiar variety in the shape of the leaf of Laurus nobilis, on a single-edged stein barte (stone broad-axe) from the Shetlands, on intermittent and reciprocating springs and the temperature of springs, on the theory of expansion, liquefaction, and vaporization, and on instinct. At the first meeting in March were read a further paper on the temperature of springs, a paper on the variations of the barometer, and one by Henry Cheyne on a milky appearance, "resembling thick whey," of the sea off the Shetlands during a period of eighteen or twenty days in the summer of 1826. Mr Coldstream attributed the appearance to "the existence of luminous animalculæ in the water." At the next meeting (March 14) the Secretary directed attention to a paper in the *Philosophical Transactions*, vol. lxii, 1772, "which tended to corroborate the appearance in the north seas," after which Allen Thomson described the circulation of the sap in vegetables. The meeting of March 20 was devoted to papers on the comparative effects of classical and scientific pursuits and on the determinate form of clouds.

Darwin is noted as having participated in the discussions on four of the evenings; it would have been particularly interesting to know what he said on the principles of natural classification, and especially on specific characters, when he spoke in the discussion on that subject.

His communication to the Plinian Society was on March 27, 1827, not "at the beginning of the year 1826" as stated in the *Autobiography* (p. 39). The record in the minute-book under that date is in the following terms:—

"Mr Darwin communicated to the Society two discoveries which he had made:

1. That the ova of the Flustra possess organs of motion.
2. That the small black globular body hitherto mistaken for the young Fucus lorius is in reality the ovum of the Pontobdella muricata.

At the request of the Society he promised to draw up an account of the facts and to lay it, together with the specimens, before the Society next evening.

Dr Grant detailed a number of facts regarding the Natural History of the Flustra."

The minute of the next meeting, on April 3, 1827, includes the record of the presentation to the Society of

"A specimen of the Pontobdella muricata, with its ova & young ones } by Mr Darwin";

but there is no mention of any account of these or of the "ova" of Flustra, and if his manuscript was placed before the Society it has not been kept.

Darwin wrote his communication in his note-book under the date April 20, three weeks after the meeting at which it was made; the account was either a "fair copy" of the manuscript or an extension of the notes he would probably have prepared for the meeting. It consists of about four and a half pages (each page 8 by 5 in.) from which the following is abstracted:—

"Having procured some specimens of the Flustra carbasea * (Lam.) [now *Carbasea carbasea* (Ellis and Sol.)] from the dredge boats at Newhaven, I soon perceived without the aid of a microscope small yellow bodies studded in different directions on it. They were of an oval shape and of the colour of the yolk of an egg, each occupying one cell. Whilst in their cells I could perceive no motion, but . . . in a watch-glass [presumably after liberation from their cells] . . . they glided to and fro with so rapid a motion as at some distance to be distinctly visible to the naked eye. When highly magnified, the cilia, which were chiefly distributed on the broader end, were seen in rapid motion; the central ones being the longest. I may mention that I have also observed ova † of the Flustra Foliacea & Truncata [now *F. securifrons* (Pallas)] in motion. That such ova had organs of motion does not appear to have been hitherto observed either by Lamarck, Cuvier, Lamouroux, or any other author. . . ."

"This and the following communication was read both before the Wernerian & Plinian Societies."

The note-book does not state the date on which Darwin first collected or observed the "ova," or as he wrote in the *Autobiography*, the "egg-cases" of Pontobdella, but immediately following the above quotation continues (Pl. II):—

"One frequently finds sticking to oyster and other old shells, small black globular bodies which the fishermen call great Pepper-corns. These have hitherto been always mistaken for the young Fucus Lorius

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* Flustra, the "sea-mat," a genus of the Polyzoa, is obtained by dredging or is found thrown up on the shore.

† These "ova," as Darwin notes in his *Autobiography*, "were in fact larvæ"; the term "ova" is used in this sense elsewhere in the note-book.

[loreus; now *Himanthalia lorea* *] to which it bears a great resemblance. Having opened some of these they at first appeared only to contain an extremely viscid fluid without any traces [of] organisation [the microscopic egg in this viscid fluid was not observed]; but on examining some others I found that this fluid, acquiring by degrees a vermicular shape, when matured was the young Pontobdella † Muricata (Lam.) which were in every respect perfect & in motion. Each ovum consisted of two parts, the outer capsule being cori[a]ceous & of considerable thickness, whilst the inner consisted merely of a thin black membrane. This bag never contained more than one animal. . . . At each end [side] there is a prominent orifice, which appears to be the outlet of [for] the young animal, but hitherto mistaken for the branches of the Fucus. . . ."

Darwin did not publish these observations.

Three days previous to Darwin's paper at the Plinian Society, that is on March 24, 1827, Dr Grant read to the Wernerian Natural History Society of Edinburgh, as the minute under that date records (*Mem.*, vol. vi, 1832, p. 564), "a Memoir regarding the Anatomy and Mode of Generation of Flustræ, illustrated by preparations and drawings. The Doctor likewise read a notice on the Existence of Ciliæ in the young of the Buccinum undatum, Purpura Lapillus, and some other molluscous animals; and also on the Mode of Generation of the Pontobdella muricata of Lamarck."

Grant's paper "on the Structure and Nature of Flustræ," which appeared in two parts in the *Edinburgh New Philosophical Journal*, vol. iii, April and October 1827, contains a more detailed account than Darwin's of the ova and of the ciliated embryos and larvæ, which Grant reared in watch-glasses into young colonies.

Grant prepared a description of the ova [or cocoons, as they should be called] of Pontobdella, published in the *Edinburgh Journal of Science*, vol. iii, July 1927, pp. 160, 161, in which he stated "the merit of having first ascertained them to belong to that animal is due to my zealous young friend Mr Charles Darwin of Shrewsbury, who kindly presented me with specimens of the ova exhibiting the animal in different stages of maturity."

Unknown to Darwin and Grant another Edinburgh naturalist, John Graham Dalyell, had observed the laying of the "eggs or capsules" of this leech, had studied their structure, and had seen the young leech as it issued from the capsule. "Beautiful drawings of the ova and young, made in July 1823 and bearing that date, are now before us" wrote the editor (Professor Jameson) of the *Edinburgh New Philosophical Journal*, vol. iii, October 1827, pp. 391, 392, who printed Dalyell's notes which accompanied the drawings. By the courtesy of Sir James Dalyell, Bart., I have recently seen these original drawings of the capsules, dated "July 1823," and of two young Pontobdella and also later drawings of this and other genera of leeches, which were published as Pl. I in the second volume (1853) of Dalyell's well-known work *The Powers of the Creator*.

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* The young stages of this brown seaweed form stalked, subspherical, and later flattened, "buttons" from the centre of which sprout the forked throng-like branches or receptacles.

† As this leech is frequently found on the skate it is often called by fishermen the skate-leech.

Another significant factor in Darwin's studies in natural history in Edinburgh was his friendship with some of the Newhaven fishermen, whom he sometimes accompanied when they dredged for oysters "and thus got many specimens." His note-book records two "sea-pens," *Pennatula mirabilis* (now *Virgularia mirabilis*) and *Pennatula phosphorea*, from the Firth of Forth, which were no doubt obtained in that way (see p. 108); the cocoons of Pontobdella which he found on oyster and other shells were probably dredged on some of these expeditions, and he procured his specimens of *Flustra carbasea* "from the dredge boats at Newhaven."

Darwin's note-book, which is inscribed (Pl. III) on the fly-leaf "Charles Robert Darwin, March 1827," provides interesting evidence of his activities as a collector and careful observer of marine animals. The first record in the book, under the date March 16, 1827, shows that he "Procured from the black rocks at Leith a large Cyclopterus Lumpus (common lump fish), length . . . 23½ inches, girth 19½. It had evidently come to the rocks to spawn and was there left stranded by the tide; its ovaria contained a great mass of spawn of a rose colour. Dissected it with Dr Grant. It appeared very free from disease and had no intestinal worms." A fairly complete examination was evidently made, for Darwin remarks on the small eyes, large stomach, liver without gall-bladder, kidneys situated some way from the vertebrae, air-bladder not seen, brain very small, optic nerves nearly as large as the spinal cord, valves in the heart very distinct, the body not covered with scales but slimy and [the skin] remarkably thick, and the sucker on the breast of a white colour ("I believe it is generally a reddish yellow"). "The plebs differ whether it is edible."

Under the same date he added: "Procured a small green A[e]olis & a Tritonia" and "Examined the ova of the Purpura Lapillus and found them out of their capsules and of this shape" [sketch appended].

Two days later (18th) he sketched three organisms [apparently calyptoblastic hydroids, one probably a Campanularia] "found growing out of [on] Alcyonium, and noted that "Some ova [see f.n. †, p. 104] from the Newhaven rocks, said to be that of the Doris, was in every respect similar to that of the Univalves & in rapid motion & continued so for 7 days."

On March 19 he "Observed ova in the Flustra Foliacea & Truncata, the former of which were in motion," and on the 28th (Pl. III) he wrote notes on "whitish circular masses of ova" which he had found on Fucus, and appended sketches of them and of the "capsules" magnified, showing the ciliated embryos. He observed "the animal & its cilia in most rapid movement." "To what animal these ova belong, I am ignorant." [They were probably those of *Lacuna vincta* (Montagu).] He noted and sketched "another mass of ova, larger and of a browner colour, the capsules also being considerably larger" [this belonged almost certainly to *Littorina littoralis* (L.)], and on the same date he "Found some ova (I believe that of the Doris Argo) resembling a piece of tape." He added sketches and noted that the "capsules had the *appearance* of being united & of an oblong shape" [two are shown so united] and "were arranged in regular rows." In these respects the spawn agrees with that of *Archidoris britannica* (Johnston)—the modern designation of Doris Argo, a species which is still common on the shore of the Firth of Forth, east of Edinburgh. The ribbon of spawn of this species is laid in a spiral coil, but Darwin did not refer to this in his note.

On April 15 Darwin noted and sketched another egg-mass on Fucus, and also one of the ciliated embryos, "on which at its anterior end are situated numerous cilia; these by their constant motion cause the young animal to turn rapidly round within the capsule, and when freed from its capsule to move to and fro in the water with the greatest ease. I kept some of these ova in a bottle of the same water and at the end of 30 days [they] were yet in motion." This egg-mass was produced by *Lacuna pallidula* (da Costa). *L. pallidula*, *L. vincta*, and *Littorina littoralis*, all relatives of the periwinkle, are common on the shore of the Forth. For the identification of their egg-masses, from the descriptions in Darwin's note-book, I am indebted to Dr Marie Lebour and Mr G. A. Steven of the Marine Laboratory, Plymouth.

About this time Grant was examining the ciliated young of gasteropodous mollusca on which he read a paper before the Wernerian Society on the 24th March (see above, p. 105); this was published in the April issue of the *Edinburgh Journal of Science* (vol. vii, 1827, pp. 121–125). He apparently did not examine the egg-masses of *Littorina littoralis* or of the two species of Lacuna observed by Darwin.

Under the date April 20, Darwin wrote his observations on the "ova" of Flustra and of Pontobdella (pp. 104, 105) after which he continued: "Procured on 15th of April, from deep water in Frith of Forth, a good many specimens of the Pennatula Mirabilis [now *Virgularia mirabilis* (O. F. Müller)]. The Polypes were situated on one side of the bony axis in alternate lunate fillets [leaves] in number from 70–80. Each fillet consisted of about 12 Polypes. Towards one end of many of the longest specimens a yellowish appearance, which upon examination turned out to be numerous ova. . . . But I could perceive no motion."

Three days later he wrote: "Procured from the Frith of Forth numerous specimens of the Pennatula Phosphorea, differs from the Mirabilis in there being only 20 rows or fillets of polypes & in these rows about 12 Polypes. These fillets are opposite & decrease in size towards both ends of the axis. The bony axis does not appear at either end of the Zoophite as it does in the Mirabilis [specimens of *V. mirabilis* are often damaged by the dredge at the ends and the axis thus exposed] but gradually tapers into two filiform soft and flexible points embedded in the fleshy matter. Enclosed in & at the base of the fillets were several ova of a large size & of a yellow colour. Could perceive no motion. . . . The whole Zoophite has a soft slimy feel, & [is] of a red colour. The long axis in both of the above-mentioned species effervesced rapidly with nitric acid."

The last zoological entry Darwin made in Edinburgh in his note-book is that he "Observed, with Mr Coldstream at the black rocks at Leith, an Asterias rubens doubled up as it were, so that the disk part formed [a] pouch in which were numerous loose ova, which the animal was in [the] act of discharging from its mouth. The double [?] tract in each of the animal's limbs contained a mass of ova of a small size & of an orange colour."

This starfish was almost certainly a specimen of *Leptasterias mülleri* (M. Sars), which closely resembles *Asterias rubens* and was first differentiated from *rubens* as a new genus and species by Michael Sars (1846). In *L. mülleri* the genital pores are ventral and difficult to see, hence the eggs might appear to issue from the mouth. This starfish incubates its eggs in the space enclosed by arching the disc and bending inwards the bases of the arms.

Observations follow on the Geranium—pollen and anthers, epidermis of petals, stamens, anthers, pistil and sepals, and on the pollen grains of *Orchis morio*. A slight difference in the writing and the inclusion of the note on the pollen of *Orchis morio*, a species which does not occur in Scotland but is common in parts of England, indicate that the botanical notes were not written in Edinburgh.

As the last zoological entry in the note-book is under the date April 23, 1827, it may be inferred that Darwin left Edinburgh soon afterwards.

With Darwin's Edinburgh notes were two lists of animals, one of "the Vermes found in the Frith of Forth and other parts of Scotland," abstracted from a paper by Professor Jameson published in 1811 (*Mem. Wernerian Nat. Hist. Soc.*, vol. i, pp. 556–565), the other of "Fishes found in Frith of Forth" from a paper by Patrick Neill of the same year (*op. cit.*, pp. 526–555). Each list was written on two double sheets, about 6 by 3½ inches, and the two sheets were fastened together by a strip of gummed paper. These lists could be carried in a pocket-book and would be convenient for reference on his collecting expeditions.

Darwin wrote a short description of a "Zoological walk" to Portobello, on a Saturday during the winter, probably of 1826–27, with Mr William Kay, one of the Presidents of the Plinian Society. The walk "was a complete failure," largely owing to the "dense and impenetrable mist." When the two naturalists reached Portobello, "the shore appeared perfectly destitute and void of everything that could interest the Zoologist," and they decided to proceed to the rocks a mile or two beyond Portobello, but they found them covered by the tide. They retraced their footsteps to Edinburgh with no specimens except a few common shells.

Darwin left Edinburgh late in April 1827 when little more than eighteen years of age, and early in the following year went to Cambridge.

Reference should be made to Darwin's friends in Edinburgh, of whom the most important was undoubtedly Dr Robert Edmund Grant (1793–1874), then about thirty-three years old. He had graduated M.D. Edinburgh in 1814, and had studied natural history and medicine from 1815 to 1820 in Paris and other continental universities. He returned to Edinburgh in 1820 and explored the coasts of Scotland and Ireland and made studies, including dissections, of many animals which he collected. He gave lectures on the comparative anatomy of invertebrates for Dr John Barclay, who was extra-mural Lecturer in anatomy in Edinburgh at that time. Grant was the author of a series of papers on sponges in 1825 and 1826, and many of the observations therein recorded were made on the shore of the Firth of Forth. In addition, he published or wrote, in the years 1825–27, more than a dozen other papers on cœlenterates, molluscs, polyzoa, crustacea, on the structure of the eye of the sword-fish, and on the anatomy of the paca of Brazil; he was at that period a most energetic collector and investigator, especially of marine invertebrates. Darwin states that he often accompanied Grant to collect animals in the tidal pools, and he records in his note-book that he dissected with Grant the Lumpsucker he had found on the Black Rocks at Leith.

It was a great advantage to Darwin to have this experienced zoologist as his friend.

In his *Autobiography* Darwin relates (p. 38) that one day when he was walking with Grant the latter "burst forth in high admiration of Lamarck and his views on evolution"; Darwin adds that he listened in silent astonishment, and, so far as he could judge, without any effect on his mind. That exposition on Lamarck acquires fuller significance when it is remembered that Grant studied in Paris during the period from 1815 to 1820, and that he would devote much of his time to work in the Museum of Natural History, where he would very probably come into association with Lamarck only a few years after the latter had published his *Philosophie Zoologique* (1809) and while he was preparing his great work on the natural history of invertebrates (1815–1822).

In the summer of 1827 Grant left Edinburgh to become the first Professor of Zoology in University College, London.

William Macgillivray (1796–1852), thirteen years Darwin's senior, was, during the time Darwin was a student in Edinburgh, Assistant and Secretary to Professor Jameson and acted as Assistant-Keeper of the University Museum of Natural History. He was known for his papers on molluscs and on birds, and was afterwards (1841) appointed Professor of Natural History in Aberdeen. With him Darwin "had much interesting natural-history talk," and, while birds are not specifically mentioned, it is scarcely possible that Darwin could be in frequent association with Macgillivray—whom the late Professor Alfred Newton (1896) regarded, after Willughby, as "the greatest and most original ornithological genius save one (who did not live long enough to make his powers widely known) that this island has produced"—without utilising such an opportunity of acquiring some of his special knowledge of the classification and structure of birds. Darwin records that in Edinburgh he had lessons on preparing birds from a negro who had travelled with Waterton, and there is extant evidence of his interest in birds at this time in two "keys" in manuscript found among his Edinburgh notes. One is a key to the genera of British birds and the other, to about one hundred genera of birds, was "copied from Brisson's *Ornithologie*, 4th Edit. . . . 1826," hence it was written during his second academic session.

Darwin states (pp. 37, 38) that he "became well acquainted with several young men fond of natural science," and he mentions Ainsworth, Coldstream, and Hardie (see pp. 111, 112). To these he might have added Kay, with whom he made the zoological excursion to Portobello (p. 109), and also Browne and Fife who, with Coldstream, proposed him for membership of the Plinian Society. During his last five months in

Edinburgh he frequently met these men at the Plinian Society. Five of them—Ainsworth, Coldstream, Kay, Browne, and Fife—were elected Presidents of the Society early in December 1826, when Darwin was chosen as a member of the Council. They were all senior to Darwin by two to four years; Browne was already L.R.C.S., and the others qualified in 1827. Brief notes of the early part of their respective careers follow.

William Francis Ainsworth (1807–1896), who is entered in the roll of the class of Anatomy as from Lancashire, became L.R.C.S. in 1827, studied geology in London and Paris for two years, and then took up medical work, chiefly in relation to cholera, in England and Ireland. He went, in 1835, as Surgeon and Geologist to the Euphrates Expedition, and published an account of his researches, including his observations in zoology.

John Coldstream (1806–1863) was born in Leith, graduated M.D. in 1827, studied in Paris for about a year and returned to practise in Leith and later in Edinburgh. He was interested particularly in marine zoology and afterwards wrote articles for Todd's *Cyclopedia of Anatomy and Physiology* on medusæ (1835), on cirripedes (1836), on animal luminousness (1841), and papers on the eggs of Sepia (1833) and on Limnoria (1834).

William Kay (1807–1861), who came from Liverpool, graduated M.D. in 1827, practised in Clifton, and was Lecturer in Forensic Medicine in the Bristol Medical School.

William Alexander Francis Browne (1805–1885) was born in Stirling and became L.R.C.S. in 1826. He attended the University classes of Chemistry and Practice of Physic in 1826–27, after which he visited the asylums of Paris and returned to Stirling to practise. He was appointed in 1834 Medical Superintendent of the Royal Asylum at Montrose and five years later the first Superintendent of the newly founded Crichton Institution at Dumfries. In 1857 he became the first Commissioner in Lunacy for Scotland.

George Fife (1807–1857), who came from Newcastle-on-Tyne, graduated M.D. in 1827 and returned to Newcastle, where he practised, and was the first Lecturer in Medical Jurisprudence and Materia Medica in the Medical School.

Darwin writes (p. 38) of Hardie as one "who would, I think, have made a good botanist, but died early in India." I have searched in the minute-book of the Plinian Society and in the class-records of the University but have failed to find any student of medicine named Hardie in the years 1825 to 1827. Included with Darwin in the list of members of the

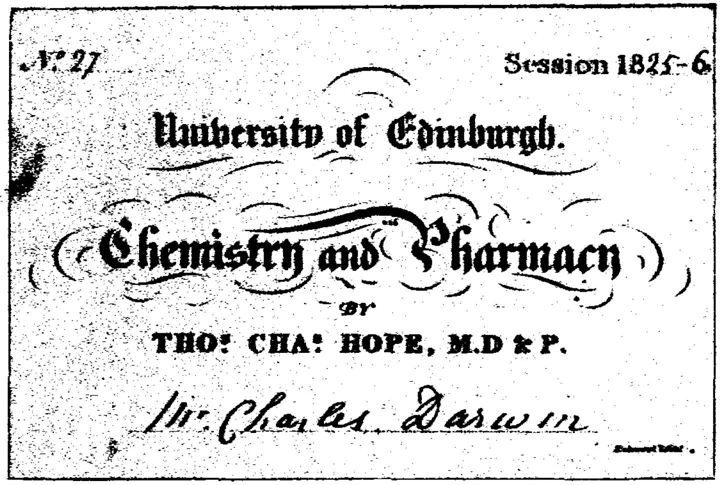
Council of the Plinian Society elected in December 1826 is Arding, who had become a member of the Society in June 1825 and is recorded as attending its meetings until the end of July 1827, after which, as he had then graduated, he left Edinburgh. No doubt this was the man Darwin had in mind.

Willoughby Arding (1805–1879) came from Wallingford, Berks, gained the gold medal in the class of Botany in 1826 for a collection of plants, graduated M.D. in 1827, and two years later became Assistant-Surgeon on the Bombay Medical Establishment. He resigned after about ten years' service and returned to Wallingford where he practised. He died there in 1879.

Darwin attended, with Grant, meetings of the Wernerian Natural History Society and heard, among others, Audubon, who spoke at the meetings in December 1826 and January and February 1827, and he was present at one meeting of the Royal Society of Edinburgh where he saw Walter Scott in the Chair as President.

I suggest that if, with the information now available, Darwin's experiences in Edinburgh are reviewed, there are good grounds for the conclusion that, while he gained comparatively little from the formal lectures, he had excellent opportunities, of which he appears to have fully availed himself, of developing his early taste for natural history and collecting, and that in Edinburgh he laid the foundation of his knowledge of the science of natural history.

Hitherto little has been known of Darwin's early work in zoology except his own brief accounts of his studies in Edinburgh and of his interest while in Cambridge in the collection of beetles. His note-book and other papers, written during his residence as a student in Edinburgh, show that he collected a number of marine animals and examined them carefully, including microscopic examination of the larvæ of some of them, that he dissected—although he regarded himself as deficient in this art, that he was interested in birds and in the use of "keys," and that he studied the records and literature of the animals he had collected. Though the results as recorded in his note-book during the last five and a half weeks of his residence in Edinburgh are perhaps slight, as judged by modern standards, it is clear that Darwin had acquired the methods of collecting and of identification of specimens and the faculty of careful observation and interpretation which he applied and developed with such remarkable success when the great opportunity came to him as Naturalist on H.M.S. *Beagle*



No. 27 Session 1825-6

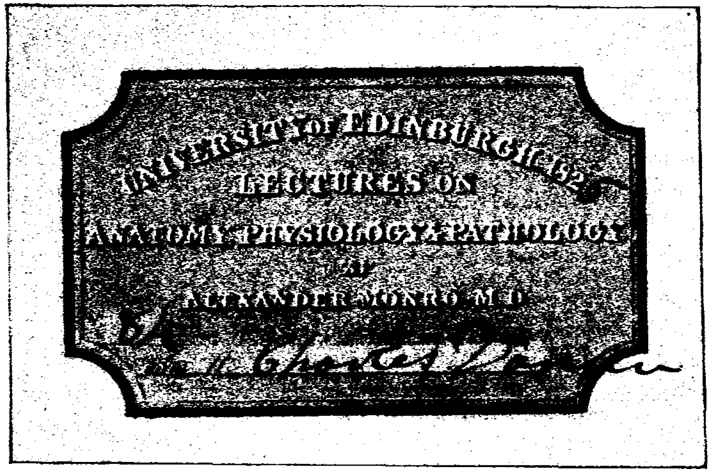
University of Edinburgh.

Chemistry and Pharmacy

BY

THO: CHA: HOPE, M.D & P.

Mr Charles Darwin



UNIVERSITY of EDINBURGH 1826

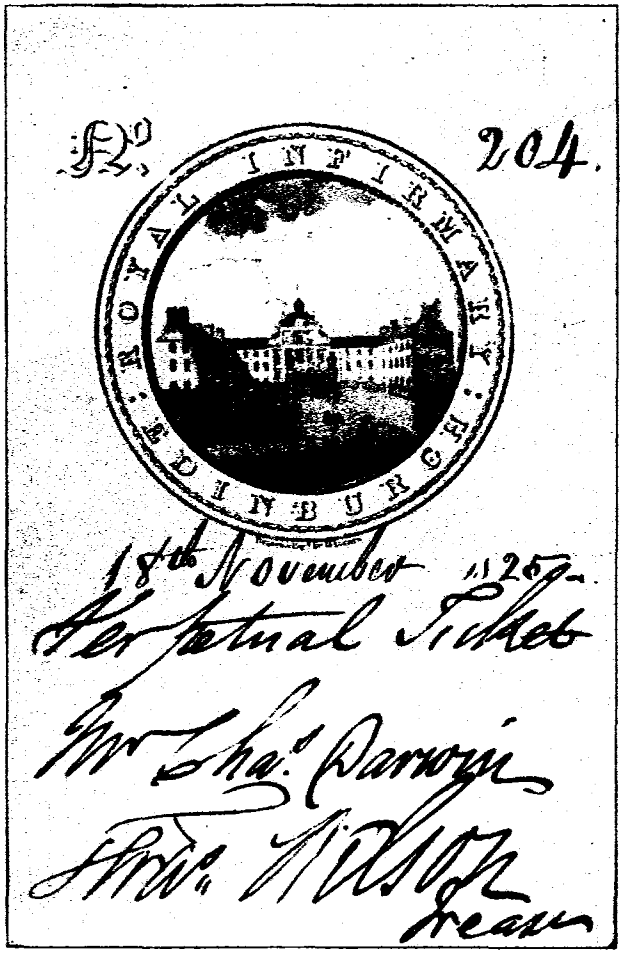
LECTURES ON

ANATOMY PHYSIOLOGY & PATHOLOGY

BY

ALEXANDER MONRO M.D

Mr Charles Darwin



No 204.

ROYAL INFIRMARY: EDINBURGH:

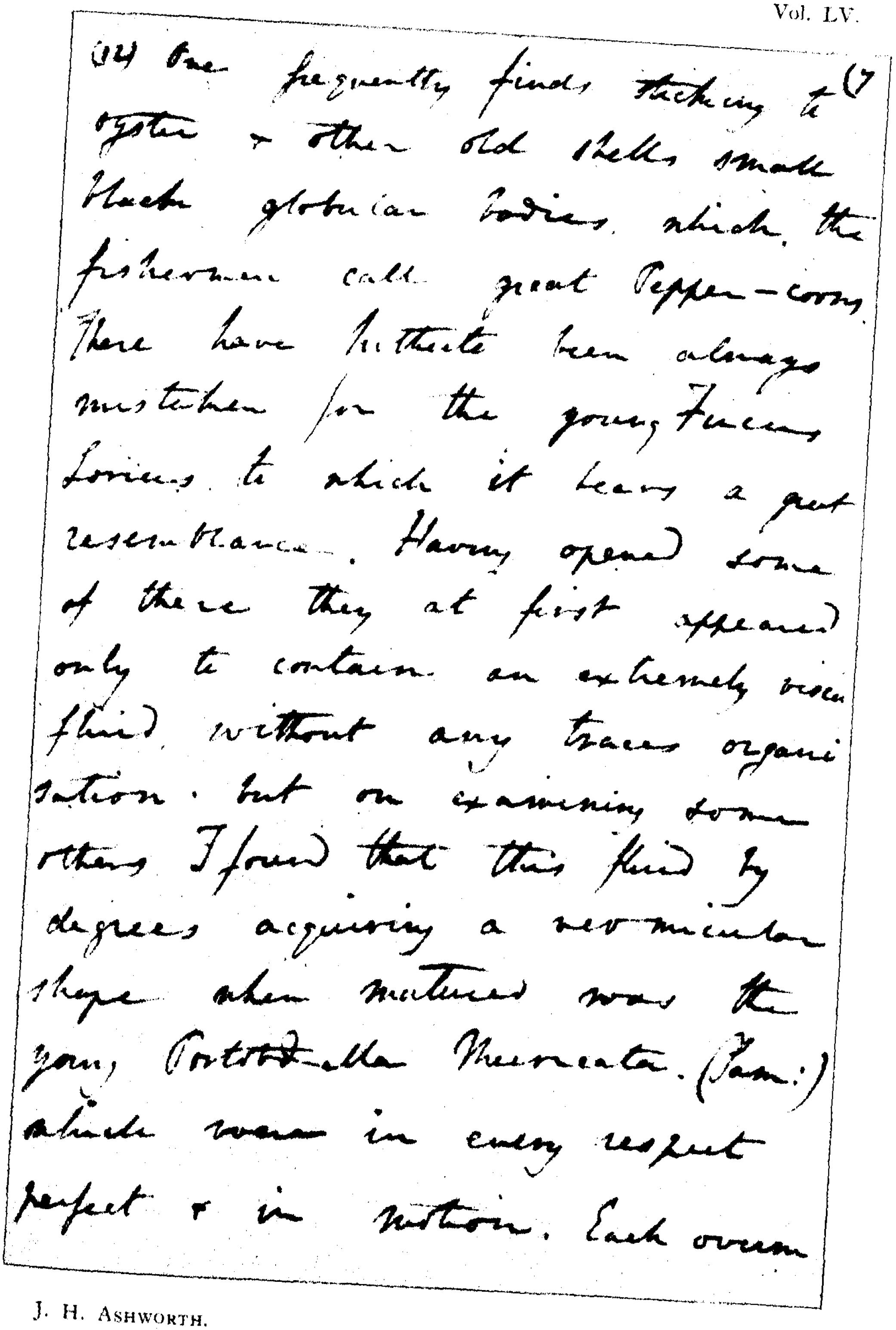
18th November 1825.

Perpetual Ticket

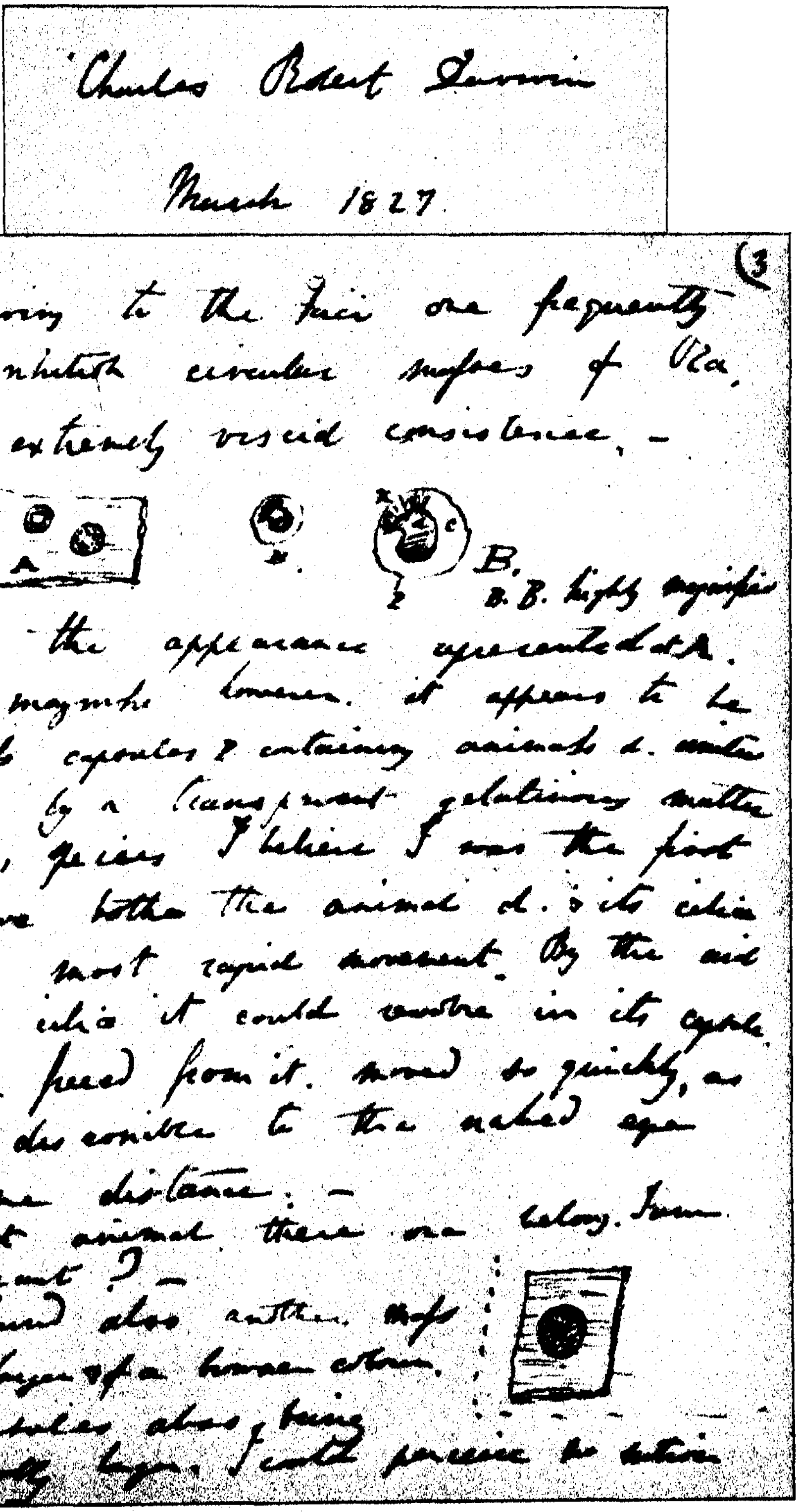
Mr Chas Darwin

Andw Wilson

Treasr



(14) One frequently finds sticking to (7 oyster & other old shells small black globular bodies, which the fishermen call great Pepper-corns. These have hitherto been always mistaken for the young Fucus loreus to which it bears a great resemblance. Having opened some of these they at first appeared only to contain an extremely visc. fluid, without any traces organisation: but on examining some others I found that this fluid by degrees acquiring a vermicular shape when matured was the young Pontobdella Muricata. (Lam:) which was in every respect perfect & in motion. Each ovum



Charles Robert Darwin

March 1827.

28th. (3

(1) Adhering to the Fuci one frequently finds whitish circular masses of Ova, of an extremely viscid consistence. —

B.B. highly magnified & have the appearance represented at A. when magnified however it appears to be a mass of capsules each containing animals which are united together by a transparent gelatinous matter. In this species I believe I was the first to observe both the animal & its cilia in most rapid movement. By the aid of these cilia it could revolve in its capsule & when freed from it moved so quickly as to be discernible to the naked eye at some distance. —

To what animal these ova belong I am ignorant? —

(2) I found also another mass from a larger of a horse [illegible]. the capsules also being considerably larger. I could perceive no cilia

REFERENCES TO LITERATURE.

CHRISTISON, ROBERT, 1885. *The Life of Sir Robert Christison, Bart.* Edited by his Sons. Vol. i, *Autobiography*. Edinburgh and London.

Commissioners for the Universities of Scotland, 1826 *and* 1830. *Evidence.* Vol. i, *University of Edinburgh*. Appendix, pp. 115–118. London, 1837.

DARWIN, CHARLES, 1887. *The Life and Letters of Charles Darwin, including an Autobiographical Chapter.* Edited by his Son, Francis Darwin. Vol. i. London.

DALYELL, JOHN GRAHAM, 1853. *The Powers of the Creator*, vol. ii, pp. 3–9, pl. i. London.

GOODSIR, JOHN, 1868. *The Anatomical Memoirs of John Goodsir.* Edited by William Turner, with a Biographical Memoir by Henry Lonsdale. Vol. i. Edinburgh.

GRANT, Sir ALEXANDER, Bart., 1884. *The Story of the University of Edinburgh during its First Three Hundred Years.* Two vols. London.

LONSDALE, HENRY, 1870. *A Sketch of the Life and Writings of Robert Knox, the Anatomist.* London.

NEWTON, ALFRED, 1896. *A Dictionary of Birds*, vol. i, *Introduction*, footnote, p. 60. London.

SARS, M., 1846. *Fauna littoralis Norvegiæ.* Erstes Heft, pp. 56–58. Christiania.

WILSON, GEORGE, and GEIKIE, ARCHIBALD, 1861. *Memoir of Edward Forbes.* Edinburgh.

DESCRIPTION OF PLATES.

PLATE I.

Two of Darwin's class-cards, reduced to about seven-eighths their actual size, and his Perpetual Ticket for the old Royal Infirmary. The originals are in the Library of the University of Edinburgh.

PLATE II.

Page 7 from Darwin's note-book, containing the first part of the description of the cocoons of *Pontobdella muricata* (see pp. 104, 105).

PLATE III.

Signature and date from the fly-leaf of Darwin's note-book and a portion of p. 3, dated March 28 (see p. 107). The note-book is in the possession of Mr Bernard Darwin.

(*Issued separately December* 19, 1935.)

XI.—On Fourfold Sampling with and without Replacement.

By **A. C. Aitken,** D.Sc., and **H. T. Gonin,** Ph.D., Mathematical Institute, University of Edinburgh.

(MS. received September 30, 1935. Read December 2, 1935.)

1. INTRODUCTORY.

WHEN each individual of a population of N members can be classified as being either A or $\bar{A}$, and at the same time either B or $\bar{B}$, the relative proportions, or probabilities, of the four types AB, A$\bar{B}$, $\bar{A}$B and $\bar{A}\bar{B}$ can be set out in the familiar "fourfold table."

$$\begin{array}{cccc} & B & \bar{B} & \\ A & p_{11} & p_{10} & p \\ \bar{A} & p_{01} & p_{00} & q \\ & p' & q' & 1 \end{array} \qquad \text{. . . . . (1)}$$

The probability of type AB is p_{11}, that of A$\bar{B}$ is p_{10}, and so on. Summing the rows and entering the sums marginally we denote by p and q the total probabilities of A and $\bar{A}$ when B and $\bar{B}$ are disregarded; thus,

$$p_{11}+p_{10}=p, \qquad p_{01}+p_{00}=q, \qquad p+q=1. \qquad \text{. . . (2)}$$

In the same way, summing the columns, we enter marginally p' and q'. The constitution of the population can also be represented by the symbolic expression

$$p_{11}AB+p_{10}A\bar{B}+p_{01}\bar{A}B+p_{00}\bar{A}\bar{B}. \qquad \text{. . . . . (3)}$$

The fourfold disjunction is the natural bivariate extension of the simple dichotomy $pA+q\bar{A}$ in one variate. It is well known that under random sampling n times this simple dichotomy gives rise to the binomial probability distribution if replacement of individuals is permitted, and to the hypergeometric distribution if replacement is not permitted or if the sample of n is drawn *en bloc*. These distributions may be considered as classical.

The bivariate distributions arising from fourfold sampling, with and without replacement, have also been studied by various authors. The chief object of the present paper is to obtain new expressions for the correlation functions of fourfold sampling in the form of series bilinear in suitable orthogonal polynomials. The polynomials appropriate to the binomial distribution prove to be slightly more general than those obtained by Gram (1879, 1882) and recently studied by Greenleaf (1932), while those appropriate to the hypergeometric distribution are new and not devoid of formal elegance. The analogies with the Hermite polynomials of the normal distribution and the Charlier polynomials of the Poisson distribution are close and helpful; and the expansions of the correlation functions are comparable with Mehler's series (1866) for the normal correlation function,

$$\begin{aligned}\phi(x, y;\ \rho) &\equiv (2\pi)^{-1}(1-\rho^2)^{-\frac{1}{2}}\exp\{-\tfrac{1}{2}(x^2-2\rho xy+y^2)/(1-\rho^2)\}\\ &= \phi(x, y;\ 0)\left\{1+\rho H_1(x)H_1(y)+\frac{\rho^2}{2!}H_2(x)H_2(y)+\ .\ .\ .\right\}, \quad . \qquad (4)\end{aligned}$$

where

$$H_r(x)e^{-\frac{1}{2}x^2} = (-)^r D^r e^{-\frac{1}{2}x^2}, \qquad D = \frac{d}{dx},$$

and the series (Wicksell, 1916; Campbell, 1932) for the Poisson correlation function,

$$\begin{aligned}\psi(x, y;\ a) &\equiv e^{-(m+m'-\overline{m})}\frac{(m-\overline{m})^x(m'-\overline{m})^y}{x!\ y!}\,{}_0F_2(-x,\ -y;\ a)\\ &= \psi(x, y;\ 0)\left\{1+aK_1(x;\ m)K_1(y;\ m')+\frac{a^2}{2!}K_2(x;\ m)K_2(y;\ m')+\ .\ .\ .\right\}, \quad . \qquad (5)\end{aligned}$$

where $K_r(x;\ m)$ is the Charlier polynomial defined by

$$K_r(x;\ m)\psi(x;\ m) = (-)^r\Delta^r\psi(x-r;\ m), \quad . \quad . \quad . \qquad (6)$$

$$a = \overline{m}(mm')^{-1},$$

m, m', and $\overline{m}$ are mean values of x, y, and xy over the range 0 to ∞, $\psi(x;\ m)$ denotes the Poisson frequency function

$$\psi(x;\ m) = e^{-m}m^x/x!, \quad . \quad . \quad . \quad . \quad . \qquad (7)$$

and ${}_0F_2$ denotes a degenerate hypergeometric series.

2. THE ORTHOGONAL POLYNOMIALS OF THE BINOMIAL DISTRIBUTION.

Under simple sampling n times with replacement the probability of x "successes" is given by

$$\phi(x) \equiv \phi(x;\ p) = n_{(x)}p^x q^{n-x}, \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

where p is the probability of success at a single drawing and $n_{(x)}$ denotes the binomial coefficient

$$n_{(x)} = n(n-1)(n-2)\ .\ .\ .\ (n-x+1)/x!.$$

We shall obtain polynomials $G_r(x) \equiv G_r(x;\ p)$ having the orthogonal properties

$$\sum_0^n x^{(s)}G_r(x)\phi(x) = 0, \qquad s < r, \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

where

$$x^{(s)} = x(x-1)(x-2)\ .\ .\ .\ (x-s+1).$$

As usual in such cases, the proof will involve summation by parts. We therefore examine the differences of $\phi(x)$ with a view to ascertaining by what function $\phi(x)$ must be multiplied in order that the rth difference of the product shall be of the form $G_r(x)\phi(x)$ where $G_r(x)$ is a polynomial of degree r. The examination shows at once that the polynomial defined by

$$G_r(x)\phi(x) = (-q)^r\Delta^r x^{(r)}\phi(x) \quad .\quad .\quad .\quad .\quad . \quad (3)$$

meets the requirements. For, applying the analogue in finite differences of Leibniz' theorem, namely,

$$\Delta^r u_x v_x = v_x\Delta^r u_x + r\Delta v_x\Delta^{r-1}u_{x-1} + r_{(2)}\Delta^2 v_x\Delta^{r-2}u_{x-2} + \ .\ .\ .\ ,$$

we have

$$\begin{aligned}(-q)^r\Delta^r x^{(r)}\phi(x) = (-)^r\phi(x)\{(n-x)^{(r)}p^r - r(n-x)^{(r-1)}p^{r-1}qx \\ + r_{(2)}(n-x)^{(r-2)}p^{r-2}q^2x^{(2)} - \ .\ .\ .\},\end{aligned} \quad (4)$$

where the bracketed factor on the right is evidently a polynomial of degree r in x. Denoting this polynomial by $G_r(x)$, substituting in the left side of (2) and applying summation by parts (Σ denoting *indefinite* summation) in the form

$$\sum u_x v_x = u_x\sum v_x - \Delta u_x\sum{}^2 v_{x+1} + \Delta^2 u_x\sum{}^3 v_{x+2} - \ .\ .\ .\ , \quad . \quad (5)$$

we derive, for $s < r$, the expression

$$\Big[(x^{(s)}\Delta^{r-1}x^{(r)}\phi(x) - sx^{(s-1)}\Delta^{r-2}(x+1)^{(r)}\phi(x+1) \\ + \ .\ .\ .\ + (-)^s s!\,\Delta^{r-s-1}(x+s)^{(r)}\phi(x+s)\Big]_0^{n+1}. \quad (6)$$

Now $\phi(x)$ vanishes for integer values of $x > n$, and so $x^{(r)}\phi(x)$ and all its differences vanish also for these values. Hence at the upper limit all the terms in (6) vanish. At the lower limit all terms except the last vanish through having x as a factor. But

$$(x+s)^{(r)}\phi(x+s) = 0, \qquad x = 0,\ 1,\ 2,\ .\ .\ .,\ \ r-s-1,$$

and so when $x=0$

$$\Delta^{r-s-1}(x+s)^{(r)}\phi(x+s) = 0.$$

Hence all terms vanish at both limits, and the orthogonal conditions (2) are satisfied.

Again, when $r=s$, summation by parts yields terms which vanish as in (6), except for the last term, which takes the form

$$\begin{aligned}(-q)^r r!\sum_0^n (x+r)^{(r)}\phi(x+r) &= r!\,n^{(r)}p^r q^r(q+p)^{n-r} \\ &= r!\,n^{(r)}p^r q^r. \quad .\quad .\quad . \quad (7)\end{aligned}$$

The orthogonal properties may therefore be expressed as

$$\sum_{0}^{n} \mathrm{G}_r(x)\mathrm{G}_s(x)\phi(x) = 0, \qquad r \neq s,$$
$$= r!\, n^{(r)} p^r q^r, \qquad r = s > 0. \quad . \quad . \quad (8)$$

The form for $\mathrm{G}_r(x)$ obtained in (4) is not the most convenient. Writing $1 - p$ for q, using Vandermondian identities,

$$n^{(r)} = (n-x)^{(r)} + r(n-x)^{(r-1)}x + r_{(2)}(n-x)^{(r-2)}x^{(2)} + \ldots,$$

and rearranging terms in the result, we may transform it into the standard form

$$\mathrm{G}_r(x) = x^{(r)} - rp(n-r+1)x^{(r-1)} + r_{(2)}p^2(n-r+2)^{(2)}x^{(r-2)} - \ldots + (-)^r p^r n^{(r)}. \quad (9)$$

Gram and Greenleaf, considering the symmetrical binomial associated with $(\frac{1}{2}+\frac{1}{2})^n$, where n is even, obtained the special case of this in which $p = \frac{1}{2}$.

When n is large but p is so small that the mean $np = m$ remains of the order of unity, $\mathrm{G}_r(x)$ tends asymptotically to

$$x^{(r)} - rmx^{(r-1)} + r_{(2)}m^2x^{(r-2)} - \ldots + (-)^r m^r, \quad . \quad . \quad (10)$$

that is, to Charlier's polynomial $\mathrm{K}_r(x;\, m)$. In § 12 we shall consider also the relation of $\mathrm{G}_r(x)$ to the Hermite polynomial $\mathrm{H}_r(x)$.

Some other properties are of use. There is a symbolic form,

$$\mathrm{G}_r(x) = (1 + p\Delta)^{-(n-r+1)}x^{(r)}, \quad . \quad . \quad . \quad . \quad (11)$$

comparable with

$$\mathrm{H}_r(x) = e^{-\frac{1}{2}\mathrm{D}^2}x_r, \qquad \mathrm{K}_r(x;\, m) = e^{-m\Delta}x^{(r)}.$$

We have also

$$\Delta \mathrm{G}_r(x) = r\mathrm{G}_{r-1}(x;\, p,\, n-1), \quad . \quad . \quad . \quad . \quad (12)$$

so that by inverting the relation (11) we derive

$$x^{(r)} = \mathrm{G}_r(x) + rp(n-r+1)\mathrm{G}_{r-1}(x;\, p,\, n-1)$$
$$+ r_{(2)}p^2(n-r+1)^{(2)}\mathrm{G}_{r-2}(x;\, p,\, n-2) + \ldots, \quad . \quad (13)$$

an expansion which will be of use later.

3. FACTORIAL MOMENTS IN THE BINOMIAL CASE.

We define the factorial moments of a function $f(x)$ by

$$\sum x^{(r)} f(x), \qquad r = 0, 1, 2, \ldots \quad . \quad . \quad . \quad (1)$$

Then the transform of $f(x)$, with binomial nucleus, defined by

$$\mathrm{F}(a) = \sum (1+a)^x f(x). \quad . \quad . \quad . \quad . \quad . \quad (2)$$

generates these moments, in the sense that the rth moment is the coefficient of $a^r/r!$ in $\mathrm{F}(a)$. Hence $\mathrm{F}(a)$ will be described as the *factorial moment* *generating function* (f.m.g.f.) of $f(x)$. A bivariate f.m.g.f. may be similarly defined by

$$F(\alpha, \beta) = \sum \sum (1+\alpha)^x (1+\beta)^y f(x, y) \quad . \quad . \quad . \quad (3)$$

THEOREM.—*The f.m.g.f. of* $G_r(x)\phi(x)$, *where*

$$\phi(x) \equiv \phi(x; p) = n_{(x)} p^x q^{n-x},$$

is

$$F(\alpha) = n^{(r)} p^r q^r \alpha^r (1+p\alpha)^{n-r}.$$

Proof.—If $F(\alpha)$ is the f.m.g.f. of a function $f(x)$, it is easily shown that $\alpha^r F(\alpha)$ is the f.m.g.f. of $(-)^r \Delta^r f(x-r)$. Applying this to the present case, we have

$$\begin{aligned} F(\alpha) &= \sum_0^n (1+\alpha)^x G_r(x)\phi(x) \\ &= \sum (1+\alpha)^x (-q)^r \Delta^r x^{(r)} \phi(x) \\ &= \sum (1+\alpha)^x q^r (x+r)^{(r)} \phi(x+r) \alpha^r \\ &= n^{(r)} p^r q^n \alpha^r \{1 + p(1+\alpha) q^{-1}\}^{n-r} \\ &= n^{(r)} p^r q^r \alpha^r (1+p\alpha)^{n-r}. \end{aligned}$$

In particular, the f.m.g.f. of $\phi(x)$ itself is $(1+p\alpha)^n$, as may be seen directly by substituting $1+\alpha$ for t in the probability g.f. $(pt+q)^n$.

4. FOURFOLD CORRELATION OF BINOMIAL TYPE.

The theorem of § 3 leads to an expression for the correlation function in fourfold sampling with replacement. Let the fourfold population of § 1 be sampled in this manner n times. The probability g.f. is

$$(p_{11}tu + p_{10}t + p_{01}u + p_{00})^n. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The substitutions $t = 1+\alpha$, $u = 1+\beta$ give the f.m.g.f. $F(\alpha, \beta)$, which in virtue of the marginal sum relations of the table takes the form

$$\begin{aligned} F(\alpha, \beta) &= (1 + p\alpha + p'\beta + p_{11}\alpha\beta)^n \\ &= \{(1+p\alpha)(1+p'\beta) + (p_{11} - pp')\alpha\beta\}^n \\ &= (1+p\alpha)^n(1+p'\beta)^n + nd\alpha\beta(1+p\alpha)^{n-1}(1+p'\beta)^{n-1} \\ &\qquad + n_{(2)} d^2\alpha^2\beta^2(1+p\alpha)^{n-2}(1+p'\beta)^{n-2} + \ldots, \quad (2) \end{aligned}$$

where $d = p_{11} - pp'$. The series terminates, since n is an integer.

But the primitive functions, of which each separate term of (2) is the f.m.g.f., are known from § 3. Hence the desired correlation function, the probability of x successes A and y successes B, is expressed by the terminating series

$$\begin{aligned} &\phi(x, y; d) = \phi(x; p)\phi(y; p') \\ &\quad \times \left\{1 + \frac{d}{npp'qq'} G_1(x; p) G_1(y; p') + \frac{d^2}{n^{(2)}(pp'qq')^2} G_2(x; p) G_2(y; p') + \ldots\right\}, \quad (3) \end{aligned}$$

bilinear in the orthogonal polynomials in x and y. When there is no correlation between A and B, $d=0$, and (3) degenerates into its first term, as is otherwise evident.

5. NATURE OF REGRESSIONS IN THE BINOMIAL CASE.

By means of § 2 (9) or (13) and the orthogonal properties § 2 (8) the regression lines can be derived immediately. Keeping y fixed, multiply § 4 (3) by 1 and also by x, and sum with respect to x. We obtain

$$\sum_x \phi(x, y;\, d) = \phi(y;\, p'), \qquad (1)$$

and

$$\sum_x x\phi(x, y;\, d) = \phi(y;\, p')\left\{np + \frac{d}{p'q'}(y - np')\right\}. \qquad (2)$$

It follows that the locus of the means of x for fixed y, that is, the x-regression, is given by

$$x - np = \frac{d}{p'q'}(y - np'), \qquad (3)$$

and in the same way the y-regression is

$$y - np' = \frac{d}{pq}(x - np). \qquad (4)$$

Thus the regression lines, as in the normal and the Poisson cases, are straight, and pass through the double mean $(np,\, np')$ of the distribution; while the correlation coefficient ρ, if defined in the usual way as the geometric mean of the regression coefficients, is given by

$$\rho = d(pp'qq')^{-\frac{1}{2}}, \qquad (5)$$

in agreement with known results (*cf.* Soper, 1922; Wicksell, 1917).

Higher moments of any order of the x and y arrays can be obtained in a similar manner. The simplest procedure is to use factorial moments with the orthogonal properties and § 2 (9) or (13).

6. FOURFOLD SAMPLING WITHOUT REPLACEMENT.

We begin from Soper's observation (1922, p. 30) that in simple sampling n times ***without replacement*** from a population of N individuals of whom Np possess the character A, the probability g.f. is the coefficient of z^n in

$$(1 + tz)^{Np}(1 + z)^{Nq}/N_{(n)}. \qquad (1)$$

The corresponding probability g.f. for the fourfold table is the coefficient of z^n in

$$(1 + tuz)^{Np_{11}}(1 + tz)^{Np_{10}}(1 + uz)^{Np_{01}}(1 + z)^{Np_{00}}/N_{(n)}. \qquad (2)$$

The probability function corresponding to (1) is the so-called "hypergeometric" one, namely,

$$\psi(x) \equiv \psi(x;\, n,\, \mathrm{N},\, \mathrm{N}p) = n_{(x)}(\mathrm{N}p)^{(x)}(\mathrm{N}q)^{(n-x)}/\mathrm{N}^{(n)}. \qquad (3)$$

Its f.m.g.f., obtained by putting $t = 1 + a$ in (1), is readily found to be

$$\mathrm{F}(a) = \mathrm{F}(-n,\, -\mathrm{N}p;\, -\mathrm{N};\, -a), \qquad (4)$$

a terminating hypergeometric series from which the factorial moments of $\psi(x)$ of any order can be written down.

7. THE ORTHOGONAL POLYNOMIALS OF THE HYPERGEOMETRIC DISTRIBUTION.

We shall obtain polynomials $\mathrm{U}_r(x) \equiv \mathrm{U}_r(x;\, n,\, \mathrm{N},\, \mathrm{N}p)$ having the orthogonal properties

$$\sum_0^n x^{(s)}\mathrm{U}_r(x)\psi(x) = 0, \qquad s < r. \qquad (1)$$

The method follows the lines of § 2. An alternative, in both cases, might be to use a theorem of E. H. Hildebrandt (1931). The outcome is that the polynomials defined by

$$\mathrm{U}_r(x)\psi(x) = (-)^r\Delta^r\{x^{(r)}(\mathrm{N}q - n + x)^{(r)}\psi(x)\}/(\mathrm{N} - r + 1)^{(r)} . \qquad (2)$$

meet the requirements. For, proceeding as in § 2, we obtain

$$\begin{aligned}\mathrm{U}_r(x) = (-)^r\{&(n-x)^{(r)}(\mathrm{N}p - x)^{(r)} - r(n-x)^{(r-1)}(\mathrm{N}p - x)^{(r-1)}x(\mathrm{N}q - n + x)\\ &+ r_{(2)}(n-x)^{(r-2)}(\mathrm{N}p - x)^{(r-2)}x^{(2)}(\mathrm{N}q - n + x)^{(2)} - \ldots\}/(\mathrm{N} - r + 1)^{(r)}, \qquad (3)\end{aligned}$$

and this is a polynomial in x, though apparently of degree $2r$. However, by Vandermondian identities such as

$$\begin{aligned}(\mathrm{N}p)^{(r)} &= (\mathrm{N}p - x)^{(r)} + r(\mathrm{N}p - x)^{(r-1)}x + r_{(2)}(\mathrm{N}p - x)^{(r-2)}x^{(2)} + \ldots,\\ (\mathrm{N}p - 1)^{(r-1)} &= (\mathrm{N}p - x)^{(r-1)} + \ldots,\end{aligned}$$

and some regroupings of terms, it may be transformed into the standard form

$$\begin{aligned}\mathrm{U}_r(x) = x^{(r)} - r\frac{(n-r+1)(\mathrm{N}p-r+1)}{\mathrm{N}-2r+2}x^{(r-1)} + r_{(2)}\frac{(n-r+2)^{(2)}(\mathrm{N}p-r+2)^{(2)}}{(\mathrm{N}-2r+3)^{(2)}}x^{(r-2)}&\\ - \ldots + (-)^r\frac{n^{(r)}(\mathrm{N}p)^{(r)}}{(\mathrm{N}-r+1)^{(r)}}.& \qquad (4)\end{aligned}$$

This may be written symbolically (*cf.* § 2 (11)) as

$$\mathrm{U}_r(x) = \mathrm{F}(n-r+1,\, \mathrm{N}p-r+1;\, \mathrm{N}-2r+2;\, -\Delta)x^{(r)}. \qquad (5)$$

To verify the orthogonal properties, we consider

$$\sum_0^n (\mathrm{N}-r+1)^{(r)}x^{(s)}\mathrm{U}_r(x)\psi(x).$$

Applying summation by parts as in § 2, we derive, for $s < r$, the expression

$$\Big[x^{(s)}\Delta^{r-1}x^{(r)}(\mathrm{N}q-n+x)^{(r)}\psi(x) - sx^{(s-1)}\Delta^{r-2}(x+1)^{(r)}(\mathrm{N}q-n+x+1)^{(r)}\psi(x+1) + \dots + (-)^s s!\,\Delta^{r-s-1}(x+s)^{(r)}(\mathrm{N}q-n+x+s)^{(r)}\psi(x+s)\Big]_0^{n+1}. \quad (6)$$

Now $\psi(x)$ vanishes for integer values of $x > n$, and so the product

$$x^{(r)}(\mathrm{N}q-n+x)^{(r)}\psi(x)$$

and all its differences vanish also for these values. Hence at the upper limit all terms in (6) vanish. At the lower limit all terms except the last vanish through having x as a factor. But

$$(x+s)^{(r)}(\mathrm{N}q-n+x+s)^{(r)}\psi(x+s)=0, \qquad x=0,\ 1,\ 2,\ \dots,\ r-s-1,$$

and so when $x=0$

$$\Delta^{r-s-1}(x+s)^{(r)}(\mathrm{N}q-n+x+s)^{(r)}\psi(x+s)=0.$$

Hence all terms vanish at both limits, and the orthogonal conditions (1) are satisfied.

Again, when $r=s$, summation by parts yields terms which vanish as in (6), except for the last term, which takes the form

$$(-)^r r!\sum_0^n (x+r)^{(r)}(\mathrm{N}p-n+x+r)^{(r)}\psi(x+r),$$

and this reduces without difficulty to

$$\frac{r!\,n^{(r)}(\mathrm{N}p)^{(r)}(\mathrm{N}-n)^{(r)}(\mathrm{N}q)^{(r)}}{\mathrm{N}^{(2r)}}\sum\psi(x;\ n-r,\ \mathrm{N}-2r,\ \mathrm{N}p-r)$$
$$=\frac{r!\,n^{(r)}(\mathrm{N}p)^{(r)}(\mathrm{N}-n)^{(r)}(\mathrm{N}q)^{(r)}}{\mathrm{N}^{(2r)}}. \quad (7)$$

The orthogonal properties may therefore be expressed as

$$\sum_0^n \mathrm{U}_r(x)\mathrm{U}_s(x)\psi(x)=0, \qquad r\neq s,$$
$$=\frac{r!\,n^{(r)}(\mathrm{N}p)^{(r)}(\mathrm{N}-n)^{(r)}(\mathrm{N}q)^{(r)}}{\mathrm{N}^{(2r)}(\mathrm{N}-r+1)^{(r)}}, \qquad r=s>0. \quad (8)$$

8. FACTORIAL MOMENTS IN THE HYPERGEOMETRIC CASE.

THEOREM.—*The f.m.g.f. of* $\mathrm{U}_r(x)\psi(x)$ *is*

$$\mathrm{F}(\alpha)=\frac{n^{(r)}(\mathrm{N}p)^{(r)}(\mathrm{N}-n)^{(r)}(\mathrm{N}q)^{(r)}}{\mathrm{N}^{(2r)}(\mathrm{N}-r+1)^{(r)}}\alpha^r\mathrm{F}(-n+r,-\mathrm{N}p+r;\ -\mathrm{N}+2r;\ -\alpha).$$

Proof.—Using the result mentioned in the corresponding part of § 3, we have

$$\begin{aligned} F(\alpha) &= \sum (1+\alpha)^x U_r(x)\psi(x) \\ &= \sum (-)^r (1+\alpha)^x \Delta^r x^{(r)} (Nq-n+x)^{(r)} \psi(x)/(N-r+1)^{(r)} \\ &= \sum (1+\alpha)^x (x+r)^{(r)} (Nq-n+x+r)^{(r)} \psi(x+r)/(N-r+1)^{(r)} \\ &= \frac{n^{(r)}(Np)^{(r)}(N-n)^{(r)}(Nq)^{(r)}}{N^{(2r)}(N-r+1)^{(r)}} \alpha^r F(-n+r,\ -Np+r;\ -N+2r;\ -\alpha). \end{aligned}$$

The case $r=0$ is in agreement with § 6 (4).

9. FOURFOLD CORRELATION OF HYPERGEOMETRIC TYPE.

The theorem of § 8 leads to an expression for the correlation function in fourfold sampling without replacement. The law governing the coefficients in the series is more complicated than in the binomial case; this is to be expected, since each drawing alters both the population and the probability of successes in subsequent drawings. The substitution of $t=1+\alpha$, $u=1+\beta$ in § 6 (2) gives the f.m.g.f. of the correlation function as the coefficient of z^n in

$$(1+z)^{Np_{00}}(1+z+\alpha z)^{Np_{10}}(1+z+\beta z)^{Np_{01}}\{1+z+(\alpha+\beta+\alpha\beta)z\}^{Np_{11}}/N_{(n)}. \qquad (1)$$

Let us write d for $p_{11}-pp'=p_{00}-qq'=-(p_{10}-pq')=-(p_{01}-p'q)$. The f.m.g.f. is then the coefficient of z^n in

$$(1+z)^{N-Np-Np'}(1+z+\alpha z)^{Np}(1+z+\beta z)^{Np'}\left\{1+\frac{\alpha\beta z}{(1+z+\alpha z)(1+z+\beta z)}\right\}^{Npp'+Nd}\Big/ N_{(n)}. \qquad (2)$$

If $d=0$, x and y are uncorrelated, and the correlation function is then the product

$$\psi(x, y; 0) = \psi(x; n, N, Np)\psi(y; n, N, Np'). \quad . \quad . \quad (3)$$

For brevity we write

$$\begin{aligned} w &= \alpha\beta z(1+z+\alpha z)^{-1}(1+z+\beta z)^{-1}, \\ \theta_r &= \frac{(Np-r)(Np'-r)}{N-2r} - Npp'. \end{aligned}$$

Now let

$$(1+w)^{Nd} = 1 + a_1 w(1+w)^{\theta_1} + a_2 w^2 (1+w)^{\theta_2} + \ldots \quad . \quad . \quad (4)$$

This gives a set of equations for the a_s in terms of Nd. The matrix of the equations is triangular in shape, and the a_s can therefore be determined successively. For example,

$$a_1 = Nd, \qquad a_2 = (Nd)_{(2)} - \theta_1 Nd, \ldots \quad . \quad . \quad . \quad (5)$$

Hence, the coefficient of z^n in (2) when $d=0$ being by (3) and § 6 (4),

$$F(-n,\ -Np;\ -N;\ -\alpha)F(-n,\ -Np';\ -N;\ -\beta),$$

the coefficient in the more general case, $d \neq 0$, by (2) and (4) above, is

$$\sum_{s=0}^{n} a_s \frac{n^{(s)}(N-n)^{(s)}}{N^{(2s)}} F(-n+s,\ -Np+s;\ -N+2s;\ -\alpha)$$
$$\times F(-n+s,\ -Np'+s;\ -N+2s;\ -\beta)\alpha^s\beta^s, \qquad (6)$$

and this, as in the binomial case of § 4, is a terminating series, of either $n+1$ or $N-n+1$ terms, whichever is the lesser.

But the primitive functions, of which each separate term of (6) above is the f.m.g.f., are known from § 8. Hence the correlation function is the terminating series

$$\psi(x, y; d) = \psi(x, y; 0)\left\{1 + a_1 \frac{N^{(2)}N}{n(N-n)NpNp'NqNq'} U_1(x; p)U_1(y; p')\right.$$
$$\left. + a_2 \frac{N^{(4)}(N-1)^{(2)}}{n^{(2)}(N-n)^{(2)}(Np)^{(2)}(Np')^{(2)}(Nq)^{(2)}(Nq')^{(2)}} U_2(x; p)U_2(y; p') + \ldots\right\}, \quad (7)$$

where the coefficients a_s are given by (4).

10. NATURE OF REGRESSIONS IN THE HYPERGEOMETRIC CASE.

The linearity of the regressions can be established as in § 5. A precisely similar use of orthogonal properties gives

$$\sum_x \psi(x, y; d) = \psi(y; p'), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and

$$\sum_x x\psi(x, y; d) = \psi(y; p')\left\{np + \frac{d}{p'q'}(y - np')\right\}. \quad . \quad . \quad (2)$$

The regression lines are therefore

$$x - np = \frac{d}{p'q'}(y - np'), \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$y - np' = \frac{d}{pq}(x - np), \quad . \quad . \quad . \quad . \quad . \quad (4)$$

formally identical with those obtained in § 5 for the binomial case; and the correlation coefficient ρ is also the same.

11. THE CENTRAL FORMS OF THE GRAM POLYNOMIALS.

We return briefly to Gram's polynomials. It is well known that as n increases the binomial probability function tends asymptotically, unless np remains small and finite, to the normal probability function. Hence the Gram polynomials (as well as the more general polynomials of § 2), when x is referred to the mean as origin and expressed in terms of the standard deviation as unit, should tend to the Hermite polynomials. We shall make this evident by expressing the Gram polynomials for the symmetrical binomial distribution, $p=\frac{1}{2}$, $q=\frac{1}{2}$, in central factorials.

With the mean, $\frac{1}{2}n=m$, as origin, let $x-m=\xi$.

We use the operators of finite differences, E, Δ, μ, δ, defined by

$$\mathrm{E}u_x=u_{x+1}, \qquad \mu u_x=\tfrac{1}{2}(u_{x+\frac{1}{2}}+u_{x-\frac{1}{2}}),$$
$$\Delta u_x=u_{x+1}-u_x, \qquad \delta u_x=u_{x+\frac{1}{2}}-u_{x-\frac{1}{2}},$$

and certain of their properties,

$$\mathrm{E}=1+\Delta, \qquad \delta=\Delta\mathrm{E}^{-\frac{1}{2}},$$
$$\mu^2=1+\tfrac{1}{4}\delta^2, \qquad \delta x^{\{r\}}=rx^{\{r-1\}},$$

where $x^{\{r\}}$ is the central factorial polynomial defined by

$$x^{\{r\}}=\left(x+\frac{r-1}{2}\right)^{(r)}=\mathrm{E}^{\frac{1}{2}(r-1)}x^{(r)}.$$

Then, by § 2 (11),

$$\begin{aligned} \mathrm{G}_r(x;\,\tfrac{1}{2}) &= (1+\tfrac{1}{2}\Delta)^{-(2m-r+1)}x^{(r)} \\ &= (\mu\mathrm{E}^{\frac{1}{2}})^{-(2m-r+1)}\mathrm{E}^{\frac{1}{2}(2m-r+1)}\xi^{\{r\}} \\ &= (1+\tfrac{1}{4}\delta^2)^{-\frac{1}{2}(2m-r+1)}\xi^{\{r\}} \qquad \qquad (1) \\ &= (1+\tfrac{1}{4}\delta^2)^{-\frac{1}{2}(2m-r+2)}\mu\xi^{\{r\}}. \qquad \qquad (2) \end{aligned}$$

Expanding the operators in (1) and (2) in powers of δ^2 and operating term by term we obtain the Gram polynomials as either Newton-Stirling or Newton-Bessel expansions. For example, the first few (with x for ξ) are as follows:—

Newton-Stirling—

$$\begin{gathered} 1, \quad x, \quad x^2-\frac{m}{2}, \quad x(x^2-1)-3\frac{m-1}{2}x, \\ x^2(x^2-1)-6\frac{m-1}{2}x^2+3\frac{m}{2}\cdot\frac{m-1}{2}, \qquad \qquad (3) \\ x(x^2-1)(x^2-4)-10\frac{m-2}{2}x(x^2-1)+15\frac{m-1}{2}\cdot\frac{m-2}{2}x, \ldots \end{gathered}$$

Newton-Bessel—

$$\begin{gathered} 1, \quad x, \quad (x^2-\tfrac{1}{4})-\frac{m-\frac{1}{2}}{2}, \quad x(x^2-\tfrac{1}{4})-3\frac{m-\frac{1}{2}}{2}x, \\ (x^2-\tfrac{1}{4})(x^2-\tfrac{9}{4})-6\frac{m-\frac{3}{2}}{2}(x^2-\tfrac{1}{4})+3\frac{m-\frac{1}{2}}{2}\cdot\frac{m-\frac{3}{2}}{2}, \qquad \qquad (4) \\ x(x^2-\tfrac{1}{4})(x^2-\tfrac{9}{4})-10\frac{m-\frac{3}{2}}{2}x(x^2-\tfrac{1}{4})+15\frac{m-\frac{1}{2}}{2}\cdot\frac{m-\frac{3}{2}}{2}x, \ldots \end{gathered}$$

The nature of the analogy with the Hermite polynomials now appears; *e.g.*

$$\mathrm{H}_4(x)=x^4-6x^2+3, \qquad \mathrm{H}_5(x)=x^5-10x^3+15x.$$

12. SUMMARY.

The bivariate probability functions derived by considering sampling with and without replacement from a fourfold population are found to be expressible as products of the corresponding univariate functions, multiplied by a terminating series bilinear in the appropriate orthogonal polynomials. The form and properties of these polynomials are studied, the linearity of the regressions is established by means of them, and analogies with the Hermite and Charlier polynomials are noted.

REFERENCES TO LITERATURE.

CAMPBELL, J. T., 1932. "The Poisson Correlation Function," *Proc. Edin. Math. Soc.*, vol. iv, ser. 2, pp. 18–26.

GONIN, H. T., 1934. "On the Orthogonal Polynomials in Binomial and Hypergeometric Distributions," a Thesis deposited in the Library of the University of Edinburgh.

GRAM, J. P., 1882. "Ueber die Entwickelung reeller Functionem in Reihen mittelst der Methode der kleinsten Quadrate," *J. für Math.*, Bd. cxiv, pp. 41–73. (Containing the results of a Thesis, Copenhagen, 1879.)

GREENLEAF, H. E. H., 1932. "Curve Approximation by Means of Functions analogous to the Hermite Polynomials," *Ann. Math. Statistics*, vol. iii, pp. 204–255.

HILDEBRANDT, E. H., 1931. "Systems of Polynomials connected with the Charlier Expansions and the Pearson Differential and Difference Equations," *Ann. Math. Statistics*, vol. ii, pp. 379–439.

MEHLER, G., 1866. "Reihenentwicklungen nach Laplaceschen Functionen höherer Ordnung," *J. für Math.*, Bd. lxvi, pp. 161–176 (174).

SOPER, H. E., 1922. *Frequency Arrays*, Cambridge, pp. 14, 18, 30.

WICKSELL, S. D., 1916. "Some Theorems in the Theory of Probability," *Svenska Aktuarieföreningens Tidskr.*, pp. 165–213 (192).

WICKSELL, S. D., 1917. "The Application of Solid Hypergeometric Series to Frequency Distributions in Space," *Phil. Mag.*, vol. xxxiii, ser. 6, pp. 389–394.

(*Issued separately December* 20, 1935.)

XII.—The Experimental Analysis of the Growth of an Insect Population.

By **D. Stewart MacLagan,** D.Sc., F.R.E.S., and **Edward Dunn,** B.Sc., Natural History Department, University of Aberdeen. *Communicated by* Professor JAMES RITCHIE, D.Sc. (With Five Figures.)

(MS. received April 30, 1935. Revised MS. received October 24, 1935. Read July 1, 1935.)

CONTENTS.

	PAGE		PAGE
1. Introduction	126	6. Effect of Density upon Utilisation of Medium for Oviposition	132
2. Life-history	127	7. Mathematical Analysis of the Data	133
3. Technique	128	8. Discussion and Conclusions	135
4. Effect of Density upon Frequency of Copulation	129	9. Summary	138
5. Effect of Density upon Rate of Oviposition	131	10. References to Literature	139

1. INTRODUCTION.

HUMAN populations have afforded favourite material for analysis by statisticians, and others, interested in mathematical theories of population-growth. Experimentally, however, human beings are not ideal biological material, so that other animals, such as protozoa, insects, and mammals, have been used. On account of their great reproductive capacity, rapid response to changes in their environment, and ease of handling in the laboratory, certain insects are very suitable for this type of work. The enormous reproductive capacity of insects has been the subject of comment by many biologists, but we are still comparatively ignorant of the exact rôle of the various factors involved in the limitation of their numbers.

The theory of biotic potential and environmental resistance (Chapman, 1928) has helped to create a fresh interest in population-studies, in that it aims to place the problem upon a quantitative experimental basis. It is now realised that even in the presence of optimum physical environmental conditions an animal population possesses a certain internal resistance to further growth. This paper is largely concerned with an analysis of those factors which constitute the internal resistance, and a preliminary notice of some of the results has appeared in *Nature* (MacLagan and Dunn, 1935).

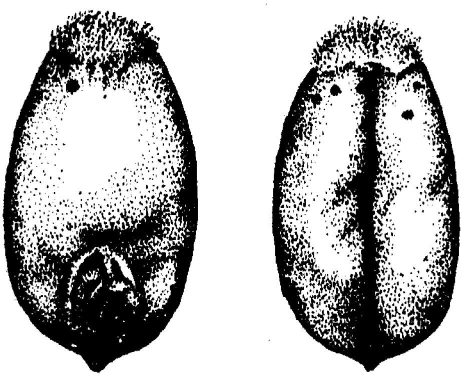
Working with *Tribolium confusum* in a constant environment, Chapman demonstrated that, irrespective of the initial density, a point of equilibrium is eventually attained, after which the population remains relatively constant, provided the flour is renewed frequently enough to remove waste-products and maintain an abundance of food. He concluded that equilibrium is attained when the biotic potential is equalled by the environmental resistance, and that the lack of population-increase is not due to the absence of eggs or their infertility, but to the eating of eggs and pupæ by the adult beetles.

Later, it was shown by one of us (MacLagan, 1932) that this explanation of the stationary character of the population is only partly correct, since, in the higher densities, there is a rapid falling off in the number of eggs oviposited, and a considerable decrease in their fertility. At the same time, it was demonstrated for both *Tribolium confusum* and *Sitophilus granarius* that there is an optimum density, above and below which reproduction takes place at a reduced rate. In attempting to explain this phenomenon, it was pointed out (*loc. cit.*, p. 449) that important factors were involved in "the frequency and chances of interruption of copulation in the various densities." These factors have now been analysed in detail, but before proceeding to consider them it will be necessary to recapitulate the salient features of the life-history of *Sitophilus oryzæ* (the subject of our experiments), and to discuss some controversial points in relation thereto.

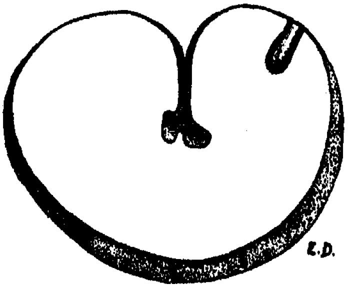
2. LIFE-HISTORY.

The adult weevil is a slender, cylindrical beetle, about 3 mm. long, but varying in size according to the temperature-humidity conditions and the food-supply. Males and females are produced in approximately equal numbers and can be distinguished by means of the proboscis, which is long and slender in the female and slightly shorter and thicker in the male. On account of structural variation this criterion is not, however, infallible. The weevils copulate within a day or two of emergence, and subsequent mating occurs at frequent intervals. Prior to oviposition the female eats a cavity in the grain of wheat, locates the cavity with the tip of her abdomen, and deposits an egg.

Regarding the site of oviposition, the female weevil exhibits a distinct preference for the hairy apex of the grain, and particularly a small area on the grooved side (fig. 1). Barnes and Grove (1916) state that they failed to observe any preference by the female weevil in regard to the site of oviposition. This was probably due to the high proportion of weevils to grains, as a preference is undoubtedly masked in high densities, when oviposition takes places indiscriminately over the grain. Damaged grains are generally avoided for oviposition, although preferred as food. Under the conditions of our experiments the highest number of eggs per female per day was 8, although Hinds and Turner (1911) state that with maize in the "gum" stage "daily records of 15 or 16 eggs were not uncommon." Given a favourable environment the average number of eggs laid per female is about 380.



Grains showing favourite site selected for oviposition.



Transverse section of grain with egg *in situ*.

FIG. 1.

The larva lives within the grain and becomes full-fed after about three weeks. At a temperature of 25° C. and relative humidity of 90 per cent. the egg-stage lasts 3–4 days, the larval stage 16–18 days, and the pupal stage 5–6 days; making a total length of 24–28 days from egg to adult. The length of life of the latter depends upon several factors, of which temperature and rate of oviposition are of profound importance, but under the above conditions the average is about four months. These outstanding facts of the life-cycle will not only elucidate what follows, but will help to convey some idea of the suitability of the insect for quantitative biological experimentation.

3. TECHNIQUE.

The weevils used in the experiments were taken from a stock-culture (derived from a few pairs) which had been maintained for six months under conditions of temperature and humidity highly favourable for rapid reproduction, and had been subjected to intense crowding, thereby ensuring an inbred strain of weevils, on account of the selective elimination which occurs in these circumstances. Such strains are very desirable in biological experimentation in order to minimise the effects of variation. The food was red wheat, and the grains were carefully selected for uniformity of size, damaged ones being rejected. In order to standardise the experimental universes and, at the same time, facilitate observation of the weevils, containers were used which held just the requisite amount of wheat when the grains were disposed in a single layer. Petri dishes of different diameters, but of similar depth, were found to be most suitable, and each was covered with cellophane held in position by a rubber-band. Each cover was perforated by numerous pin-holes to allow the air in the container to come rapidly into equilibrium with the air in the incubator.

It was decided to preclude the possibility of mistakes by selecting pairs *in copula* from the stock-culture, at least for all experiments in which less than one hundred weevils were utilised. When more than fifty pairs were required the weevils were selected at random. As the sex-ratio is about 1 : 1, the chances of obtaining equal numbers of males and females would be fairly good, when more than a hundred individuals were used. Judging from the results obtained, the method of random selection appears to have been quite satisfactory.

All the experiments were conducted at a temperature of 25° C. ± 0·5, and relative humidity of 90 per cent., conditions which are highly favourable for rapid reproduction of *S. oryzæ*.

4. Effect of Density upon Frequency of Copulation.

The various densities were arranged as indicated in Table I. Each experiment extended for a period of five days, and six hourly observations of the number of weevils *in copula* were made each day, the observations being made always at the same times of day. The weevils are very sensitive to various stimuli, and they exhibit strong negative phototropism and geotropism, especially if they have been roughly handled. For this reason the experiments were conducted in the dark, the copulating pairs quickly counted through the cellophane cover, and the container immediately returned to the incubator—all with a minimum of disturbance to the insects. The results are contained in Table I.

Table I.

Number of weevils .	4	8	16	32	128	128	128	128
Number of grains .	800	400	400	400	200	100	50	25
Weevils per grain .	·005	·02	·04	·08	·64	1·28	2·56	5·12
Grains per weevil .	200	50	25	12·5	1·56	·78	·39	·19
Copulation-frequency *	22·8	31·97	33·06	24·23	5·89	2·56	2·01	1·60

* Average percentage of females *in copula* (hourly observations).

Each experiment was performed twice, and the above results represent the average of the two experiments. The data, which are presented graphically in fig. 2, indicate the extraordinary precision with which the density of the population affects the frequency of copulation. It will be observed that the latter increases rapidly when the density increases from ·005 to ·04 weevils per grain, but beyond this degree of crowding there occurs an equally rapid decrease; thereafter the frequency of copulation diminishes at an ever-decreasing rate, until, at a density of 10 weevils per grain, the frequency almost reaches zero. The latter conclusion is derived by extrapolation of the copulation-frequency curve, but there is good reason to presume its accuracy, as at this degree of crowding the weevils form a ceaselessly crawling mass. The existence of an optimum density for frequency of copulation is clearly demonstrated.

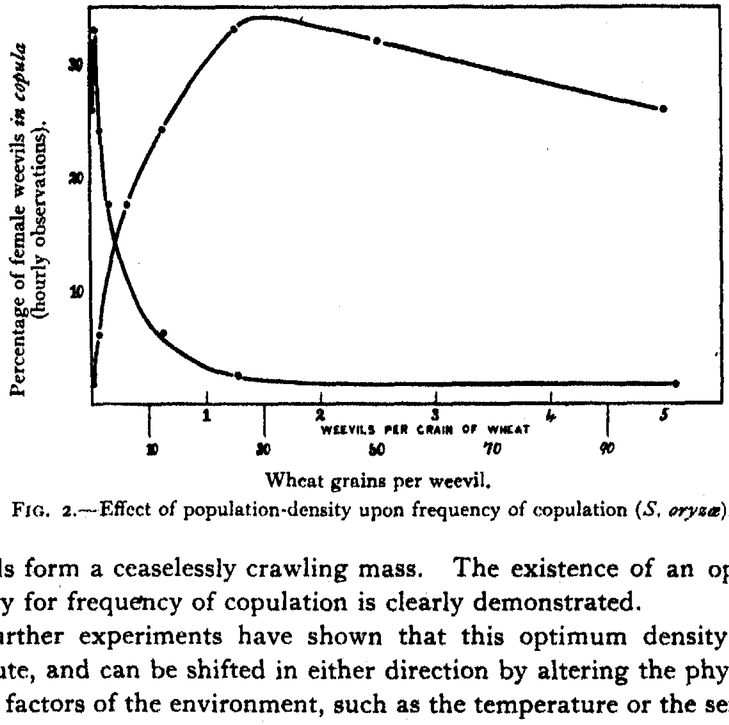


FIG. 2.—Effect of population-density upon frequency of copulation (*S. oryzæ*).

Further experiments have shown that this optimum density is not absolute, and can be shifted in either direction by altering the physical or biotic factors of the environment, such as the temperature or the sex-ratio. For instance, by halving the number of males, the optimum is shifted to a more intense degree of crowding, to produce a copulation-frequency of 16·7 at a density of ·06 weevils per grain. This means a big reduction in the copulation-frequency in the lower densities, but there is an increase in the higher ones, since densities of ·48 and 1·9 weevils per grain gave copulation-frequencies of 12 and 5·9 respectively. Conversely, by doubling the number of males the optimum is shifted to a less intense degree of crowding, to produce a copulation-frequency of 48·3 at a density of ·007 weevils per grain; and the percentage of females *in copula* is increased in the lower densities but reduced in the higher ones.

5. EFFECT OF DENSITY UPON RATE OF OVIPOSITION.

The influence of crowding upon the frequency of copulation having been demonstrated, it was equally desirable to determine whether there was a similar effect upon the rate of oviposition, and, if so, what relationship existed between the latter and frequency of copulation. Hence, another set of experiments was prepared. The experiments were for five days as before, and at the end of this period the weevils were removed and the grains divided into two equal batches. One was returned to the incubator and left there for six days to allow the eggs to develop. The grains constituting the other batch were examined with a view to determining the number of eggs in each. In all but the very low densities (where the number of eggs was determined by counting the plugs with which the female seals the egg-cavity) every grain had to be carefully hand-sectioned. This was a laborious task, involving a total of almost 3000 grains. The oviposition experiments were all repeated in order to reduce the experimental error. Although only a 50 per cent. sample was examined, and errors might have been introduced in counting the eggs within the grains, the results proved satisfactory. They are contained in Table II and presented graphically in figs. 3 and 4.

TABLE II.

Number of weevils .	4	8	32	128	128	128	128
Number of grains .	800	400	400	200	100	50	25
Weevils per grain .	·005	·02	·08	·64	1·28	2·56	5·12
Eggs/female/day .	6·75	3·52	3·02	1·60	1·30	·96	·59
Larvæ/female/day .		3·70	2·82	1·42	1·18	·80	·42
Adults/female/day .		3·54	2·25	·71	·44	·24	·08
Eggs per grain . .	·08	·17	·60	2·56	4·16	6·14	7·55
Larvæ per grain .		·18	·56	2·24	3·84	5·1	5·3
Adults per grain .		·18	·45	1·14	1·42	1·56	1·04
Grains (per cent.) with no eggs.	92·0	84·2	61·0	5·0	1·0	2·0	0

The great precision with which the density affects the rate of oviposition is amply demonstrated. It will be observed that egg-production is high when the population is low, and *vice versa.* This marked reduction in fecundity is accompanied by a reduction in the fertility of the eggs, for at a density of ·005 weevils per grain almost 100 per cent. of the eggs were viable, at ·08 weevils per grain the percentage viability was 93·3, and at 5·12 weevils per grain it was reduced to 70·6. These results were obtained from the batch of grains returned to the incubator for six days, by which time all viable eggs should have hatched. Each grain was carefully sectioned under a microscope and all larvæ were counted.

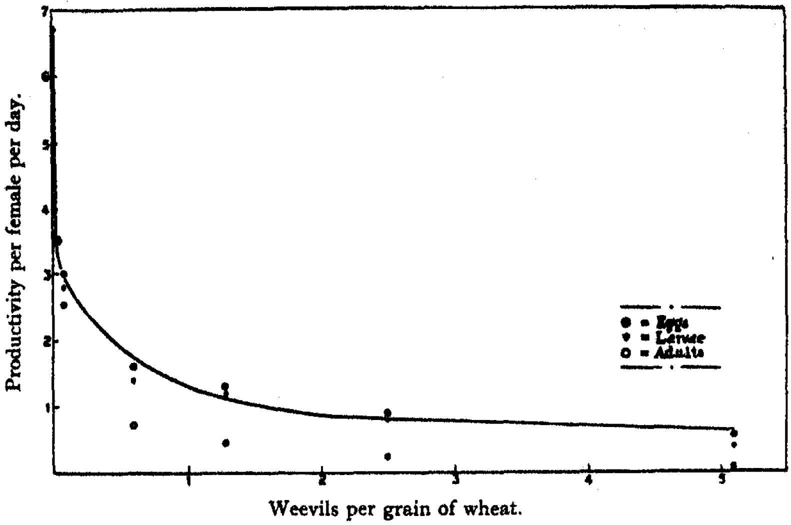


FIG. 3.—Effect of population-density upon rate of reproduction.

6. EFFECT OF DENSITY UPON UTILISATION OF MEDIUM FOR OVIPOSITION.

Examination of the data shows that the number of eggs deposited in a grain of wheat is dependent upon the intensity of crowding. With increasing density the number of eggs per grain increases, and does so according to a definite mathematical formula (see next section). When, under the conditions of these experiments, the weevils deposit an average of one egg per grain, it is interesting to note that 36 per cent. of the grains do not contain any eggs. Of the 64 per cent. which are utilised slightly less than half contain one egg per grain, while the remainder contain numbers varying from 2–5. A distribution of this nature occurs when there are 12·5 wheat grains to each female weevil, and the population consists of equal numbers of both sexes. An interesting point is that up to this particular density the percentage of grains utilised for oviposition rises extremely rapidly, but thereafter the percentage continues to rise at an ever-decreasing rate until eventually *all* the grains contain one or more eggs. This occurs when there is an average of 2·5 female weevils to each grain of wheat. The persistence of a small percentage of grains which are not utilised for oviposition suggests that there is probably always a certain number of grains which are not of desirable quality, and are therefore avoided by the females until excessive crowding forces them to utilise every available grain.

The frequency-distributions of eggs per grain, at different densities, exhibit a gradual shifting from marked positive skewness to a distribution which is approximately normal in the highest densities. The correct interpretation of an analysis of the frequency-distributions is somewhat obscure, but it is at least certain that if, in low densities, the female weevils are capable of distinguishing grains which already contain one or more eggs from those which contain none, this capacity is impaired by excessive crowding. Eventually it breaks down completely, and the eggs are distributed through the medium at random.

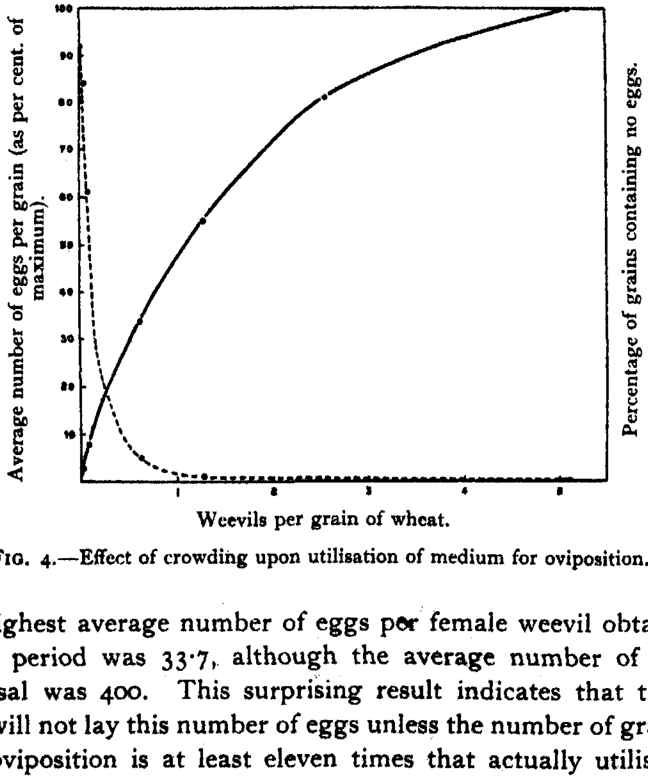


FIG. 4.—Effect of crowding upon utilisation of medium for oviposition.

The highest average number of eggs per female weevil obtained over the 5-day period was 33·7, although the average number of grains at her disposal was 400. This surprising result indicates that the female *S. oryzæ* will not lay this number of eggs unless the number of grains available for oviposition is at least eleven times that actually utilised. Any reduction in this number of grains is accompanied by a reduction in the number of eggs laid per female, despite the fact that she is utilising only a small proportion of the number of available grains.

7. MATHEMATICAL ANALYSIS OF THE DATA.

At this point a brief mathematical analysis of the data is desirable in order to facilitate interpretation of the results. It has already been shown (MacLagan, 1932) that the effect of crowding upon the net reproductive rate of *Sitophilus granarius* can be described by the equation $Y = aX^b$, where Y represents progeny per weevil, X the number of grains per weevil, and a and b are constants. Essentially the same equation, $Y = aX^{-b}$, describes the relation between population-density and frequency of copulation in *S. oryzæ*, at least for densities higher than the optimum. When presented in logarithmic form the above equation becomes

$$\log Y = \log a - b \log X,$$

where Y represents frequency of copulation and X the number of weevils per grain. Inspection shows that if the data conform to this formula a straight line should be obtained when log Y is plotted against log X.

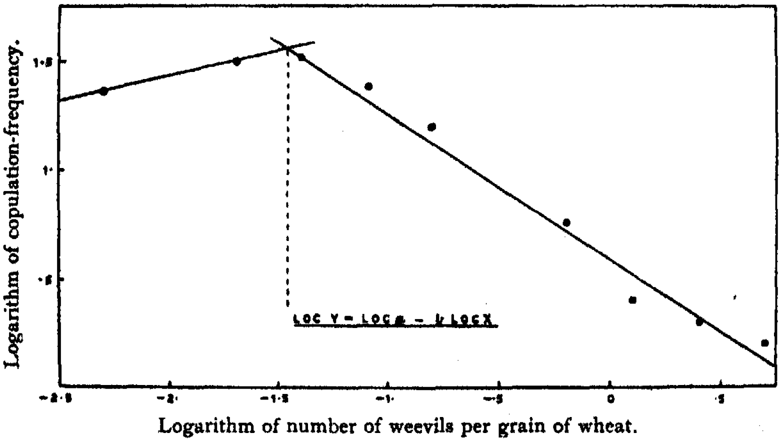


FIG. 5.—Mathematical relationship between population-density and copulation-frequency.

Hence, a convenient method is available for testing the accuracy of the experimental data, and their degree of conformity to the theoretical relationship represented by the formula.

The plot of the two variables, population-density and copulation-frequency, is represented on the double logarithmic scale in fig. 5. It is apparent that, for all densities tested above the optimum, the situation can be described by a straight line, *i.e.* the logarithm of the copulation-frequency is inversely proportional to the logarithm of the population-density. Under the conditions of these experiments Y attains a maximum value when X has a value of ·03, and approaches zero when X is 10. The parameter b describes the rate of change of copulation-frequency with increasing density, and may be used in a manner corresponding to that of a "temperature-coefficient." Reduction in the rate of oviposition with increasing density follows the same general law.

8. DISCUSSION AND CONCLUSIONS.

Within the group of biotic environmental factors, those which operate directly through the activities of members of the same species, and result in an inhibition of the rate of population-growth, may be termed *autobiotic* factors of control. As the previous experiments have shown, these factors become more intensive in their action as the population-density increases. While it is comparatively easy to describe the changes in the activities of the insects with increasing density of population, even with the precision of a mathematical formula, the reasons underlying the changes present a difficult problem, involving the question of "purposiveness."

Dealing first with the effects of density upon the frequency of copulation, it is fairly obvious that the reduction of frequency in densities below the optimum is due to the decreased opportunities of meeting of the sexes. The reason for the reduction in frequency when the population attains a certain density is apparently due, primarily, to the effects of excessively numerous mutual contacts. Direct observation of the weevils shows that in sufficiently low densities they remain comparatively quiescent, but under crowded conditions of existence, or if roughly handled, the insects become very active. The weevils are, therefore, extremely sensitive to mechanical stimulation, including contacts with other individuals, and it seems not unreasonable to assume that excessive contact-stimulation, in addition to preventing favourable opportunities for copulation, may provoke a reaction which depresses the stimulus to copulation.

In the lower densities the effect of raising the sex-ratio (males/females) above unity is to increase the percentage of females *in copula*, whereas, beyond a certain degree of crowding, the increased proportion of males causes a reduction in the percentage of females *in copula*. This relationship can be reversed by lowering the sex-ratio to less than unity. The obvious conclusions are, firstly, that copulation-frequency of the weevils is dependent upon the density and sex-ratio of the population, an excess of males over females being advantageous to this end in low densities, but the reverse conditions being the more favourable above a certain degree of crowding; secondly, in depressing the copulation-frequency, crowding has a differential effect upon the sexes, operating with greater intensity upon the males than upon the females. Why this should be so is not clear, unless a greater sensitivity to crowding causes greater activation of the males; or there may be competition amongst the males for the females.

Regarding the decrease in rate of oviposition with increasing density, mathematical analysis shows that it obeys the same general law as that for the copulation-frequency, but the optimum density for rate of oviposition is slightly lower than the optimum for frequency of copulation. However, when the copulation-frequency begins to decrease it does so at a greater rate than the decrease in rate of oviposition. This greater decrease in copulation-frequency relative to the number of eggs laid is partly responsible for the decrease in fertility with increasing density; but another factor concerned in the production of infertile eggs, in the highest densities, is the puncturing of eggs by the rostra of the weevils in their probing of the grains for food and for the purpose of oviposition. Sectioning of the grains revealed many of these punctured eggs.

The continuous fall in the rate of oviposition despite an optimum for frequency of copulation nullifies any attempt to explain the fall as a direct result of the decrease in copulation-frequency. This factor may be in operation, but its effect is masked by a more powerful influence depressing the oviposition-rate, namely, the increased contacts between individual weevils. Although the primary effect is mechanical, it operates organically through the reduction of the times available for feeding, ovipositing, and resting; thereby causing adverse effects upon the physiological processes of reproduction.

Since the population-density productive of the maximum copulation-frequency is higher than that productive of the greatest number of fertile eggs, it follows that copulation may take place in excess of physiological requirements, and is largely a matter of the chances of meeting of the sexes. This provides another illustration of a previous suggestion (MacLagan, 1932 *a*) that living organisms are not always the highly co-ordinated physiological units they are often represented to be, and that some should be regarded as heterogeneous assemblages of diverse physiological processes, each with its own optimum, rather than harmonious physiological units.

The most recent attempt to explain why intermediate densities of *Tribolium confusum* have a greater initial increase than higher or lower densities is that of Park (1933), who says: "This was found to be due to the interaction of two factors. The first was egg-eating . . .; the second factor was the question of frequency of copulations. Since more copulations were occurring in concentrated populations, this fact favoured greatest increase in maximal-sized groups. The interaction of these factors . . . would cause an intermediate-sized population to have greatest initial increase." Unfortunately, Park overlooked the possibility of a decreased copulation-frequency above a certain degree of crowding, although this provides the key to the situation, as the decrease in population is greater than can be completely accounted for by eating of the eggs.

The fact that, as the space available for oviposition diminishes, the female weevil utilises it more efficiently, may give the impression of a "space-consciousness" on the part of the weevils, but it is more apparent than real, and is a result of the combined effects discussed above. Similarly, any attempt to construe purposive activity by the weevils, respecting the limitation of their numerical increase according to the conditions imposed by the environment, is immediately abandoned, on considering the percentage of eggs which give rise to adult insects, in the different densities.

Thus, at a density which produces an average of 0·17 eggs per grain all the eggs produce adults; when the average is 6·1 eggs per grain only one-fourth produce adults; and when the average is 7·5 eggs per grain slightly over one-eighth produce adults. The mortality with increasing density is, therefore, enormous, although there is actually a continual rise in the number of survivors up to a density productive of 6·1 eggs per grain, when the average number of surviving adults is 1·5 per grain. It might be said that the main purpose of the female is accomplished if she succeeds in increasing the total adult survivors irrespective of the total mortality, but by continuing to increase the average number of eggs per grain above 6·1 her object is defeated, as this results in a decrease in the number of surviving adults. From whatever angle the behaviour of the weevils may be regarded, then, it can only be described as irrational, the effective control of population-growth with increasing density being brought about by influences which are, primarily, thigmotropic, and therefore operate automatically. The profound importance of animate contacts in the lives of animals receives support from the work of Uvarov (1928) and of Faure (1932).

The increasingly heavy mortality with increasing density which occurs between the first instar larva and the adult weevil is the result of competition for food, accidental injury of one larva by another within the grain, and cannibalism; the last two factors being the most important of the three, as they allow a certain number of larvæ to survive at the expense of the others, so minimising direct competition for food. When subjected to detailed analysis, the so-called density-effect is thus seen to be due to a complex of factors affecting the actual and the potential numbers of the population. Moreover, as the population increases these factors tend to come into operation in a definite sequence, such that the inhibiting effect of environment upon the potential number of individuals precedes the decimating effect of environment upon the actual number; the lowering of fecundity and fertility being eventually accompanied by a rise in the amount of mutual injury and the intensity of competition for food. The latter may ultimately resolve into cannibalism.

The ecological significance of crowding, in relation to migration and the epidemiology of animals, is becoming increasingly realised, and there is little doubt that this factor comes into operation long before there is any shortage in the food-supply. Suffice it to say, that, in the limitation of numbers, the more favourable the physical factors the less significant do they become, and the greater the importance of the rôle of autobiotic factors. In Nature, these will be of greatest moment when the population approaches "plague" dimensions. It seems, therefore, that natural populations can exert an automatic check on their numerical increase, and that the organism itself imposes the ultimate limit to its own abundance when all other factors normally inhibiting population-increase have failed.

9. SUMMARY.

1. Animal populations offer a certain internal resistance to their own growth. The factors which constitute the internal resistance vary in their significance, depending upon the species, the population-density, and the sex-ratio.

2. In *S. oryzæ* the resistance is brought about by a lowering of fecundity and fertility, and an increase in the mortality of the young larvæ and the eggs.

3. There is no indication of any "conscious" effort of the weevils to limit their rate of reproduction as the population-density increases; the limitation of numbers results from influences which are, primarily, thigmotropic, and thus operate automatically.

4. The copulation-frequency increases up to a certain density, and then decreases according to a definite biological law, such that $\log Y = \log a - b \log X$, where Y is the copulation-frequency and X the population-density.

5. In virtue of the internal resistance (which is a complex of factors affecting the actual and potential numbers of the population) natural populations can exert an automatic check on their numerical increase; and hence, the organism itself imposes the ultimate limit to its own abundance in the absence of other factors of control.

10. REFERENCES TO LITERATURE.

BARNES, J. H., and GROVE, A. G., 1916. *Mems. Dept. Agric. India* (Chem. Ser.), vol. iv, p. 192.
CHAPMAN, R. N., 1928. *Ecology,* vol. ix, p. 111.
FAURE, J. C., 1932. *Bull. Ent. Res.*, vol. xxiii, p. 293.
HINDS, W. E., and TURNER, W. F., 1911. *Journ. Econ. Ent.*, vol. iv, p. 230.
MACLAGAN, D. S., 1932. *Proc. Roy. Soc.*, B, vol. cxi, p. 437.
MACLAGAN, D. S., 1932 *a*. *Bull. Ent. Res.*, vol. xxiii, p. 188.
MACLAGAN, D. S., and DUNN, E., 1935. *Nature*, vol. cxxxv, p. 33.
PARK, T., 1933. *Journ. Exp. Zool.*, vol. lxv, p. 17.
UVAROV, B. P., 1928. "Locusts and Grasshoppers," *Imp. Bur. Ent.*, London.

(*Issued separately January* 11, 1936.)

OBITUARY NOTICES.

Henry Briggs, O.B.E., D.Sc.

THE premature and unexpected death of Henry Briggs in a London nursing home on August 26, 1935, was a loss to the Society of one of its active members, and to the University of a widely known and highly esteemed professor.

Henry Briggs was born at Stott Park, Lakeside, Lancashire, in 1883. His early education was at Bradford. At the age of sixteen he gained a National Scholarship, and Queen's Prizes in mathematics and applied mechanics, which enabled him to proceed to the Royal School of Mines, of which, after experience in several mines, he passed as Associate, afterwards receiving the very exceptional distinction of Honours Associate. He then served as Research Assistant and Demonstrator at the School of Mines under a very inspiring chief, Sir Clement le Neve Foster. In 1905 he became Lecturer in Mine Surveying at Birmingham University under Professor (now Sir Richard) Redmayne, and also Lecturer in Mining for Warwickshire. From these positions he was appointed in 1907 the first Lecturer in Mining at the Heriot-Watt College, Edinburgh, and soon began to make his mark as a teacher and investigator, gathering round him pupils and other friends, and developing vigorously the organisation and equipment of his department. In 1919 his lectureship was converted into a chair, and in 1924 Mr James Hood, Chairman of the Lothian Coal Company, who was impressed by the value of Briggs's work, presented to the University an endowment for a University Chair of Mining, which was held by Briggs, along with the Chair in the Heriot-Watt College, till his death.

Briggs was at no time a mere specialist. On looking at the long list of papers which he published, what is perhaps most striking is the variety of the subjects which he had investigated and written about. This variety brought him into contact with mining engineers and investigators of mining subjects throughout the country; and the Edinburgh school of coal-mining soon began to take a high place as a centre of knowledge and training place for mining engineers.

In a short memoir such as the present it would be impossible to give an adequate account of Briggs's many investigations, but I may perhaps refer to some in which I was specially interested. Before the War Briggs had begun to study the physiology of respiration in its relation to mining and mine-rescue apparatus, and we made some joint experiments on this subject at Newbattle Colliery, where ample facilities were given us by the management. As a development of these observations and others made in his laboratory with a bicycle ergometer, Briggs discovered that in men who are not in good physical condition the percentage of carbon dioxide in expired air does not rise, and may fall, during the increased breathing produced by muscular exertion, unless oxygen is added to the inspired air, whereas with men who are physically fit there is a very marked rise, which is not added to when oxygen is breathed. The physically fit man breathes, therefore, less air with the same physical exertion, and this is apparently due to active secretion of oxygen inwards by the lungs. On those observations Briggs based a method of testing recruits for physical fitness, and this was used in Edinburgh during the War.

A subject to which he gave much attention was the development and proper use of self-contained breathing apparatus for use in poisonous or irrespirable air in mines. A central mine-rescue station was established on a site off the Grassmarket, belonging to George Heriot's Trust, and this not only served the Lothian mining district but led to the establishment of efficient stations in other Scottish mining districts. New forms of breathing and reviving apparatus were also developed and tested at the rescue station or at the Royal Infirmary in connection with medical investigators there.

During the War a small Committee on Mine-rescue Apparatus was appointed by the newly appointed Department of Scientific and Industrial Research. Briggs took a leading part in the work of this Committee, and carried out numerous experiments published by it in three successive Reports.

Another subject on which he contributed several papers was the adsorption of gases by coal and other substances, and the sudden outbursts of gas which sometimes occur from disintegrated coal which is highly charged with adsorbed methane.

Further subjects on which Briggs wrote valuable papers were surveying, including the surveying of bore-holes; the laws of surface subsidence brought about by mining; the laying of dust on mine roadways; and testing for oxygen with a flame-lamp. One of his last experimental papers, and one of several published in the *Transactions* and *Proceedings* of the Society, was on the contraction which is produced in coal when adsorbed gas escapes. Mention should also be made of a number of interesting historical notes and several short popular books on mining.

Early in his scientific career Briggs married Myfanwy Williams, daughter of a well-known Welsh writer; and she became almost as well known as her husband at scientific meetings south of the border and as a genial hostess in Edinburgh. Deep sympathy for her is associated with the widespread regret caused by her husband's death.

He was elected a Fellow of the Society in 1916, and served on the Council from 1923 to 1926.

J. S. H.

David Lawrence Bryce.

DAVID LAWRENCE BRYCE was born in Edinburgh in 1852 and received his education at the Royal High School. While still under twenty he went to London, where, except for a year in Germany, he remained till he retired from business in 1925.

In his earlier years he was interested generally in natural history, and used a microscope which he is credited with having constructed himself. His admission to the Hackney Microscopical Society in 1884 opened up a new field to him—the Rotifera; and his enthusiasm was raised to the pitch of making a manuscript copy of the monograph by Hudson and Gosse for his own use. The Bdelloida chiefly engaged his attention and form the subject of numerous communications appearing in the *Journal of the Quekett Microscopical Club* and elsewhere, a noteworthy contribution being his new classification of the Bdelloid rotifers (*Journ. Q.M.C.*, November 1910). The article on Rotifers in the last edition of the *Encyclopædia Britannica* bears his initials.

His ambition was to write a comprehensive work on the rotifers of Scotland, for which he collected much material, but this he did not accomplish, as the material reaching him from various parts of the world, and involving a heavy correspondence with specialists in other countries, absorbed most of his available time.

Of his admissions to scientific societies none seems to have afforded so much gratification as his election as a Fellow of the Royal Society of Edinburgh in 1927, which he regarded as a reward for the hours, after a long day's work at business, spent in his studies. He died on October 26, 1934.

W. W.

James Chatham.

THE death of Mr James Chatham, one of the most distinguished actuaries of his day, calls for more than a passing remark, although with the lapse of time he was but little known personally to the present generation.

Born in Edinburgh in 1857 and educated here, it was not originally intended that he should follow an insurance career. He entered a merchant's office in Leith and remained there for some years. At the age of twenty-two he was offered and accepted a post in the Head Office of The Scottish Equitable Life Assurance Society, where for fourteen years he was assistant to that great actuary, Dr T. B. Sprague. In 1893 he received his first official appointment, that of an Assistant-Actuary to the Edinburgh Life Assurance Company. Then followed, in 1895, further promotion, when he became Secretary and Actuary, and second officer, of The Scottish Life Assurance Company—a post which suited him admirably and gave full scope to his great talents.

A Fellow of the Statistical Society and also a Fellow, by examination, of both the Faculty and the Institute of Actuaries, he gained the first prize in two open competitions of the latter body—in 1890 the one offered by Dr Sprague, and in 1895 the Messenger Prize, for the best essays on Mortality Experiences and Valuation Methods respectively. The *Journal of the Institute of Actuaries* and the *Transactions of the Faculty of Actuaries* contain ample evidence of the value of his contributions to actuarial science, which are too numerous to mention, but his name will always be associated with two developments in life assurance practice. The first relates to the valuation of the liabilities of an office, and the second to the preparation of renewal premium receipts and notices. By the methods he devised the former can be accomplished in as many days as it used to take months; and the latter in as many hours as it used to take weeks. This second reform consisted of the adaptation, in 1904, of the addressograph to the printing of receipts, etc.—a practice which has spread, it is believed, all over the world.

James Chatham was by no means only an actuary. He was an all-round capable man of business: an able administrator, an authority on investments, a wise counsellor, and a man of high personal character.

It was these qualifications that, in 1906, led the Directors of the North British and Mercantile Insurance Company to appoint him Manager of the Company in Edinburgh. This important post he held only for some six years, as unfortunately his health had become impaired and necessitated his retirement.

On his resignation he went to live at Melrose, on Tweedside, where for more than twenty years he led a pleasant, health-restored life, contentedly retired from the arena of official life.

He was elected a Fellow of the Society in 1899, and died after a short illness on December 28, 1934.

L. P. O.

Shepherd Dawson, M.A., D.Sc.

By the sudden and unexpected death of Dr Shepherd Dawson, at the height of his powers, British psychology has suffered an irreparable loss. Born at Whitehaven, Cumberland, and educated at Owens College, Manchester, and King's College, London, Shepherd Dawson's main life-work was done in Glasgow, at the University and at Jordanhill Training College, but his influence in his science extended far beyond Glasgow. Though his main activities were involved in educational psychology, as principal Lecturer in Psychology, Logic, and Ethics at Jordanhill, Shepherd Dawson was recognised as one of the ablest experimental psychologists in the country, and his work on the frontiers between psychology and medicine must also be regarded as of the first importance. Articles in the *British Journal of Psychology* on the phenomena of binocular vision, particularly on phenomena of brightness and of colour-mixing, are models of their kind of work; the same may be said of articles in *Archives of Diseases of Childhood* on the influence on intelligence of encephalitis lethargica, epilepsy, etc.

During his period of work in London Shepherd Dawson studied statistics under Karl Pearson, and his interest in this field remained with him, and issued ultimately in the publication, in 1933, of his *Introduction to the Computation of Statistics*, a book which is quite invaluable for anyone who seeks to apply statistical methods to psychological and educational problems.

As an old and active member of the British Association—Secretary, Recorder, and, in 1934, President of Section J—Shepherd Dawson was widely known among men of science in this country; as Contributing Editor for Britain and the Colonies of the *Psychological Index* for many years, he was no less widely known in other countries. As a member of the Scottish Council for Research in Education since its institution, and as Convener of its Publications Committee and of its International Examinations Enquiry Committee, he has left a deep mark on contemporary educational and psychological science in Scotland.

A colleague of Shepherd Dawson's, writing to the author of the present notice, says of him: "He acted as a 'scientific conscience' to teams of research workers in a variety of problems connected with education, children's diseases, housing, and the like," and this "strict 'scientific conscience' has proved of incalculable value to research work in Scotland and may long serve as a standard for work in these fields." No estimate could be truer or juster.

He was elected a Fellow in 1931, and died on March 26, 1935.

J. D.

Thomas Cuthbert Day, F.C.S.

MR T. CUTHBERT DAY died in Edinburgh on June 13, 1935, in his eighty-third year. He was born at Burton-on-Trent, where his father, the Rev. Henry Day, was headmaster of the Grammar School. In his early years Day was a keen student of chemistry, and to various practical applications of that science he devoted himself enthusiastically throughout a long and active career.

For many years Day was engaged in the brewing industry, and about forty-five years ago he came to Edinburgh to join the staff of one of the city's brewing firms. During this part of his career he wrote a number of papers dealing for the most part with experiments on germinating wheat and barley. These were published in the *Journal of the Chemical Society*, the *Transactions of the Botanical Society of Edinburgh*, and the *Proceedings of the Royal Physical Society of Edinburgh*.

On account of a change in his religious outlook Day decided to sever his connection with the brewing industry, and he later found a congenial outlet for his chemical skill when, in the late nineties, he joined the firm of Hislop and Day at a time when many new developments were impending in photo-engraving. Day visited the Continent in order to study the newest processes, particularly in colour reproduction, and he was mainly responsible for developing the firm's work on thoroughly scientific lines.

Day was a Fellow of many scientific societies, including the Royal Society of Edinburgh, to which he was elected in 1917, the Edinburgh Geological Society, of which he was President in 1921 and 1922, the Botanical Society of Edinburgh, the Royal Physical Society of Edinburgh, and the Scottish Natural History Society. During the last thirty years of his life Day devoted himself enthusiastically to geological research, contributing fully a score of papers to the *Transactions of the Edinburgh Geological Society*. His most noteworthy achievements in field geology are recorded in his papers dealing with his successive discoveries of many unrecorded volcanic necks in East Lothian. It was along chemical lines, however, that Day worked most profitably for the advancement of geological knowledge. His long series of chemical analyses of the igneous rocks of Fife and the Lothians forms a contribution of outstanding merit, and will be of great service to future workers engaged in petrological research.

Day did much to popularise geology by lecturing and by leading field excursions. He wrote also a beautifully illustrated volume on "*Arthur Seat. A Ruined Volcano*" as a guide for beginners.

Ever ready to place his skill as chemist and photographer at the service of others, Cuthbert Day was held in the highest esteem by his fellow-workers. His genial presence will be missed at the meetings of the many societies of which over a long period he had been such an active member.

R. C.

Sir James Alfred Ewing, K.C.B., M.A., D.Sc., LL.D., F.R.S.

JAMES ALFRED EWING, a great engineer, scientific man, and charming Scot, was distinguished in many fields of activity. Born in 1855 at Dundee, where his father was a minister of the Free Church of Scotland, he grew up in an intellectual and spiritual atmosphere in which his early education was greatly influenced by his mother, whose critical taste in letters was passed on to her children.

From Dundee High School Ewing proceeded in the early seventies to the University of Edinburgh, where his great ability soon manifested itself in the classes of Tait and Fleeming Jenkin, the latter of whom brought him into touch with Sir William Thomson (afterwards Lord Kelvin), then engaged in the development of submarine telegraph cables, an introduction which led to Ewing taking part in three cable-laying voyages to Brazil and the River Plate. Fleeming Jenkin was a man of wide culture, and his literary and scientific circle included Ewing and another young student, Robert Louis Stevenson, then at the commencement of an engineering training, soon to be abandoned for a literary career of the greatest distinction, the beginnings of which have been happily described by Ewing in "The Fleeming Jenkins and Robert Louis Stevenson."

In 1878, at the early age of twenty-three, Ewing went to Japan as Professor of Mechanical Engineering in the University of Tokyo, a post which he occupied for five years, and there he made his classical researches in the widely different fields of magnetism and seismology.

While in Tokyo he married his first wife, Miss Washington, a great-great-grand-niece of the first President of the United States of America. Their two children, born in Japan, reached mature years before their mother's death.

In 1883 he returned to Scotland, becoming Professor of Engineering at University College, Dundee, where he continued his research work, the great value of which was recognised by his election into the Royal Society in 1887.

At this period the importance of engineering science as a suitable field of activity within a university was becoming recognised by the older English universities, and at Cambridge a school of engineering, somewhat thinly disguised under a non-committal name, had been brought into being by the activities of James Stuart, who later, becoming more interested in politics than engineering, resigned his Chair. By great good fortune Ewing was appointed to succeed him, mainly on the advice of John Hopkinson, the eminent electrical engineer and mathematician. Up to that time (1890) only a pass degree was available for students of engineering, and it says much for the tact and organising ability of Ewing that, within a very short time, he was able to persuade the University to establish a Mechanical Sciences Tripos and also to obtain funds to build an engineering laboratory next door to that great centre of physical science, the Cavendish Laboratory.

The new tripos proved a great attraction to men who, with a year to spare, had already taken the Mathematical or Physical Sciences Tripos examination, while the number of students who came to devote three years to an organised course of study of engineering science began to grow steadily and continued to do so throughout the thirteen years during which Ewing continued as Professor in Cambridge; and so well was the course planned that it continued to attract increasing numbers until it is now the largest University School of Engineering in this country.

A good deal of administrative work fell necessarily on Ewing, but he had a very sure instinct in the selection of assistants who could give material help, among the early ones being Dalby, Lamb, and Peace, and he found time to build up a research school and to contribute much himself to applied science by researches either alone or in collaboration with his pupils, especially on magnetic hysteresis and the crystalline structure of metals in the plastic state, described in a Bakerian Lecture with Rosenhain as co-author and subsequently developed in other papers.

This period marks the development of many new instruments of precision for use in engineering laboratories, and even more notably the writing of text-books for undergraduate students of engineering, distinguished for their clarity and literary style. Probably the best known of these works is a text-book on the steam-engine and other prime movers, which has passed through many editions and translations. His *Thermodynamics for Engineers* was another fine contribution for more advanced study.

In addition, his help and advice were much sought after by eminent engineering firms and also by the Government. It was mainly on his advice that a new scheme of naval education was drawn up, and this ultimately led to his becoming the first Director of Naval Education under the Selborne scheme, whereby engineering training became an integral part of the education of all naval officers.

This was a great loss to the Cambridge School of Engineering, although fortunately a man was available in Bertram Hopkinson to carry on its programme and to extend it. The new Admiralty scheme, involving great changes, was a complete success owing to Ewing's administrative gifts combined with a tactfulness which disarmed some opposition to the considerable alterations then made. The commencement of the War found Ewing still Director of Naval Education and brought him an opportunity which he described as a wonderful piece of good fortune, but might equally well be described as a happy inspiration on the part of the authorities in choosing the right man for a post of singular delicacy. He was asked to undertake the deciphering of enemy cipher, a task which he and the staff he gathered round him brought to great perfection.

Readers of tales of mystery may well imagine the veil of secrecy in which the now famous "Room 40" was enshrouded. One pictures an almost inaccessible chamber approached through locked doors and past armed guards. In reality it was quite an ordinary room, near the Admiralty entrance, opening into a main corridor, with the door wide open. In it Ewing and three or four others were seated at desks of a pattern used by the humbler members of the Civil Service and covered with bundles of papers. No doubt the means of ensuring secrecy were there, but there were no traces of this. It was all part of the great war game then being played out.

It was while carrying on this work that Ewing was invited to return to his *alma mater* as Vice-Chancellor and Principal, and although somewhat reluctant to commence so great a task at the age of sixty-two he was persuaded to do so.

During his twelve years in Edinburgh he strove with great success to adapt and widen the University for the many new duties which the post-War era brought with it. Many chairs and lectureships were founded for which new buildings were required on an extensive scale, but Ewing's zeal and persuasive tongue carried through a scheme involving an expenditure of more than three-quarters of a million sterling, most of which was contributed by private benefactors and public trusts without impairing the general finances of the University.

In the no less important social activities of the University he and his second wife, a daughter of John Hopkinson, were a great accession to Edinburgh, but even all this did not complete the tale, for Ewing found time to engage in a good many outside activities such as the Chairmanship of the Bridge Stress Committee, and another on the testing of timber. Perhaps the most notable of these was his Presidency of the British Association at York in 1932, where in a masterly survey of the progress of physical science during the last century, and especially in the latter half of it, he sounded a note of warning and expressed a feeling that man was ethically unprepared for the bounty bestowed upon him.

Ewing was the recipient of many honours. He became a Knight Commander of the Order of the Bath in 1911, and held honorary degrees of many universities. His retirement from the University was marked by the conferment of the freedom of the City of Edinburgh, and later by that of his native city, Dundee, on the occasion of a visit of the Institution of Mechanical Engineers, of which body he was an honorary life-member.

Many other distinctions were showered upon him, and it is of especial interest here to note that his connection with the Royal Society of Edinburgh was long and intimate, for he became a Fellow in 1878, was a frequent contributor to its *Transactions* and *Proceedings*, served as Vice-President from 1920 to 1923, and finally became its President from 1924 to 1929.

He died in Cambridge on January 7, 1935. (See also *Obituary Notices of Fellows of the Royal Society*, No. 4, 1935, p. 475.)

E. G. C.

James Gordon Gray, D.Sc., M.I.E.E.

PROFESSOR GRAY died suddenly at his home at Dowanhill on Tuesday, November 6, 1934. From the beginning of the session he had been absent intermittently, but he had delivered two lectures on the previous Monday, and his death came as a great shock to everyone.

Professor Gray was born in Glasgow in 1876. He was the second son of the late Professor Andrew Gray, F.R.S., and was educated at Bangor where his father was Professor of Physics. But when his father was appointed to the Chair of Natural Philosophy in the University of Glasgow in 1899, Gray entered as a student at Glasgow, and took the B.Sc. course in electrical engineering. In 1904 he was appointed to his father's staff as assistant, in 1908 he became senior lecturer in the department, and in 1920 he was appointed to the new Chair of Applied Physics founded by Sir John Cargill for the teaching of physics to students of medicine and engineering. In 1911 appeared the *Treatise on Dynamics*, written in collaboration with his father, which is well known as a text-book for University students.

His first scientific investigations were on magnetism, for which he obtained the D.Sc. degree in 1908; some of these are described in the Society's *Proceedings*. In this work he was associated with A. D. Ross, who afterwards became Professor of Mathematics and Physics in the newly founded University of West Australia. In 1910, at his father's suggestion, he took up the study of gyrostats, a subject which had been associated with the department since the time of Lord Kelvin, and this became his life-work. At first he contented himself with constructing models for demonstrating the properties of gyrostats, walking and climbing gyrostats, gyrostatic bicycles and tricycles, pole-balancing tops, etc., but later proceeded on to original devices, and during the War achieved his greatest success, the gyrostatic stabiliser. This provided a platform for use in aeroplanes, amongst other purposes, which remained absolutely steady no matter how the aeroplane rocked, and from which bombs could be dropped with very much greater accuracy than from an ordinary aeroplane.

He gave frequent demonstrations of his inventions before the Royal Societies of London and Edinburgh and other scientific bodies, and he was in great request as a lecturer on the subject, being celebrated as an authority on gyrostats both in this country and in America.

As a teacher he was clear and painstaking. He retained throughout life an ability to see matters from the students' point of view, and he had a gift of humour which endeared him to the weaker members of the class. He inherited from his father a strong fund of common sense, and from him gained a knowledge of University affairs which made his judgment very reliable. His death breaks a direct connection of thirty-five years with the University of Glasgow, and severs a link, through his father, with the classic days of Lord Kelvin's laboratory.

He was elected a Fellow of the Society in 1909.

R. A. H.

Hector Munro Macdonald, O.B.E., M.A., LL.D., F.R.S.

Hector Munro Macdonald was born on January 19, 1865, in Edinburgh. His father, Donald Macdonald, and mother were both Highland and belonged to the Ross-shire parish of Kiltearn, near the Cromarty Firth. Kiltearn is largely in the estate of the Munros of Foulis, the lairds of which were frequently called Hector. Hence, indirectly, the names given to the boy are accounted for.

He received his earliest education in Edinburgh, where his parents remained till he was ten or eleven years old. Then they went to a farm at Fearn, Easter Ross, and his education was continued at the Hill of Fearn Public School.

For his secondary education he went to Tain Royal Academy, whose rector, John Scott, was a mathematician. Rector and pupil worked at mathematics together. A short period followed at the Grammar School in Old Aberdeen. He entered the University of Aberdeen in 1882, and graduated in 1886 with First Class Honours in Mathematics and Natural Philosophy, winning the Simpson Mathematical and Arnott Experimental Physics Prizes and a Fullerton Scholarship.

Proceeding to Clare College, Cambridge, he graduated in 1889 as Fourth Wrangler. He became a Fellow of Clare in 1890, won the Smith's Prize in 1891 and the Adams' Prize in 1901. The Royal Society of London elected him a Fellow in 1901, and in 1916 awarded him a Royal Medal.

At Clare Macdonald was in close touch with Mr Ewbank, the College bursar, an able administrator of the College estates, and for some years had an intimate knowledge of the College property and its administration. This and his acquaintance with agriculture, as a farmer's son, were turned to good account when, in 1904, he became Professor of Mathematics in Aberdeen, and, in a few years, took over much of the administrative work of the University, especially in all matters connected with buildings.

In July 1916 Professor Macdonald joined the Ministry of Munitions and was attached to the Wages Section of the Labour Department. He soon became head of this section, to which wages disputes found their way and in which the officials were under almost constant strain. At a later date he took over additional duty as chairman of an important committee set up to regulate the demands of certain Government departments for building labour. Macdonald's knowledge of labour conditions, especially in the building trades, his unusual memory for figures, his inexhaustible patience, and his powers of physical endurance made him an invaluable member of the administration, and particularly fitted him for the work he was called upon to do. He was not infrequently required to go into the provinces and effect a settlement of some dispute. His services were of such importance that he was asked and consented to continue in the Ministry of Labour for the period of reconstruction that followed the War. He did not return to Aberdeen until September 1919.

Mountaineering was Professor Macdonald's great recreation. He climbed most of the highest peaks of the Bernese and Valaisian Alps, his favourite guide being Emmanuel Byrgener of Sass. On the march he was tireless, not given to an excess of speech any more than at other times, but a fine view or colour-effect he would recall long afterwards. He joined the Alpine Club on February 6, 1906. The Scottish Mountaineering and Cairngorm Clubs he never joined, but the Cairngorms and the Ross-shire hills he knew. Perhaps one of the last long expeditions he made was an all-night cross-country walk from Alltdourie to Inchrory by the Sneck and the summit of Ben Avon on June 30, 1934. At that time he was going quite well with his long mountaineering stride.

Though a true Highlander, he was never a fluent Gaelic speaker. His knowledge of Gaelic was gained during visits to his grandmother, and he would run into the house to ask her the meaning of Gaelic phrases used by other boys. She taught him to read the Gaelic Bible. He was a regular attender at the annual Gaelic Service in King's College Chapel, Aberdeen, and was often at students' Gaelic society meetings. His chief work for the Highlands was as Governor, from February 1916, of the Highlands and Islands Educational Trust, of which his intimate knowledge made him a most valuable member. He became Convener of the Educational Committee in 1924, and in 1930 Chairman of the Trust. His recollection of the circumstances of the various beneficiaries was remarkable.

Until the Spring of 1935 he enjoyed good health. Characteristically, when he agreed to enter a nursing home on Tuesday, May 7, for examination, he refused to let his relatives and friends know. He was elected a Fellow of this Society in 1905 and died on Thursday, May 16, 1935.

For an appreciation of Professor Macdonald's scientific work reference may be made to the *Obituary Notices of Fellows of the Royal Society*, No. 4, 1935, p. 551.

J. G.

Sir William Leslie Mackenzie, M.A., M.D., LL.D.

William Leslie Mackenzie was born in Ross-shire and educated mainly at the University of Aberdeen, where he studied the classics, philosophy, and medicine, graduated M.A., M.B., C.M., and M.D. with Honours, and qualified as D.P.H.

After preliminary terms of service in Galloway and Leith he was appointed Medical Inspector under the Local Government Board for Scotland. In 1904 he was promoted to the major position of Medical Member. In 1919, on a rearrangement of offices, he joined the Scottish Board of Health. He retired from the Civil Service in 1928, but in his retirement still kept touch with his lifelong interests. He visited the State of Kentucky to inaugurate there a frontier nursing service on the Scottish model. He remained the Crown nominee for Scotland on the General Medical Council till his death on February 28, 1935.

While much of Mackenzie's administrative work was carried on under the self-denying anonymity observed by state officials, his name can be directly associated with a number of *Reports* and *Proceedings* which were the precursors of health legislation previously unexampled in Scotland. The evidence which he gave, with Dr Matthew Hay, before the Royal Commission on Physical Training, his report on Scottish Mothers and Children, his revelation of the forlorn condition of the Highlands and Islands, and his work as a member of the Committee on Tuberculosis were all instrumental in promoting statutory and other remedial measures In his own view his membership of the Royal Commission on Housing was his most valuable effort in the cause of public health.

He brought to his duties as a public official acute and original powers of mind, already exercised, as above indicated, in the traditional culture of his native land. His philosophy became to him a compelling force, justifying his innate humanity and stimulating his will to action for the welfare of the people. When his major period of public service began, health activities in Scotland were chiefly environmental in outlook. When it ended, policy was being strongly guided, under the promptings of the national conscience, along the path of social service.

Mackenzie, if not the sole creator—for others also rendered valuable aid—was the inspiration of the new régime thus brought into being. Alike by his convincing disclosures of existing health deficiencies, his lucid and fervent understanding of their threat to the future of the race, and his administrative rôle in devising the remedies, he made his pervasive influence felt almost at every turn, and may without any stressing of the term be fitly ranked, in his own demesne, among the makers of modern Scotland.

He received his knighthood in 1919. He was LL.D. of Aberdeen (1912), and an honorary medallist of the University of Brussels.

He was elected a Fellow of the Society in 1904.

J. R. C.

John James Rickard Macleod, M.B., Ch.B., D.Sc., LL.D., F.R.S.

PROFESSOR MACLEOD was born at Cluny, near Dunkeld, on September 6, 1876. He was educated at Aberdeen Grammar School and at the University of Aberdeen. After a distinguished undergraduate career he qualified M.B., Ch.B., with Honours, in 1898 and was awarded the Anderson Travelling Fellowship. After a year's work in the Physiology Institute of the University of Leipzig, he was appointed first a Demonstrator in Physiology and later Lecturer in Biochemistry at the London Hospital Medical College, holding at the same time the McKinnon Research Studentship of the Royal Society of London. In 1903 he was appointed Professor of Physiology at the Western Reserve University, Cleveland, Ohio, a post he held until 1918 when he was selected as Professor of Physiology at the University of Toronto. He remained at Toronto until, in 1928, he was appointed Regius Professor of Physiology in the University of Aberdeen, a post which he filled with acceptance, despite constant ill-health, until his early death at the age of fifty-eight on March 16, 1935.

Macleod's name will always be identified with research work on the metabolism of carbohydrate and more especially with the epoch-making discovery and isolation of insulin. This was no mere chance discovery. Macleod had first-hand knowledge of the whole literature of carbohydrate metabolism in general and of diabetes in particular, he had equally a first-hand knowledge of the necessary experimental technique, and his outlook was catholic. His interest in the fate of carbohydrate in the body began as far back as 1905, and before the experimental work which led ultimately to the discovery of insulin in 1921 was commenced he had published thirty-seven papers on problems relating to the metabolism of carbohydrate. Thus when F. G. Banting came to discuss with him in 1921 the possibility of attacking the problem of obtaining an active extract from the pancreas, using duct-ligated gland, no more helpful and better equipped adviser could have been selected. Macleod encouraged Banting to engage in the research and offered him all the facilities of his laboratory. Banting, in conjunction with C. H. Best, accordingly set to work and obtained within a few months an extract which did exert a beneficial effect on experimental diabetes. This discovery was made public in December 1921. The difficulty which now confronted the investigators was that of obtaining a pure reliable substance in sufficient amount. A trained chemist, J. B. Collip, now joined the team, and soon the discovery was made that the active material could be extracted from normal ox pancreas in sufficient amount to render it practicable to treat patients suffering from diabetes mellitus. This discovery has literally saved thousands from an unduly early death. The original idea which started the train of research work and led up to the isolation of insulin was undoubtedly Banting's, but it may be said without fear of contradiction that, if it had not been for the active co-operation of Macleod and others, the result, if it had been attained at all, would certainly not have been attained so quickly. This investigation is one of the best testimonials to the value of team-work directed to a single end which could be selected.

Although the discovery of insulin was an important step in the understanding of the utilisation of carbohydrate in the body, it raised almost as many difficulties as it solved. Almost up to the end of his life Macleod was engaged in an attempt to elucidate many of these new problems.

In addition to his carbohydrate research work Macleod had pursued many other lines of investigation. Between 1899 and 1933 he, either alone or in collaboration, had published nearly two hundred papers. In addition to these papers he also published eleven books, the best known of which are his text-book *Physiology and Biochemistry in Modern Medicine*, which is now in its seventh edition, *Carbohydrate Metabolism and Insulin* (1926), and the *Fuel of Life* (1928). Further, he actively engaged in editorial duties, being, for example, one of the associate editors of *Physiological Reviews* and of *Nutrition Abstracts and Reviews*. He also found time to act as Dean of the Faculty of Medicine at Aberdeen and to serve from 1929 to 1933 on the Medical Research Council.

During his later years Macleod was the recipient of many honours. He was elected a Fellow of the Royal Society of Canada in 1919, of the Royal Society of London in 1923, of the Royal Society of Edinburgh in 1932, and of the Royal College of Physicians of London in 1930, and was elected a member of many foreign learned societies.

In 1923, jointly with F. G. Banting, he was awarded the Nobel Prize in Physiology and Medicine, and in the same year was elected Cameron Prizeman of the University of Edinburgh. In 1928 he was Vanuxem Lecturer at the University of Princeton, and in 1933 Herter Lecturer at Johns Hopkins University, Baltimore.

Macleod was an admirable lecturer, clear, happy, and interesting. As a director of research he was equally good. Although he would not tolerate for an instant careless work, he would listen to any kind of proposal for research work provided the applicant was honest and in earnest.

He was a sympathetic, enthusiastic leader who inspired devotion. As a man he was endowed with a most attractive personality and had wide interests outside his laboratory work. Honest, loyal, and just himself, he resented intensely all suggestion of chicanery or meanness. He was devoid of affectation and false pride. He was a cheery optimist who met everyone with a happy smile. During his last years, when crippled with arthritis, he required all his optimism. And it did stand him in good stead. These years displayed his true courage and greatness of heart to the utmost. To the very end his serene and happy spirit triumphed. (See also *Obituary Notices of Fellows of the Royal Society*, No. 4, 1935, p. 585.)

E. P. C.

James Musgrove, M.D., LL.D., F.R.C.S., F.R.C.S.E.

JAMES MUSGROVE, emeritus Professor of Anatomy in the University of St Andrews, was born in Kendal in 1862, and died in St Andrews on February 6, 1935, after a prolonged period of ill-health.

Musgrove was a graduate of the University of Edinburgh, taking the degrees of M.B., C.M., in 1886 and that of M.D. in 1888. He became a Demonstrator of Anatomy under Sir William Turner, and held that office for six years, and Turner's personality made a deep and permanent impression upon him. His ability and industry at that time were shown by taking the Fellowship of the Royal College of Surgeons of England, and, at a later date, that of the corresponding Royal College in Edinburgh.

He was elected a Fellow of this Society in 1907 and he was also a Fellow of the Anthropological Institute of Great Britain.

In 1894 Musgrove left the department of Anatomy in the University of Edinburgh and began extra-mural classes of Anatomy in Bristo Street, where he carried on active and vigorous teaching, and attracted considerable numbers of students to his classes.

In 1896 he was appointed to the newly instituted Lectureship on Anatomy in the United College of St Andrews University. The work of this post was arduous, and numerous difficulties had to be faced.

The early years of his work were fruitful, and in 1901 the third Marquis of Bute provided a permanent home for the teaching of medical subjects in St Andrews and endowed a Chair of Anatomy. Musgrove was appointed the first Bute Professor of Anatomy.

During his tenure of this office Musgrove's technical skill and artistic taste found scope in the preparation and arrangement of a Museum of Anatomy in the new buildings. He was a clear and lucid teacher, and he assisted and encouraged other workers in his department in their researches.

He was for a time the Dean of the Science Faculty, and in 1911, before the celebration of the quincentenary of the foundation of the University, he and Mrs Musgrove presented to the University a magnificently furnished Museum in memory of Professor Bell Pettigrew

His health began to fail, and in 1914 he resigned his Chair. The University conferred upon him the degree of LL.D. in 1916 in recognition of his services.

D. W.

Robert Alexander Robertson, M.A., B.Sc.

WITHIN four months of his retirement from the Chair of Botany in the University of St Andrews, Professor R. A. Robertson passed away. Thus terminated forty-four years of strenuous service for the University and a life devoted to the advancement of botany in Scotland.

Born in Rattray, Perthshire, in 1873, Robertson went from the village school to the University of Edinburgh, where he graduated in arts and science. Physiology under Rutherford and botany under Dickson and Bayley Balfour were the subjects most frequently mentioned by him as holding his interest while a student. In 1889 he came to St Andrews to assist Professor W. C. McIntosh in the practical zoology class, but he lectured in botany and in physiology. In 1891 he was appointed Lecturer in Botany, his status being raised to that of Reader in 1915. In 1929 a Chair of Botany in the University was instituted, and Professor Robertson became its first occupant. He retired from the Chair in September 1934, and passed away on January 22, 1935. To his pioneer work the University owes its present flourishing department of Botany, whose alumni have gained recognition for their chief across the seven seas.

Robertson was a great teacher. Labouring incessantly to keep himself proficient, he had the power of imparting knowledge, developing initiative, and creating enthusiasm. Pioneer work in the identification of timbers illustrated by microphotographs, the use of anatomy as an aid to taxonomy in the Genus *Potentilla*, together with advocating the conception of "functional inertia" by examples drawn from the vegetable kingdom stand out as lines of investigation during Robertson's early years at St Andrews. His fame as a teacher had its penalty. Classes increasing in numbers and conducted single-handed, in accommodation no longer adequate, rendered personal research impossible. A commodious new department of Botany was provided in 1932, but in the intervening years a steady stream of young enthusiasts worked under his guidance. The flow of publications continued under the names of the investigators, Robertson refusing to countenance joint authorship.

Captain Robertson will be remembered almost as much for his services to the Officers' Training Corps. He was an enthusiast for military training, and at the outbreak of the World War was Captain of the St Andrews

O.T.C. Due to his efficiency in training, St Andrews was able to take a worthy share when the crisis came.

Robertson, who was elected a Fellow of the Botanical Society of Edinburgh in 1890, became its President in 1915. He was elected a Fellow of the Royal Society of Edinburgh in 1902, and of the Linnean Society in 1903. He was elected to the Honorary Committee of Management of the Imperial Bureau of Mycology from its inception in 1922.

He was a man of high principles, who never stooped to subterfuge, steadfast in his loyalty to justice and honour, and was lacking neither in sympathy nor in humour.

R. J. D. G.

Sir Edward Albert Sharpey-Schafer,
Kt., M.D., D.Sc., LL.D., F.R.S.

THE death of Sir Edward Albert Sharpey-Schafer on March 29, 1935, in his eighty-fifth year, has taken from our midst the doyen of British physiologists and a man of outstanding personality and world-wide reputation. Sir Edward Sharpey-Schafer was elected a Fellow of the Royal Society of Edinburgh in 1900, shortly after his appointment to the Chair of Physiology in the University of Edinburgh, and from that time onwards took an active part in the work of the Society. He was a member of Council for three periods, Vice-President from 1913 to 1917, and President from 1929 to 1934. The Society awarded him the Neill Prize in 1921 for his contributions to our knowledge of physiology, and in recognition of his published work extending over a period of fifty years. His high scientific attainments, wide experience, and business capacity were freely given and were invaluable to the Society.

Born in London in 1850, Sir Edward was the son of John William Henry Schäfer and was of German origin on his father's side. He was educated at Clewer House School and at University College, London, where he came under the influence of William Sharpey, the first occupant of its Chair of Physiology. The training he received there, more particularly in histology and its technique, laid a fitting foundation to his life's work. His undergraduate career was of great promise, and he became the first Sharpey Scholar in Physiology in 1871. An unfortunate experience led him to abandon his intention of graduating in medicine in the University of London, but he took the qualification of M.R.C.S. in 1874. Thereafter he sought no degree—honorary degrees were to come to him in plenty in later days—but began his work in physiology as Assistant-Professor to Burdon Sanderson in University College. On the appointment of the latter to the Chair of Physiology in the University of Oxford, Schäfer became Jodrell Professor of Physiology in University College in 1883, and remained there until 1899, when he succeeded Professor William Rutherford in the University of Edinburgh. He resigned in 1933, barely two years before his death, and had been successively the head of two great departments of physiology, in London and in Edinburgh, for a period of fifty years, and had been actively engaged in teaching and research for sixty years.

As a teacher Sir Edward was logical and clear, his subject-matter well arranged, and as often as possible illustrated by diagrams or lantern slides. But it was research which attracted him most and in which he excelled. Keenly observant and critical, little escaped his notice. His publications were many and varied, his outlook wide, and there were few branches of his subject to which he had not made valuable contributions. He was a pioneer in the field of the ductless glands and internal secretions. With Oliver in 1894 he was the first to discover the pressor activity of the medulla of the suprarenal capsules and of the pituitary body. He also discovered the immediate diuretic effect of extracts of the posterior lobe of the pituitary body, though he failed to recognise their subsequent anti-diuretic property. His work laid the foundation of the physiology of the ductless glands, and his writings maintained his position as the authority in this branch of physiology. His book on *The Endocrine Organs*, an extension of the Lane Medical Lectures delivered at Stanford University, California, was published in 1916, and a second edition in two volumes, Part I appearing in 1924, and Part II in 1926. In it he defined internal secretion and introduced the term autacoid to include all the drug-like or medicinal substances produced by the body, dividing them into stimulants, for which he kept the name hormone previously introduced by Bayliss and Starling, and inhibiting agents or chalones. It is of interest to note that in the section upon the internal secretion of the pancreas he remarked that it would be convenient to refer to the hypothetical autacoid as "insuline." This was six years before Banting and Best isolated from the pancreas the active substance now known as insulin.

While at University College Schäfer did much valuable work upon the central nervous system, investigating tracts and the localisation of function in the cerebral cortex and other parts of the brain and spinal cord. Microscopical observations upon insect muscle led him to propound a theory of muscular contraction which he linked up with the action of cilia and amœboid movement. He also described the histological details of the absorption of fat by the intestinal villi. Latterly he had been engaged in the study of respiratory movements and the physiology of the pulmonary circulation. He also experimented upon himself, and within recent years had had afferent nerves divided in his left hand to ascertain the sensory changes which resulted from various methods of nerve lesion.

Shortly after going to Edinburgh Schäfer investigated the means of resuscitation in asphyxia, especially after apparent death by drowning. His prone pressure method, now known as the Schäfer method of treatment, proved highly successful and a great advance upon existing methods, and has been widely adopted by life-saving societies all over the world

(*Proc. Roy. Soc. Edin.*, vol. xxv, 1903, pp. 39–50). The Royal Life Saving Society awarded him their Distinguished Service Medal in 1909, and the great merit of his work has been recognised by the public.

In 1899 Schäfer edited the *Advanced Text-book of Physiology* in two volumes, a standard work to which he contributed largely. His other books include *A Course of Practical Histology*, *The Essentials of Histology*, now in its thirteenth edition, a *Text-book of Microscopic Anatomy*, which forms part 1 of vol. ii of Quain's *Anatomy*, and *Experimental Physiology*, now in its fifth edition. He wrote in conjunction with Professor Symington vol. iii, parts 1 and 2 of Quain's *Anatomy*, upon Neurology and the Peripheral Nerves and Sense Organs. His *History of the Physiological Society 1876 to 1926* appeared in 1927. Sir Edward was then the only surviving member of the small band of distinguished men who founded the Physiological Society in 1876. He founded the *Quarterly Journal of Experimental Physiology*, and edited it until his retirement in 1933. Volume xxiii of the journal consisted of papers written by his past and present assistants in 1933, and was dedicated to him.

Schäfer was one of the General Secretaries of the British Association from 1895 to 1900, and its President at the meeting in Dundee in 1912. His address on that occasion upon life and its origin was a notable one, and in some respects far in advance of the time in which it was delivered.

Honours came to him freely. He was elected a Fellow of the Royal Society in 1878, and was awarded a Royal Medal in 1902 and the Copley Medal in 1924. The Royal College of Physicians awarded him the Baly Medal in 1897. He was knighted in 1913. The Universities of Aberdeen in 1897, McGill in 1908, St Andrews in 1911, and Edinburgh in 1933 conferred upon him the honorary degree of LL.D.; Trinity College, Dublin, in 1905, Cambridge University in 1914, Oxford University in 1926, and the National University of Ireland in 1933 conferred upon him the honorary degree of D.Sc. The University of Berne gave him their M.D. in 1910, the University of Groningen their M.D. in 1914, and the University of Louvain their D.Sci. Med. in 1930. The Royal College of Physicians of Edinburgh elected him an honorary Fellow in 1931. He was President of the International Physiological Congress at the meeting in Edinburgh in 1923.

Sir Edward built up a school of physiology in Edinburgh, and his pupils fill Chairs in many parts of the world. He encouraged his staff to devote part of their time to research work, and in a well-ordered department he saw that their teaching duties did not prevent their attention to what he considered of more importance. He set the pace and never wasted time.

Sir Edward suffered a cruel blow in the loss of his two sons while on active service in the War, the elder in the Navy and the younger in the Army. Outwardly he remained the same, and carried on bravely and efficiently. In memory of William Sharpey, who had inspired his life's work, he had named his elder son John Sharpey Schäfer, and soon after the loss of this son he assumed the name Sharpey, and was henceforward known as Sharpey-Schafer.

Soon after he came to Edinburgh Sir Edward made his residence at North Berwick, where he built Marly Knowe and laid out its charming grounds. He was very fond of gardening, and here and at his later home at Park End his talent was very evident in the beauty of his gardens. Sir Edward and Lady Sharpey-Schafer were most hospitable, and during the summer frequently entertained staff and students on a large scale. Sir Edward was also an enthusiastic golfer, and until recently played round the courses of North Berwick and Muirfield.

Sir Edward Sharpey-Schafer was one of the great physiologists of all time. His work was continuous for sixty years, and good to the end. At heart he was very human. He was a good friend, thinking nothing of personal inconvenience whenever he could be of any help. To the Royal Society of Edinburgh he gave much of his time and thought. We owe much to him, and we revere his memory. (See also *Obituary Notices of Fellows of the Royal Society*, No. 4, 1935, p. 401.)

P. T. H.

Theobald Smith, For. Mem. R.S.

By the death of Theobald Smith medicine and veterinary science have lost one of their greatest pioneers and discoverers. He started his scientific career in 1883 as a Doctor of Medicine of the Albany Medical College, U.S.A., but turning from human to animal medicine in 1884, was appointed to the pathological laboratory of the Bureau of Animal Industry. Influenced by the work of Koch, he devoted himself to the bacteriological and experimental study of animal diseases. At that time Texas fever of cattle was causing serious losses to the cattle industry in the United States, and to Theobald Smith fell the important duty of investigating the disease. His efforts were crowned with brilliant success. By 1893, after four years of intensive work in the laboratory and in the field, he had solved the problem of its ætiology and spread. He discovered the causal parasite, *Piroplasma* (or *Babesia*) *bovis*, and its transmission by the tick *Boophilus* (or *Margaropus*) *annulatus*. For the first time a protozoan infection had been shown to be spread by an arthropod vector, and this was undoubtedly the prototype of the later discoveries by other famous investigators who elucidated the causation of such insect-borne diseases as malaria and trypanosomiasis. Many other brilliant observations in animal pathology and more particularly in the field of bacteriology and immunity were due to him, but some of the most noted of these at first received little attention from contemporary workers, and he himself made little effort apparently to follow up his discoveries, leaving it to others to explore their possibilities. This was one of the outstanding features of his scientific work, and illustrates his great personal modesty and his pursuit of research for its own sake. It has come to light that along with Salmon, in 1886, he pointed out how a dead culture of a bacterium produced immunity against the related disease, but it was only after the lapse of several years that the immunising properties of killed bacteria were completely studied by Pfeiffer and others and applied in practice. Theobald Smith was the first to demonstrate (1895–96) the experimental production of a deficiency disease corresponding to scurvy. He made the primary observations which revealed the existence of serum-anaphylaxis, but contented himself by communicating his findings in a letter to Ehrlich. The first differentiation of the human and bovine types of tubercle bacilli may be attributed to him. He discovered the *Vibrio fœtus*, the causal organism of a form of infectious abortion in cattle. Along with Reagh, in 1903, he showed that different types of antigen pertain to the flagella and bodies respectively of motile bacteria, but this most important observation was lost sight of for many years. It is unnecessary to refer to the many other researches of fundamental importance which he made during his career; his work has greatly enriched our knowledge of bacteriology and comparative pathology.

Theobald Smith occupied with great distinction several scientific positions in the United States, including professorships of bacteriology and comparative pathology and—his most important post—the directorship of the Department of Animal Pathology of the Rockefeller Institute, which he held from the foundation of the Institute in 1914 until he resigned in 1929. In this last position he built up a great research centre for the investigation of animal diseases and established himself as an inspiring leader. His life's work illustrates the close relationship of human and veterinary medicine.

Many well-deserved honours were awarded to him. He was a foreign member and Copley Medallist of the Royal Society of London, a foreign member of the Paris Academy of Sciences, and an honorary member of many other scientific societies. He was the recipient of honorary degrees in law, science, and medicine from American and European Universities, including the LL.D. of the University of Edinburgh, which was conferred on him *in absentia* in 1934.

He was elected an Honorary Fellow of this Society in 1934.

This distinguished veteran of science passed away on December 10, 1934, in his seventy-fifth year, but his name will live in the history of human and animal medicine. (See also *Obituary Notices of Fellows of the Royal Society*, No. 4, 1935, p. 515.)

T. J. M.

Robert Rattray Tatlock, F.C.S.

ROBERT RATTRAY TATLOCK was born in Glasgow on May 18, 1837, and died at Helensburgh on December 22, 1934. He was the elder son of Robert Tatlock, a manufacturer, and on the maternal side he could trace his ancestors, who were engaged in commerce, back to the early seventeenth century.

After his education at school, Tatlock's ambition was to be an analytical chemist, and he studied chemistry under Dr Penny, in Anderson's College. He was appointed assistant to Dr Penny, a post which he held for about eight years, and during that period his most important undertaking was the analysis of over a hundred "productions" in the case of Dr Pritchard, who had poisoned his wife and mother-in-law with an antimony compound.

He was for a time manager of a department of the Kames Gunpowder Company, Kyles of Bute, and in 1867 he began his career as an analytical and consulting chemist in Glasgow. In his laboratory commercial analysis was carried out, and there was a class for students, one of whom was the late Sir William Ramsay. He continued in active work, in partnership, until 1917.

In 1867 he was elected Fellow of the Chemical Society, in 1870 Fellow of the Royal Society of Edinburgh, in 1877 Fellow of the Institute of Chemistry of Great Britain and Ireland, and from 1883 to 1890 he acted as Examiner in Chemistry in the University of Glasgow.

During his years of active service he contributed a number of papers to chemical journals on "The Estimation of Potassium," "Some Sources of Error in Volumetric Analysis," on tea, coffee, the estimation of small proportions of bromine and chlorine in iodine, etc.

He was appointed Public Analyst in Glasgow in 1877, and later Gas Examiner and Sewage Analyst, and he was an expert witness in many cases.

R. T. T.

Hugo de Vries, For. Mem. R.S.

HUGO DE VRIES, who died on May 21, 1935, at the age of eighty-seven, was one of the founders of the modern science of genetics. From his earliest youth he was an enthusiastic botanist, and he broke away from family tradition in order to study natural science at the University of Leiden. After receiving his doctorate in 1870 he went to Germany, where he studied first under Hofmeister and later under Sachs. The next few years were divided between Holland and Germany, but in 1878 he was appointed to the staff of the University of Amsterdam where he remained as Professor of Botany until his retiral in 1918.

Whilst still a student at Leiden, de Vries was profoundly influenced by reading Darwin's *Origin of Species*, and during the whole of his subsequent career he took a keen interest in the problems of evolution. Viewed superficially, his work can be divided into two distinct phases: first, researches in experimental physiology; and secondly, the investigation of evolutionary problems. De Vries himself, however, explained that one fundamental idea underlay all his work, namely, the desire to carry Darwin's work a stage further and to learn more about the causes of the vital phenomena.

The classic researches on osmosis and plasmolysis which he carried out during the period 1870–85 won him a great reputation as a plant physiologist, but it is primarily as a pioneer in the experimental study of evolution that his name will go down to posterity. His theory of intracellular pangenesis, published in 1889, was an important addition to the existing knowledge of the mechanism of heredity, and all the subsequent work in this field has only served to demonstrate the essential accuracy of de Vries' conclusions. But probably his most famous contribution to science was *The Mutation Theory*, published in 1901–3, which was based on his exhaustive studies of the Evening Primrose, *Œnothera Lamarckiana*. In addition to these outstanding researches, de Vries published a vast amount of other valuable work. His collected scientific papers, republished in book form during the years 1918–27, fill seven volumes, with a total of more than 4300 pages.

After his retiral in 1918 de Vries lived in the little village of Lunteren, where he had a private experimental garden and a laboratory attached to his house. For the remaining seventeen years of his life he devoted his time to the further study of *Œnothera*. He possessed a boundless enthusiasm for science and a tireless energy that enabled him, in spite of his failing strength, to continue his work until a day or two before his death.

He received many distinctions from universities and scientific societies both in Holland and in other countries, and was elected an Honorary Fellow of this Society in 1910. (See also *Obituary Notices of Fellows of the Royal Society*, No. 4, 1935, p. 371; and "Aus dem Leben und Wirken von Hugo de Vries," by T. Stomps in *Tübing. naturw. Abh.*, pt. 12, 1928, pp. 7–16.)

W. W. S.

Sir James Walker, Ph.D., D.Sc., LL.D., F.R.S.

JAMES WALKER was born at Dundee on April 6, 1863, and received his elementary education at Dundee High School. Entering the University of Edinburgh in 1882, he graduated as B.Sc. in 1885 and as D.Sc. in 1886. He then proceeded to Germany for research work, first under Baeyer at Munich and later under Ostwald at Leipzig. After several years' service as an assistant to Crum Brown at Edinburgh and to Ramsay at University College, London, he was elected to the Chair of Chemistry at University College, Dundee. This position he held until 1908, when he was appointed to succeed Crum Brown at the University of Edinburgh.

Throughout his whole teaching career Walker was recognised as the protagonist of physical chemistry in Great Britain, and he did more to advance this new branch of science, by his research and by his text-books, than any of his contemporaries. His tenure of the Chair of Chemistry at Edinburgh was distinguished by valuable work during the Great War in connection with the manufacture of high explosives, and by the foundation of the new department at King's Buildings. He retired in 1928, and died on May 6, 1935.

He was elected a Fellow of the Society in 1891, served on the Council in 1903–5, 1910–13, 1922–25, and 1928–31, and was Vice-President from 1916 to 1919. He was awarded the Makdougall-Brisbane Prize in 1894 and the Gunning Victoria Jubilee Prize in 1932. (See also *Obituary Notices of Fellows of the Royal Society*, No. 4, 1935, p. 537.)

J. K.

Bertram Martin Wilson, M.A., D.Sc.

It is with much regret that we record the death after a few weeks' illness, on March 18, 1935, of Bertram Martin Wilson, Professor of Mathematics in the University College, Dundee, in but the second year of his tenure of the Chair.

Wilson, who was born on November 14, 1896, was educated at King Edward School, Birmingham, and Trinity College, Cambridge, where he was Senior Scholar and a distinguished pupil of the school of mathematical analysts inspired by Hardy and Littlewood. After gaining high mathematical honours at Cambridge he went to the University of Liverpool in 1920, and six years later became Senior Lecturer in Pure Mathematics. In 1930 he married Margaret Fancourt Mitchell, and in 1933 was elected to be Professor of Mathematics at Dundee, in the University of St Andrews, upon the retirement of Professor Steggall, the first occupant of the Chair. During the two short years of his life in Scotland, Wilson won the admiration and warm regard both of his colleagues and of his students. He willingly entered into the particular and also the broader duties of his office both in Dundee and in the University as a whole; and perhaps the students will best remember him for his love of music, which led him to co-operate actively in a scheme for arranging a notable series of musical recitals in the College.

Wilson, quiet and unassuming in demeanour, was a careful and brilliant expositor of mathematical analysis. He was not only a sound and lucid teacher of specialists, but he could on occasions give a thoroughly interesting account of an intricate subject to a general mathematical audience. His inaugural Dundee lecture on the Prime Number Theorem, as well as his lecture on Divergent Series to the Edinburgh Mathematical Society, will be remembered by his hearers as models of their kind. Besides producing many original papers he gave freely of his time to mathematical editorial work, and was chosen to be one of the three editors of the *Compositio Mathematica*, a recently founded international journal. But he will best be remembered for his part in the publication, in 1927, of the *Collected Papers of S. Ramanujan*, and more recently still for his elucidation of the labyrinth of arithmetical wonders left in the manuscript notebooks of that genius from India, a task which he undertook in collaboration with Professor G. N. Watson of Birmingham.

Many hours of unstinted devotion were given to this work upon which Wilson was still engaged up to his illness. As a friend has said of him, we mourn the loss of a man of rare charm, taken from us in the spring of a career rich in promise.

He was elected a Fellow of the Society in 1934.

H. W. T.

FRANCIS JAMES BLIGHT, formerly Chairman and Managing Director of Charles Griffin and Co., Ltd., was born in London on April 22, 1858. He was a freeman of the City of London.

He was elected a Fellow of the Society in 1918, and died on January 27, 1935.

CECIL CARUS-WILSON, J.P., F.G.S., F.R.G.S., a former Mayor of Twickenham, was elected a Fellow of the Society in 1898, and died on September 24, 1934, at Bristol.

JAMES MAIN DIXON, M.A., was born in Scotland in 1856, and was a graduate of the University of St Andrews, where he lectured for a period on English Literature. In 1880 he was appointed Professor of English and Secretary in the Imperial College of Engineering, Tokyo, where he remained until 1886. From 1892 to 1901 he was Professor of English at Washington University, Saint Louis, and from 1905 to 1931 was Professor of Comparative Literature at the University of Southern California.

He was elected a Fellow of the Society in 1885, and died on September 27, 1933.

REV. WILLIAM GALLOWAY DONALDSON, Minister of the Old Parish Church, Forfar, was elected a Fellow of the Society in 1905, and died on November 25, 1934.

WILLIAM RICHARD HODGKINSON, C.B.E., M.A., Ph.D., was born near Sheffield, in 1851, and was educated at the Royal Grammar School, Sheffield, and the Royal School of Mines. He was assistant to the late Sir E. Frankland, and Principal Demonstrator in Chemistry at the Royal College of Science under Sir E. Frankland and Sir T. E. Thorpe. Later he was appointed Professor of Chemistry and Metallurgy at Artillery College, Woolwich, from which he retired in 1918. He was joint author with Valentine of *Practical Chemistry* and of other technical publications.

He was elected a Fellow of the Society in 1885, and died on April 8, 1935.

SIR HORACE LAMB, Kt., M.A., Sc.D., D.Sc., LL.D., F.R.S., formerly Professor of Mathematics in the University of Manchester, was born at Stockport on November 27, 1849. From Stockport Grammar School he went to Owens College, Manchester, and thence to Trinity College, Cambridge, where he became Second Wrangler and Second Smith's Prizeman. He was elected a Fellow of his College, and quickly began those hydrodynamical investigations on which his fame so securely rests. He served for two periods as Vice-President of the Royal Society, received a Royal Medal in 1902, and its highest honour, the Copley Medal, in 1924. He was President of the London Mathematical Society from 1902 to 1904, and President of the British Association at Southampton in 1925. Several Universities honoured him with their doctorates.

He was elected an Honorary Fellow of the Society in 1913, and died on December 4, 1934. (See also *Obituary Notices of Fellows of the Royal Society*, No. 4, 1935, p. 375.)

SIR JOHN S. SAMUEL, K.B.E., D.L., J.P., Private Secretary to the Lord Provost, and Ceremonial Secretary to the Corporation of Glasgow, rendered honorary services to many public movements and enterprises in that city. He was born at Kippen, Stirlingshire, on April 2, 1870. He was the author of several books on historical and literary subjects.

Sir John was elected a Fellow of the Society in 1903, and died on November 10, 1934.

ALFRED C. WILSON, of Bloomfield House, Sadberge, near Darlington, was elected a Fellow of the Society in 1900, and died on July 15, 1935.

JOHNSTONE CHRISTIE WRIGHT, late of Northfield, Colinton, was born in 1852, and from 1871 to 1892 was a surveyor with the North British and Mercantile Insurance Co., Ltd. He was elected a Fellow of the Society in 1890, and died on June 1, 1935.

APPENDIX.

CONTENTS.

PAGE

PROCEEDINGS OF THE STATUTORY GENERAL MEETING, OCTOBER 1934 . 183

PROCEEDINGS OF THE ORDINARY MEETINGS, SESSION 1934–1935 . 186

PROCEEDINGS OF THE STATUTORY GENERAL MEETING, OCTOBER 1935 . 191

THE KEITH, MAKDOUGALL-BRISBANE, NEILL, GUNNING VICTORIA JUBILEE, JAMES SCOTT, BRUCE, AND DAVID ANDERSON-BERRY PRIZES, AND THE BRUCE-PRELLER LECTURE FUND 194

AWARDS 198

ACCOUNTS OF THE SOCIETY, SESSION 1934–1935 . . . 206

VOLUNTARY CONTRIBUTORS UNDER LAW VI (END OF PARA. 3) . . 214

THE COUNCIL OF THE SOCIETY AT OCTOBER 28, 1935 . . . 215

FELLOWS OF THE SOCIETY AT OCTOBER 28, 1935 . . . 216

HONORARY FELLOWS OF THE SOCIETY AT OCTOBER 28, 1935 . . 242

CHANGES IN FELLOWSHIP DURING SESSION 1934–1935 . . . 244

FELLOWS OF THE SOCIETY ELECTED DURING SESSION 1934–1935 . 244

LAWS OF THE SOCIETY 245

ADDITIONS TO THE LIBRARY—PRESENTATIONS, ETC.—1934–1935 . 253

INDEX 258

INDEX, UNDER AUTHORS' NAMES, OF PAPERS PUBLISHED IN "TRANSACTIONS" 260

PROCEEDINGS OF THE STATUTORY GENERAL MEETING

Beginning the 152nd Session, 1934–1935.

At the Statutory General Meeting of the Royal Society of Edinburgh, held in the Society's Rooms, 24 George Street, on Monday, October 22, 1934, at 4.30 P.M.

Professor C. G. Darwin, M.A., F.R.S., Vice-President, in the Chair.

The Minutes of the Statutory Meeting held on October 23, 1933, were read, approved and signed.

The Secretary read a letter from the President expressing his regret that, by his doctor's orders, he could not be present that day to demit his office.

The Chairman nominated as Scrutineers Lt.-Colonel J. C. LAMONT and Mr W. J. M. MENZIES.

The Ballot for the Election of Office-Bearers and Council was then taken.

The General Secretary submitted the following report:—

GENERAL SECRETARY'S REPORT, OCTOBER 22, 1934.

During the session four addresses have been given to the Society by request of the Council. On October 23, 1933, in terms of the award of the BRUCE-PRELLER LECTURE FUND, Professor C. H. LANDER addressed the Society on "The Liquefaction of Coal"; on December 4, 1933, Mr JAMES M. STAGG gave an address on "The British Polar Year Expedition to Fort Rae, 1932–33"; on February 5, 1934, Dr R. A. FLEMING gave an address on "The Psychology of Crime and Criminals, with special reference to Measures for Reformation"; on May 7, 1934, the Society commemorated the completion of its 150th year, and Professor D'ARCY W. THOMPSON, C.B., F.R.S., in an address entitled "Fifty Years Ago," referred to many of the notable personalities of that period and the following four decades. This address is published in the *Proceedings* (vol. 54, pp. 145–157). The President's remarks, which preceded the address, are printed in the Proceedings of the Meetings (pp. 232–233).

On the evening of the same day the Lord Provost, Magistrates, and Council of the City of Edinburgh gave a Reception to the Fellows of the Society in the Galleries of the Royal Scottish Academy (by courtesy of the President and Council). After receiving the Fellows and their friends, the Lord Provost in a short speech of welcome recalled that the Society had been housed in the "Royal Institution" for eighty-three years until in 1909 it removed into the present premises. He referred to the long line of eminent men who had presided over the destinies of the Society and to the services to science rendered by the President, Sir EDWARD SHARPEY-SCHAFER. The President thanked the Lord Provost and Corporation.

28 papers were read, as compared with 41 in the previous session. The papers were divided among subjects as follows:—Mathematics, 4; physics, 1; geology, 3; palaeobotany, 3; botany, 1; zoology, 8; animal genetics, 1; physiology, 5; anatomy, 2. 14 papers have been published in the *Transactions*, and 14 in the *Proceedings*. 1 paper has been withdrawn by the author, 9 have been declined as unsuitable for publication by the Society, and several are still under the consideration of the Council.

HIS ROYAL HIGHNESS THE DUKE OF YORK, having graciously consented to be nominated, was elected an Honorary Fellow of the Society at the Ordinary Meeting on March 5, 1934.

The Society has lost by death 21 Fellows and 4 Honorary Fellows; 1 Fellow has resigned. 28 Fellows and 6 Foreign and 6 British Honorary Fellows have been elected.

In commemoration of the completion of the 150th year of the Society, 6 Foreign and 6 British Honorary Fellows were elected. To make the election possible, Law I was altered at the Ordinary Meeting on February 5, 1934, to read as follows:—"The number of Honorary Fellows shall not exceed sixty-six, of whom not more than twenty-two may be British subjects, and not more than forty-four subjects of Foreign States." Formerly the numbers, according to statute, were twenty and thirty-six respectively.

Invitations were received, and the Society was represented as follows on the occasions mentioned:—

1. University of Edinburgh. 350th Anniversary, October 28, 1933. The President, Sir E. A. Sharpey-Schafer, represented the Society, and the Honorary Degree of Doctor of Laws was conferred upon him.
2. Royal Asiatic Society of Bengal. 150th Anniversary, Calcutta, January 15, 1934. Dr A. Macmillan Heron, Director of the Geological Survey of India.
3. Ninth International Congress of Pure and Applied Chemistry, Madrid, April 5, 1934. Professor George Barger.
4. Dinner of the Scottish Anthropological Society. Edinburgh, May 22, 1934. Professor J. H. Ashworth, General Secretary.
5. Commemoration of Antonio Pacinotti. Pisa, May 24, 1934. Letter sent.
6. 20th Annual Conference of the National Association for the Prevention of Tuberculosis. London, June 14–16, 1934. Sir Robert Philip.
7. 90th Anniversary of the Naturhistorische Verein d. Preussischen Rheinlande u. Westfalens zu Bonn. July 1, 1934. Letter sent.
8. Congrés Internationale des Sciences Anthropologiques et Ethnologiques. London, July 30 to August 4, 1934. Professor J. C. Brash and Professor V. G. Childe.
9. Edinburgh Geological Society Centenary Celebrations. September 3, 4, 1934. Professor J. H. Ashworth, General Secretary.
10. Deuxième Congrès des Mathematiciens des Pays Slaves. Prague, September 23–28, 1934. Letter sent.
11. University of Catania. 500th Anniversary, October 19–22, 1934. Professor D'Arcy W. Thompson presented address.

The undernoted Fellows were appointed to fill vacancies in the Society's representation on the following National Committees, to serve until December 31, 1939:—

Astronomy . . .	Mr T. L. MacDonald, Glasgow.
Biology . . .	Professor D'Arcy Wentworth Thompson.
Geodesy and geophysics	Professor R. A. Sampson.
Physics . . .	Professor H. S. Allen.
Radio-telegraphy . .	Professor C. G. Darwin.

Dr A. Crichton Mitchell has been re-appointed as the Society's representative on the Governing Body of the Heriot-Watt College for three years as from January 1, 1934.

Professor R. K. Hannay has been appointed to succeed the late Lord Sands as the Society's representative on the Council of the National Trust for Scotland.

The KEITH PRIZE for the period 1931–1933 was awarded to Dr A. Crichton Mitchell, and the NEILL PRIZE for the same period to Dr G. W. Tyrrell.

During the session £225 has been expended on the binding of books, and additional steel and wooden shelving costing approximately £100 has been installed.

A number of publications were added to the Library by exchange, gift, and purchase.

The issue of the *Proceedings* has been increased from 1500 to 1520.

During the session the "Instructions to Authors" on the preparation of papers for the *Transactions* and *Proceedings* have been revised, and the new regulations are printed inside the covers of parts of *Proceedings*. The Council urges on the attention of authors the need for brevity in statement, and for economy in tables and illustrations.

The acknowledgment of the Society is due to the Carnegie Trust for the Universities of Scotland for grants to authors towards the cost of tabular matter and illustrations of papers published in the *Transactions* and *Proceedings* amounting to £155, 12s. 7d.; to the Oxford University Exploration Club Expedition for £12 towards the cost of the paper by Drs Tyrrell and Sandford in the *Proceedings*; to Bedford College for Women, London, for £10 towards the cost of Miss E. Dix's paper in the *Transactions*; and for a sum of £300 received from the Royal Society of London, from the Government Publication Grant, in aid of the cost of the Society's publications during the session 1933–1934.

Treasurer's Report:—

The Treasurer, in submitting the Accounts of the past year, referred briefly to the changes in the leading items of Receipts and Expenditure.

Professor E. P. Stebbing moved the adoption of the Reports, and the reappointment of Messrs Lindsay, Jamieson & Haldane, C.A., as auditors for the ensuing session. These motions were approved.

The Scrutineers reported that the Ballot Papers were in order, and the Chairman declared that the following Office-bearers and Council had been duly elected:—

Professor D'ARCY W. THOMPSON, C.B., D.Litt., D.Sc., LL.D., F.R.S., President.	
Principal Sir THOMAS H. HOLLAND, K.C.S.I., K.C.I.E., D.L., Hon. D.Sc., LL.D., F.R.S.	Vice-Presidents.
Professor C. G. DARWIN, M.A., F.R.S.	
Professor R. A. SAMPSON, M.A., D.Sc., LL.D., F.R.S.	
Principal O. CHARNOCK BRADLEY, M.D., D.Sc.	
Professor P. T. HERRING, M.D., F.R.C.P.E.	
The Most Hon. THE MARQUIS OF LINLITHGOW, K.T., G.C.I.E., D.L.	
Professor J. H. ASHWORTH, D.Sc., F.R.S., General Secretary.	
Professor F. A. E. CREW, M.D., D.Sc., Ph.D.	Secretaries to Ordinary Meetings.
Professor JAMES P. KENDALL, M.A., D.Sc., F.R.S.	
JAMES WATT, W.S., LL.D., Treasurer.	
LEONARD DOBBIN, Ph.D., Curator of Library and Museum.	

MEMBERS OF COUNCIL.

Professor E. B. BAILEY, M.C., M.A., F.R.S.
Professor J. C. BRASH, M.C., M.A., M.D.
Professor A. J. CLARK, M.C., B.A., M.D., F.R.S.
Professor A. G. OGILVIE, O.B.E., M.A., B.Sc.
Professor E. M. WEDDERBURN, O.B.E., M.A., D.Sc., LL.B., W.S.
Lt.-Col. A. G. M'KENDRICK, M.B., D.Sc., F.R.C.P.E.
Emeritus Professor JAMES MACKINNON, M.A., Ph.D., LL.D.
Professor WILLIAM PEDDIE, D.Sc.
ALEXANDER C. AITKEN, M.A., D.Sc.
Principal J. C. SMAIL, A.Inst.E.E.
Sir HAROLD J. STILES, K.B.E., M.B., F.R.C.S.E., LL.D.
Professor JOHN WALTON, M.A., D.Sc.

The CHAIRMAN thanked the Scrutineers for their services.

The Public were then admitted.

The CHAIRMAN, in the course of his remarks, said that we were unfortunate in the absence of both the retiring President and the new one. Professor D'ARCY WENTWORTH THOMPSON was absent abroad representing the Society at the 500th Anniversary of the University of Catania, and Sir EDWARD SHARPEY-SCHAFER was prevented by his doctor's orders from coming to-day to demit office. Under these circumstances he gave a short review of the session's activities, and cited certain names from the List of Fellows who had died during the session.

In speaking of the retiring President and Curator, and of the work of the General Secretary and Treasurer, the CHAIRMAN said:—

> "I must speak of the services of our retiring officers, and here I count it an advantage that the Chair should be occupied by a mere Member of Council, for if either the old or the new President had been in the Chair it would not have been possible for him to tell of the virtues of *all* of the officers.
>
> "We are losing the services of Professor D'ARCY THOMPSON as Curator on his transference to a higher sphere of activity, and it is not possible to let the occasion pass without referring to the great improvements he has made in our library during his period of office. This has been largely due to what I may call the energetic good taste which he has shown in seeing what was needed next.
>
> "We must also congratulate ourselves on the performance of our General Secretary. The continued existence of a society like ours depends more than anything else on the efficiency of its secretary, who does all the work, takes all the blame when things go wrong, but is often not publicly given the credit when things go right. I can assure you that the present prosperity of our Society is mostly due to our General Secretary, and to the assistance he receives from Mr STEWART and the rest of the staff. As to the Treasurer, I shall leave his report to speak for him, but I cannot forbear from pointing out that our grant in aid of publications has been increased by the Royal Society of London to £300. I do not know how much of the increase is due to the persuasive tongue of our Treasurer and the charm of our Secretary, but it may surely be taken as an index that our publications are held in good esteem. In this connection I may digress to remind you how deeply our Society is indebted to the Carnegie Trust for the Universities of Scotland. Its generous grants make it possible for us to publish in our journals many important tables, and especially illustrations for papers which otherwise we should have to refuse on the grounds of expense.
>
> "And last I come to our retiring President. It is not possible for one who has chiefly followed other branches of science to give a reasoned account of his work, but it takes no expert to say that he is the doyen of all our physiologists. He is one of the very few men of science who have had the distinction of starting a great new field of knowledge—I refer to the study of the endocrine organs—and have survived to see, years later, their subject advanced to such a state that the mystery has been emptied out of it, so that it takes its place as one of the ordered branches of science. We may rejoice that it has been possible for us to bring a culmination to his life's work by honouring ourselves in having him for our President. Throughout his period of office he has shown a vigour in conducting our business that would be expected in a man of sixty rather than eighty years. It is a matter of great regret that he was not allowed to come here to-day, and in concluding I would like to propose a motion—That the following message be sent to the retiring President:—
>
> "'On the occasion of your retiring from the Presidency, the Society desires to record its gratitude to you for your services during your term of office, and expresses the hope that your health will be soon restored.'"
>
> This motion was carried unanimously.

PROCEEDINGS OF THE ORDINARY MEETINGS, Session 1934–35.

FIRST ORDINARY MEETING.

Monday, November 5, 1934.

Professor D'Arcy Wentworth Thompson, President, in the Chair.

The Minutes of the previous Ordinary Meeting were taken as read, and signed.

Mr GEORGE FRASER and Mr DAVID HALDANE signed the Roll, and were admitted to the Society by the President.

The President referred to the fact that this was his first occupancy of the Chair, as President of the Society, and dwelt upon the services of the retiring President, Sir EDWARD SHARPEY-SCHAFER, to the Society and to Science.

The following Communications were submitted:—

1. The Geology of Inchkeith. By Lt.-Col. L. M. DAVIES, M.A., F.G.S. *Trans.*, vol. 58.
2. The Tertiary Geology of Raasay (Inner Hebrides). By CHARLES F. DAVIDSON, B.Sc. Communicated by Dr FREDERICK WALKER. In the absence of Mr DAVIDSON, the paper was presented by Dr A. G. MACGREGOR of the Geological Survey (Scotland). *Trans.*, vol. 58, pp. 375–407.

Read by title:—

1. The Duration of Life in an Albino Rat Population. By B. P. WIESNER, D.Sc., and N. M. SHEARD. *Proc.*, vol. 55, pp. 1–22.
2. The Invariant Theory of the Correlation. By Professor H. W. TURNBULL, F.R.S. *Proc.*, vol. 55, pp. 27–41.

SECOND ORDINARY MEETING.

Monday, December 3, 1934.

Professor D'Arcy Wentworth Thompson, President, in the Chair.

The Minutes of the previous Ordinary Meeting were taken as read, and signed.

Professor IVAN DE BURGH DALY signed the Roll, and was admitted to the Society by the President.

At the request of the Council an Address was delivered by Lt.-Col. R. B. SEYMOUR SEWELL, C.I.E., F.R.S., on "The John Murray Expedition."

At its close the President thanked Lt.-Col. Sewell in the name of the Society.

THIRD ORDINARY MEETING.

Monday, January 7, 1935.

Professor D'Arcy Wentworth Thompson, President, in the Chair.

The Minutes of the previous Ordinary Meeting were taken as read, and signed.

The President referred to the death of Sir ALFRED EWING, and moved that the following telegram be sent to Lady EWING:—

> "The Royal Society of Edinburgh at their Meeting to-day expressed deep regret at the death of Sir ALFRED EWING, and resolved to send you this expression of their sincere sympathy."

The Meeting expressed its wish that this telegram should be sent.

The following Communications were submitted:—

1. On the Anatomy of a Marine Copepod *Calanus finmarchicus*, (Gunnerus). By ESTHER LOWE, M.Sc. Communicated by Professor H. GRAHAM CANNON, Sc.D. *Trans.*, vol. 58, pp. 561–603.
2. Dating of Late-Glacial Clay Varves in Scotland. By Professor BARON DE GEER, Stockholm. Communicated by J. B. SIMPSON, D.Sc. *Proc.*, vol. 55, pp. 23–26.
3. Scottish Lower Carboniferous Plants: the Fossil Hollow Trees of Arran and their Branches (*Lepidophloios Wünschianus*, Carruthers). *With a Demonstration of the Material and Methods used in Section Making.* By Professor J. WALTON, D.Sc. *Trans.*, vol. 58, pp. 313–337.

Read by Title:—

4. The Mathematical Representation of the Energy Levels of the Secondary Spectrum of Hydrogen: II. By IAN SANDEMAN, Ph.D. *Proc.*, vol. 55, pp. 49–61.

FOURTH ORDINARY MEETING.

Monday, February 4, 1935.

Professor D'Arcy Wentworth Thompson, President, in the Chair.

The Minutes of the previous Ordinary Meeting were taken as read, and signed.

The President announced the nomination by the Council of Professor RICHARD ANSCHÜTZ, of the University of Bonn, as a Foreign Honorary Fellow of the Society.

At the request of the Council and in terms of the BRUCE-PRELLER LECTURE FUND an Address was delivered by Professor W. L. BRAGG, O.B.E., M.A., F.R.S., University of Manchester, on "The New Crystallography." *Proc.*, vol. 55, pp. 62–71.

The President, in introducing the lecturer, said that:—

> Dr WILLIAM LAWRENCE BRAGG, Fellow of Trinity, Langworthy Professor in the University of Manchester, medallist of the Royal Society, Nobel Prizeman, is and has long been a physicist of great distinction. He began so young that he shared the Nobel Prize with his Father before he was five-and-twenty years old, for the work of Father and Son on X-rays and Crystal-structure.
>
> It was the Son who first suggested that Laue's diffraction spectra were due to deflection from planes in the crystal-lattice—a wonderful simplification of an obscure matter. It started two new lines of physical investigation, as the President of the Royal Society said when he handed the Hughes Medal to Dr BRAGG. One was the measurement of X-ray wave-lengths which led on to Moseley's discovery and all its developments; the other was the visible demonstration of crystal-structure, which in the hands of the Braggs has thrown undreamed-of light on crystalline architecture, even to the revelation of actual inter-atomic distances and linkages.
>
> Dr BRAGG has made a special study of the rock-forming silicates, commonest of minerals, of which the earth's crust is mostly composed. He has shown their minute configuration or molecular architecture to be mainly determined by the arrangement or "close-packing" of their oxygen-atoms; many diverse types of structure result from ringing the changes on this simple principle, and make an elegant and fascinating picture.
>
> Recently Professor BRAGG has been studying the structure of metals, especially of alloys or inter-metallic compounds. On this difficult subject he delivered the Bakerian Lecture last year. Altering and simplifying all former views, he has shown a "phase-structure" to be characteristic of an alloy; the atoms fit into the sites of this phase-structure in more and more orderly arrangement, depending on the previous history of the alloy, the heat to which it may have been subjected or other treatment which it may have received. Such growing or graded orderliness suggests comparison with what is found in magnates; for the genesis of structural order in the alloy is analogous to the onset of ferro-magnetic properties, both of them tending to appear as the temperature falls.
>
> I had better not attempt to tell you of Dr BRAGG'S prowess in alpine sports, his skill as a gardener, his knowledge and love of birds. But let us not forget that he delivered the Children's Christmas Lectures at the Royal Institution a few weeks ago. No more

> delightful course has ever been given, so I am told, in all the hundred years since Faraday set the great example. We shall listen to him to-day just as the children did at Christmas-time.

At the conclusion of Professor BRAGG'S Address, which included applications of the results of X-ray analysis to problems in organic chemistry, inorganic chemistry, biochemistry, geology and metallurgy, a vote of thanks to the lecturer was moved by Professor C. G. DARWIN, seconded by Professor J. P. KENDALL, and carried.

FIFTH ORDINARY MEETING.

Monday, March 4, 1935.

Professor C. G. Darwin, F.R.S., Vice-President, in the Chair.

The Minutes of the previous Ordinary Meeting were taken as read, and signed.

Mr JAMES L. BEGG and Mr JAMES WRIGHT signed the Roll and were admitted to the Society.

The Ballot for Ordinary Fellows then took place. The Chairman nominated Dr MURRAY MACGREGOR and Dr FERGUSON WATSON as Scrutineers.

The following gentlemen were duly elected:—

JAMES LAW BROWNLIE, ROBERT SELBIE CLARK, SAMUEL HUNTER COWAN, CHARLES FINDLAY DAVIDSON, MAXWELL DAVIDSON, BHIMBHAI NICHHABHAI DESAI, RONALD GRANT, ALEXANDER MILLAR MEEK GRIERSON, ARTHUR CYRIL WILLIAM HUTCHINSON, JOHN HENRY KENNETH, PETER MACCALLUM, WALTER ALAN MOZLEY, JAMES MUNNOCH, BASUDEVA NARAYANA, CHARLES SOLOMON PICHAMUTHU, THOMAS ROWATT, MAURICE GEORGE SAY, ERIC STEVENSON, JOHN DERG SUTHERLAND, JOHN EDWARD TOUCHE, SIR WILLIAM EDWARD WHYTE.

The following Communications were submitted:—

1. The Geology of St Kilda. By A. M. COCKBURN, B.Sc., Ph.D. Communicated by Professor T. J. JEHU, M.D., F.G.S. *Trans.*, vol. 58, pp. 511–547.

2. The Anatomy of the Adhesive Apparatus in the Tadpoles of *Rana afghana* Günther, with special reference to the Adaptive Modifications. By JNANENDRA LAL BHADHURI, M.Sc. Communicated by Dr S. L. HORA. *Trans.*, vol. 58, pp. 339–349.

Read by Title:—

3. On Least Squares and Linear Combination of Observations. By A. C. AITKEN, D.Sc. *Proc.*, vol. 55, pp. 42–48.

SIXTH ORDINARY MEETING.

Monday, May 13, 1935.

Professor D'Arcy Wentworth Thompson, President, in the Chair.

The Minutes of the previous Meeting were taken as read, and signed.

The following gentlemen signed the Roll and were admitted to the Society by the President: Mr MAXWELL DAVIDSON, Dr A. C. W. HUTCHINSON, Dr J. H. KENNETH, Mr J. MUNNOCH, Mr THOMAS ROWATT, Mr ERIC STEVENSON, and Mr J. EDWARD TOUCHE.

The General Secretary read the Address submitted by the Council on behalf of the Society to His Majesty The King on the occasion of the celebration of His Silver Jubilee (see p. 192).

The President announced that the Council had awarded the MAKDOUGALL-BRISBANE PRIZE for the period 1932 to 1934 to Dr A. E. CAMERON, Lecturer in Entomology, University of Edinburgh, for his publications in Entomology, including his recent paper in the *Transactions*, "The Life-History and Structure of *Hæmatopota pluvialis* Linné (Tabanidæ)."

The following Communications were submitted:—

1. An Introduction to the Ecology of Intertidal Rock Surfaces on the Coast of Argyll. By J. A. KITCHING, M.A., Ph.D. Communicated by Professor J. GRAHAM KERR, F.R.S. *Trans.*, vol. 58, pp. 351–374.
2. Rare and New Ostracoderm Fishes from the Downtonian of Shropshire. By Professor LEONARD J. WILLS, M.A., Sc.D., F.G.S. Communicated by Professor W. T. GORDON, D.Sc., and read by Dr T. M. FINLAY. *Trans.*, vol. 58, pp. 427–447.

Read by Title:—

3. The Mathematical Representation of the Energy Levels of the Secondary Spectrum of Hydrogen: III. By IAN SANDEMAN, Ph.D. *Proc.*, vol. 55, pp. 72–84.

SEVENTH ORDINARY MEETING.

Monday, June 3, 1935.

Professor D'Arcy Wentworth Thompson, President, in the Chair.

The Minutes of the previous Ordinary Meeting were taken as read, and signed.

Mr RICHARD A. ROBB, elected to Fellowship in March 1929, signed the Roll and was admitted to the Society by the President.

The following Communications were submitted:—

1. The Female Reproductive System in the Guinea-Pig: Intravitam Staining; Fat Production; Influence of Hormones. By THOMAS NICOL, M.B., D.Sc., F.R.C.S.E. *Trans.*, vol. 58, pp. 449–486.
2. The Endodermis in Light-grown and Etiolated Shoots of the Leguminosæ: a Contribution to the Causal Study of Differentiation in the Plant. By G. BOND, B.Sc., Ph.D. Communicated by Dr S. WILLIAMS. *Trans.*, vol. 58, pp. 409–425.
3. The Lunar Atmospheric Inequalities at Glasgow. By R. A. ROBB, M.A., M.Sc., and T. R. TANNAHILL, M.A., B.Sc. *Proc.*, vol. 55, pp. 91–96.

EIGHTH ORDINARY MEETING.

Monday, July 1, 1935.

Professor D'Arcy Wentworth Thompson, President, in the Chair.

The Minutes of the previous Ordinary Meeting were taken as read, and signed.

The following gentlemen signed the Roll and were admitted as Fellows by the President: Professor S. L. BHATIA, Dr R. GRANT, and Dr GEORGE TAYLOR.

The Society proceeded to ballot for RICHARD ANSCHÜTZ, Professor of Chemistry in the University of Bonn, proposed as an Honorary Fellow.

The President nominated Mr J. W. BUTTERS and Dr A. C. STEPHEN as Scrutineers.

After the Ballot the President declared Professor ANSCHÜTZ unanimously elected.

The President presented the MAKDOUGALL-BRISBANE PRIZE to Dr A. E. CAMERON, Lecturer in Entomology, University of Edinburgh, the Council of the Society having made this award to him, for the period 1932–1934, for his publications in Entomology, including his recent paper in the *Transactions*, "The Life-History and Structure of *Hæmatopota pluvialis* Linné (Tabanidæ)."

In making this presentation the President said:—

> Dr A. E. CAMERON'S work in entomology has not only ranged over various subjects, but also over a wide area in Britain and in North America. Among his earlier papers was one on "The Insect Association of a Local Environmental Complex," published in 1917 in the *Transactions* of this Society, which contains the results of a careful inquiry,

quantitative and qualitative, into the distribution of insects in the soil of the area examined, and the relations of such insects to each other, to the soil as an environment and to the vegetation which it supports.

Dr CAMERON'S work in economic entomology includes the demonstration that the Sugar-beet or Mangold Fly, *Pegomyia hyoscyami*, exists in at least three races according as it is bred on different species of Solanaceæ and Chenopodiaceæ. This discovery was followed by a study of the Pear-thrips, which at one time threatened the cultivation of deciduous fruits in Vancouver Island until successful methods of control were devised. He then turned to the investigation of various insects and ticks injurious to live stock, and particular reference may be made to his paper on "The Bionomics of the Tabanidæ of the Canadian Prairie" (1926); in this paper the pre-imaginal stages of more than twenty species of these large blood-sucking flies are studied and recorded. His latest work is an intensive study of the Cleg, *Hæmatopota pluvialis*, forming an important memoir in our *Transactions* for 1934. This insect is the first European species of the Tabanidæ to be successfully reared through all its stages in the laboratory.

To mark their approval of Dr CAMERON'S many contributions to entomology, and their especial appreciation of his success in rearing the Cleg and in describing and illustrating its larval and pupal structures the Council award him the MAKDOUGALL-BRISBANE PRIZE, and wish him all success in his further work, now in progress, on other species of Scottish Tabanidæ.

The following Communications were submitted:—

1. The Freshwater and Terrestrial Mollusca of Northern Asia. By ALAN MOZLEY, Ph.D. *Trans.*, vol. 58, pp. 605–695. The communication was made by Dr C. H. O'DONOGHUE.

2. Ordovician Submarine Disturbances in the Girvan District. By S. M. K. HENDERSON, Ph.D. Communicated by Professor E. B. BAILEY, M.C., F.R.S. *Trans.*, vol. 58, pp. 487–509.

3. Further Observations on the Genus *Lyginorachis*, Kidston. By MARY G. CALDER, Ph.D. Communicated by Professor J. WALTON, D.Sc. *Trans.*, vol. 58, pp. 549–559.

4. The Experimental Analysis of the Growth of an Insect Population. By D. STEWART MACLAGAN, D.Sc., F.R.E.S., and EDWARD DUNN, B.Sc. Communicated by Professor JAMES RITCHIE, D.Sc. *Proc.*, vol. 55, pp. 126–139.

Read by Title:—

5. Some Series and Integrals involving Associated Legendre Functions, regarded as Functions of their Degrees. By Professor T. M. MACROBERT, M.A., D.Sc. *Proc.*, vol. 55, pp. 85–90.

PROCEEDINGS OF THE STATUTORY GENERAL MEETING

Ending the 152nd Session, 1934–1935.

At the Statutory General Meeting, held in the Society's Rooms, 24 George Street, on Monday, October 28, 1935, at 4.30 P.M.

Professor D'Arcy Wentworth Thompson, President, in the Chair.

The Minutes of the Statutory Meeting held on October 22, 1934, were read, approved, and signed.

The President nominated as Scrutineers, for the election of Office-Bearers and Council, Professor G. BARGER and Dr R. SCHLAPP.

The Ballot was then taken.

The General Secretary submitted the following report:—

GENERAL SECRETARY'S REPORT, OCTOBER 28, 1935.

The following Address was submitted through the Secretary of State for Scotland to His Majesty The King on the Twenty-fifth Anniversary of His Accession to the Throne:—

TO THE
KING'S MOST EXCELLENT MAJESTY

May it please Your Majesty,

We of the Royal Society of Edinburgh bring to Your Majesty and to Her Most Gracious Majesty the Queen our humble and hearty congratulations on the happy issue of the five-and-twenty years during which Your Majesty has ruled over a loyal and devoted people.

These years of Your Majesty's reign mark a great epoch in History. War has shaken the world and changed the lives and thoughts and circumstances of men. But the devotion of the people to Your Majesty's Throne and Person has deepened and strengthened through all the changes and tumults of the age.

In this Royal Society the Natural and Physical Sciences are our daily occupation and task. Never have these Sciences flourished more than under Your Majesty's protection nor have they ever been more diligently applied to the service and advantage of mankind. Now for the first time in all the world men go their daily journeys above the clouds: all nations and languages send speech and music through the air: and Your Majesty's voice is grown familiar in Your people's ears even to the ends of the earth throughout Your world-wide Empire and Dominions.

And that Your Majesty may long be spared in health and wealth to live and reign this Society will ever pray.

(*Signed*) D'ARCY W. THOMPSON, *President.*
J. H. ASHWORTH, *Secretary.*

EDINBURGH, *May* 1, 1935.

The following letter was received from the Under-Secretary of State for Scotland:—

"SCOTTISH OFFICE,
WHITEHALL, S.W. 1,
7th May 1935.

"SIR,

I am directed by the Secretary of State to say that he has been Commanded by The King to convey to you His Majesty's thanks for the loyal and dutiful Address from the Royal Society of Edinburgh on the completion of the Twenty-fifth Year of His Majesty's Reign and to assure you that His Majesty deeply appreciates the sentiments of loyalty and affection to which it gives expression.

I am, Sir,
Your obedient Servant,
(Signed) JOHN JEFFREY."

During the session two addresses have been given to the Society by request of the Council: on December 3, 1934, by Lt.-Col. R. B. SEYMOUR SEWELL, C.I.E., F.R.S., on "The John Murray Expedition," and on February 4, 1935, in terms of the award of the BRUCE-PRELLER LECTURE FUND, by Professor W. L. BRAGG, O.B.E., F.R.S., on "The New Crystallography." The latter address was published in the *Proceedings* of the Society (vol. 55, 1935, pp. 62–71).

22 papers were read, as compared with 28 in the previous session. The papers were in the following subjects:—Mathematics, 5; Physics, 1; Geology, 6; Palaobotany, 2; Botany, 1; Zoology, 4; Animal Genetics, 2; Physiology, 1. 12 papers have been published in the *Transactions*, and 8 in the *Proceedings*. 3 papers have been withdrawn, and 6 have been declined as unsuitable for publication. Several papers are still under the consideration of the Council.

The Society has lost by death 2 Past Presidents—Sir ALFRED EWING and Sir E. A. SHARPEY-SCHAFER—22 Fellows and 3 Honorary Fellows. 3 Fellows have resigned, and 3 were removed from the Roll of Fellows. 21 Fellows and 1 Foreign Honorary Fellow were elected.

The Society was represented at the Service in St Giles' Cathedral on May 6, 1935, commemorating the Silver Jubilee of His Majesty The King.

The Society was officially represented, for the first time, at the Levee of His Majesty's Lord High Commissioner to the Church of Scotland at the Palace of Holyroodhouse (Wednesday, May 22, 1935).

Invitations were received, and the Society was represented as follows on the occasions mentioned:—

1. Academia Nacional de Ciencias Antonio Alzate, Mexico. 50th Anniversary, October 4, 1934. Letter sent.
2. Royal Scottish Geographical Society. Jubilee Celebrations, October 24, 1934. Professor J. H. ASHWORTH, General Secretary.
3. L'Académie Royale de Belgique, Commission Royale d'Histoire. Centenaire, Brussels, November 28, 29, 1934. Dr D. S. JERDAN.
4. Société Royale des Sciences et des Lettres de Bohême. 150th Anniversary, December 8, 1934. Letter sent.
5. Association Guillaume Budé, Paris. Deuxième Congrès, Nice, April 23–27, 1935. Letter sent.
6. Académie Française, Paris. Troisième Centenaire, Paris, June 17–20, 1935. Address sent.
7. Musée National d'Histoire Naturelle. Troisième Centenaire, Paris, June 24–29, 1935. The President attended and presented an Address.
8. 21st Annual Conference of the National Association for the Prevention of Tuberculosis. Southport, June 27–29, 1935. Sir ROBERT PHILIP.
9. Geological Museum, London. Opening, July 3, 1935, and the Celebration of the Centenary of the Geological Survey of the United Kingdom on following days. Dr G. W. TYRRELL.
10. International Botanical Congress. Amsterdam, September 2–7, 1935. Professor JOHN WALTON.
11. Congrès Internationale de Zoologie. Lisbon, September 15–21, 1935. Letter sent.
12. Royal Hungarian "Peter Pázmány" University of Sciences, Budapest. Tercentenary Celebration, September 25–29, 1935. Letter sent.

The Council of the Eugenics Society, London, has formed an independent Committee to select candidates for Leonard Darwin Studentships in Eugenics, which carry an emolument of £250 per annum. The Society is represented on this Committee for the present year by Professor D. F. FRASER-HARRIS.

Various MSS. and a number of reprints of his papers were bequeathed to the Society by Sir THOMAS MUIR, C.M.G., F.R.S. Dr A. C. AITKEN is making an examination of the MSS., and will report upon them.

The Polar Year Committee of the Royal Society of London has refunded the outstanding balance of the outlays made in connection with the auroral work carried out under the auspices of the Society during the Polar Year (1932–1933).

The MAKDOUGALL-BRISBANE PRIZE for the period 1932 to 1934 was awarded to Dr A. E. CAMERON, Lecturer in Entomology in the University of Edinburgh.

The issue of the *Proceedings* has been increased from 1520 to 1545.

During the session the sum of £201 has been expended on the binding of books in the Library.

The acknowledgments of the Society are due to the Carnegie Trust for the Universities of Scotland for grants to authors towards the cost of tabular matter and illustrations of papers published in the *Transactions* and *Proceedings* amounting to £197, 2s. 11d.; to the University of St Andrews for £18, 19s. 1d. towards the cost of the paper by Professor DAVID WATERSTON in the *Transactions*; to the Smithsonian Institution, Washington, for £20, 3s. towards the publication of Dr ALAN MOZLEY'S paper in the *Transactions*; to the Birkbeck College, London, for £6 towards Dr J. A. KITCHING'S paper in the *Transactions*; to Professor W. T. GORDON, Professor L. J. WILLS, and Dr ALAN MOZLEY for contributions of £30, £30, and £10 respectively towards the publication of their papers in the *Transactions*; and for a sum of £300 received from the Royal Society of London, from the Government Publication Grant, in aid of the cost of the Society's publications during the session 1934–1935.

TREASURER'S REPORT:—

The TREASURER, in submitting the Accounts of the past session, referred briefly to the leading items of receipts and expenditure, and mentioned that a sum of £200 had been set aside in the Accounts as a reserve for belated printing.

Professor F. G. BAILY moved the adoption of the Reports, and the reappointment of Messrs LINDSAY, JAMIESON and HALDANE, C.A., as Auditors for the ensuing session. These motions were approved.

The Scrutineers reported that the Ballot Papers were in order, and the PRESIDENT declared that the following Office-bearers and Council had been duly elected:—

Professor D'ARCY W. THOMPSON, C.B., D.Litt., D.Sc., LL.D., F.R.S., President.
Professor C. G. DARWIN, M.A., F.R.S.
Professor R. A. SAMPSON, M.A., D.Sc., LL.D., F.R.S.
Principal O. CHARNOCK BRADLEY, M.D., D.Sc.
Professor P. T. HERRING, M.D., F.R.C.P.E. } Vice-Presidents.
The Most Hon. THE MARQUIS OF LINLITHGOW, P.C., K.T., G.C.I.E., D.L.
Professor E. B. BAILEY, M.C., M.A., F.R.S.
Professor J. H. ASHWORTH, D.Sc., F.R.S., General Secretary.
Professor F. A. E. CREW, M.D., D.Sc., Ph.D. } Secretaries to Ordinary Meetings.
Professor JAMES P. KENDALL, M.A., D.Sc., F.R.S.
JAMES WATT, W.S., LL.D., Treasurer.
LEONARD DOBBIN, Ph.D., Curator of Library and Museum.

MEMBERS OF COUNCIL.

Lt.-Col. A. G. M'KENDRICK, M.B., D.Sc., F.R.C.P.E.
Emeritus Professor JAMES MACKINNON, M.A., Ph.D., LL.D.
Professor WILLIAM PEDDIE, D.Sc.
ALEXANDER C. AITKEN, M.A., D.Sc.
Principal J. C. SMAIL, C.Inst.E.E.
Sir HAROLD J. STILES, K.B.E., M.B., F.R.C.S.E., LL.D.
Professor JOHN WALTON, M.A., D.Sc.
Professor EDWIN BRAMWELL, M.D., F.R.C.P.E.
Emeritus Professor T. H. BRYCE, M.A., M.D., F.R.S.
Professor I. DE BURGH DALY, M.A., M.D., B.Ch.
JOHN ALEXANDER INGLIS, K.C., M.A., LL.B.
Professor A. D. PEACOCK, D.Sc.

The PRESIDENT thanked the Scrutineers for their services.

The Visitors were then admitted.

The PRESIDENT, in the course of his remarks, gave a short review of the session's activities, and referred to the loss the Society had sustained, during the session, by the deaths of two former Presidents—Sir ALFRED EWING and Sir EDWARD SHARPEY-SCHAFER, and of a number of distinguished Fellows.

The PRESIDENT announced that the Council had made the first award of the DAVID ANDERSON-BERRY PRIZE (1935) to CHARLES MELVILLE SCOTT, M.A., M.B., D.Sc., Lecturer in Materia-Medica, University of Edinburgh, for his essay "On the Action of X- and Gamma-Rays on Living Cells."

The PRESIDENT then called upon Professor J. H. ASHWORTH, F.R.S., to deliver his address "On Charles Darwin as a Student in Edinburgh, 1825–1827." *Proc.*, vol. 55, 1935, pp. 97–113.

The PRESIDENT, in the name of the Society, cordially thanked Professor ASHWORTH for his address.

THE KEITH, MAKDOUGALL-BRISBANE, NEILL, GUNNING VICTORIA JUBILEE, JAMES SCOTT, BRUCE, AND DAVID ANDERSON-BERRY PRIZES, AND THE BRUCE-PRELLER LECTURE FUND.

The above Prizes will be awarded by the Council in the following manner:—

I. KEITH PRIZE.

The KEITH PRIZE, consisting of a Gold Medal and about £30 in Money, will be awarded in the Session 1937–1938 for the "best communication on a scientific subject, communicated,* in the first instance, to the Royal Society of Edinburgh during the Sessions 1935–1936 and 1936–1937." Preference will be given to a Paper containing a discovery. (See also Council's resolutions at the end of these regulations.)

II. MAKDOUGALL-BRISBANE PRIZE.

(*Amended June* 7, 1926.)

This Prize is to be awarded biennially by the Council of the Royal Society of Edinburgh to such person, for such purposes, for such objects, and in such manner as shall appear to them the most conducive to the promotion of the interests of science; with the *proviso* that the Council shall not be compelled to award the Prize unless there shall be some individual engaged in scientific pursuit, or some Paper written on a scientific subject, or some discovery in science made during the biennial period, of sufficient merit or importance in the opinion of the Council to be entitled to the Prize.

1. The Prize, consisting of a Gold Medal and a sum of Money, will be awarded before the close of the Session 1936–1937, for an Essay, Paper, or other work having reference to any branch of scientific inquiry, either material or mental.
2. It is open to all men of science.
3. The specific subjects taken into consideration in the current award are governed by the resolutions of the Council as stated at the end of these regulations.
4. For the current period the Committee is representative of Group *A*.
5. The Committee will consider Papers presented to the Society within the Sessions 1934–1935 and 1935–1936, and will make a recommendation.

It is empowered to recommend either:—

(*a*) An award to the Author of an Essay or Paper considered as above, or
(*b*) That no award be made on the ground that, within its group, no Paper of sufficient merit has been presented, or

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* For the purposes of this award the word "communicated" shall be understood to mean the date on which the manuscript of a Paper is received in its final form for printing, as recorded by the General Secretary or other responsible official.

(*c*) That the Prize be awarded to some distinguished man of learning, who may not have presented a Paper to the Society within the period considered, but who is willing to deliver an address.

III. NEILL PRIZE.

The Council of the Royal Society of Edinburgh having received the bequest of the late Dr PATRICK NEILL of the sum of £500, for the purpose of "the interest thereof being applied in furnishing a Medal or other reward every second or third year to any distinguished Scottish Naturalist, according as such Medal or reward shall be voted by the Council of the said Society," hereby intimate:—

1. The NEILL PRIZE, consisting of a Gold Medal, will be awarded during the Session 1937-1938.

2. The Prize will be given for a Paper of distinguished merit, on a subject of Natural History, by a Scottish Naturalist, which shall have been presented * to the Society during the two years preceding the fourth Monday in October 1937,—or failing presentation of a Paper sufficiently meritorious, it will be awarded for a work or publication by some distinguished Scottish Naturalist, on some branch of Natural History, bearing date within five years of the time of award. (See also Council's resolutions at the end of these regulations.)

IV. GUNNING VICTORIA JUBILEE PRIZE.

This Prize, founded in the year 1887 by Dr R. H. GUNNING, is to be awarded quadrennially by the Council of the Royal Society of Edinburgh, in recognition of original work in Physics, Chemistry, or Pure or Applied Mathematics.

Evidence of such work may be afforded either by a Paper presented * to the Society, or by a Paper on one of the above subjects, or some discovery in them elsewhere communicated or made, which the Council may consider to be deserving of the Prize.

The Prize consists of a sum of money, and is open to men of science resident in or connected with Scotland. The first award was made in the year 1887. The next award will be made in Session 1936-1937.

In accordance with the wish of the Donor, the Council of the Society may on fit occasions award the Prize for work of a definite kind to be undertaken during the three succeeding years by a scientific man of recognised ability.

V. JAMES SCOTT PRIZE.

This Prize, founded in the year 1918 by the Trustees of the JAMES SCOTT Bequest, is to be awarded triennially, or at such intervals as the Council of the Royal Society of Edinburgh may decide, "for a lecture or essay on the fundamental concepts of Natural Philosophy."

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* For the purposes of this award the word "presented" shall be understood to mean the date on which the manuscript of a Paper is received in its final form for printing, as recorded by the General Secretary or other responsible official.

VI. BRUCE PRIZE.

The Society is trustee of a fund, instituted in 1923, to commemorate the work of Dr W. S. BRUCE as an explorer and scientific investigator in polar regions.

The Committee of Award is appointed jointly by the Royal Society of Edinburgh, the Royal Physical Society, and the Royal Scottish Geographical Society.

The Prize consists of a Bronze Medal and sum of Money. It is open to workers of all nationalities, with a preference, *ceteris paribus*, for those of Scottish birth or origin, and is to be awarded biennially for some notable contribution to Natural Sciences, such as Zoology, Botany, Geology, Meteorology, Oceanography, and Geography; the contribution to be in the nature of new knowledge, the outcome of a personal visit to polar regions on the part of the recipient. The recipient shall preferably be at the outset of his career as an investigator.

The next award will be made in 1936. Papers for the consideration of the Committee should be in the hands of the General Secretary of the Royal Society, 22 George Street, Edinburgh 2, not later than March 31 of that year.

VII. BRUCE-PRELLER LECTURE FUND.

The Council of the Royal Society of Edinburgh having received in 1929 the bequest of the late Dr CHARLES DU RICHE PRELLER of the sum of £500, decided that the income thereof be applied by the Council biennially as an honorarium for a special BRUCE-PRELLER Lecture or Address by an outstanding man of science, its subject to be Geology or Electrical or Physical Science, or in the discretion of the Council some other branch of science. The next award will be made in Session 1936–1937.

VIII. DAVID ANDERSON-BERRY FUND.

The Council of the Royal Society of Edinburgh having received in the year 1930, free of duty, the capital sum of one thousand pounds (£1000), to be used in terms of the will of the late Dr DAVID ANDERSON-BERRY, dated 23rd April 1926, decided that the income thereof be applied triennially, "in the first place in the presentation of a gold medal, and in the second place in the payment of a sum of money to the winner for the year of such gold medal, the winner being the person who, in the opinion of the Society, shall be the producer for the year of the best essay on the nature of X-rays and their therapeutical effect on human diseases."

The second award will be made in July 1938.

RESOLUTIONS OF COUNCIL IN REGARD TO THE MODE OF AWARDING PRIZES.

(*See Minutes of Meeting of January* 18, 1915.)

I. With regard to the Keith and Makdougall-Brisbane Prizes, which are open to all Sciences, the mode of award will be as follows:—

1. Papers or essays to be considered shall be arranged in two groups, *A* and *B*—Group *A* to include Astronomy, Chemistry, Mathematics, Metallurgy, Meteorology, and Physics; Group *B* to include Anatomy, Anthropology, Botany, Geology, Pathology, Physiology, and Zoology.
2. These two prizes shall be awarded to each group in alternate biennial periods, provided Papers worthy of recommendation have been communicated to the Society.
3. Prior to the adjudication the Council shall appoint, in the first instance, a Committee composed of representatives of the group of Sciences which did not receive the award in the immediately preceding period. The Committee shall consider the Papers which come within their group of Sciences, and report in due course to the Council.
4. In the event of the aforesaid Committee reporting that within their group of subjects there is, in their opinion, no Paper worthy of being recommended for the award, the Council, on accepting this report, shall appoint a Committee representative of the alternate group to consider Papers coming within their group and to report accordingly.
5. Papers to be considered by the Committees shall fall within the period dating from the last award in Groups *A* and *B* respectively.

II. With regard to the Neill Prize, the term "Naturalist" shall be understood to include any student in the Sciences composing Group *B*, namely, Anatomy, Anthropology, Botany, Geology, Pathology, Physiology, Zoology.

AWARDS OF THE KEITH, MAKDOUGALL-BRISBANE, NEILL, GUNNING, JAMES SCOTT, BRUCE, AND DAVID ANDERSON-BERRY PRIZES, AND THE BRUCE-PRELLER LECTURE FUND.

I. KEITH PRIZE.

1st Biennial Period, 1827-29.—Dr **Brewster**, for his papers "on his Discovery of Two New Immiscible Fluids in the Cavities of certain Minerals," published in the Transactions of the Society.

2nd Biennial Period, 1829-31.—Dr **Brewster**, for his paper "on a New Analysis of Solar Light," published in the Transactions of the Society.

3rd Biennial Period, 1831-33.—**Thomas Graham**, Esq., for his paper "on the Law of the Diffusion of Gases," published in the Transactions of the Society.

4th Biennial Period, 1833-35.—Professor **J. D. Forbes**, for his paper "on the Refraction and Polarization of Heat," published in the Transactions of the Society.

5th Biennial Period, 1835-37.—**John Scott Russell**, Esq., for his researches "on Hydrodynamics," published in the Transactions of the Society.

6th Biennial Period, 1837-39.—Mr **John Shaw**, for his experiments "on the Development and Growth of the Salmon," published in the Transactions of the Society.

7th Biennial Period, 1839-41.—Not awarded.

8th Biennial Period, 1841-43.—Professor **James David Forbes**, for his papers "on Glaciers," published in the Proceedings of the Society.

9th Biennial Period, 1843-45.—Not awarded.

10th Biennial Period, 1845-47.—General Sir **Thomas Brisbane**, Bart., for the Makerstoun Observations on Magnetic Phenomena, made at his expense, and published in the Transactions of the Society.

11th Biennial Period, 1847-49.—Not awarded.

12th Biennial Period, 1849-51.—Professor **Kelland**, for his papers "on General Differentiation, including his more recent Communication on a process of the Differential Calculus, and its application to the solution of certain Differential Equations," published in the Transactions of the Society.

13th Biennial Period, 1851-53.—**W. J. Macquorn Rankine**, Esq., for his series of papers "on the Mechanical Action of Heat," published in the Transactions of the Society.

14th Biennial Period, 1853-55.—Dr **Thomas Anderson**, for his papers "on the Crystalline Constituents of Opium, and on the Products of the Destructive Distillation of Animal Substances," published in the Transactions of the Society.

15th Biennial Period, 1855-57.—Professor **Boole**, for his Memoir "on the Application of the Theory of Probabilities to Questions of the Combination of Testimonies and Judgments," published in the Transactions of the Society.

16th Biennial Period, 1857-59.—Not awarded.

17th Biennial Period, 1859-61.—**John Allan Broun**, Esq., F.R.S., Director of the Trevandrum Observatory, for his papers "on the Horizontal Force of the Earth's Magnetism, on the Correction of the Bifilar Magnetometer, and on Terrestrial Magnetism generally," published in the Transactions of the Society.

18th Biennial Period, 1861-63.—Professor **William Thomson**, of the University of Glasgow, for his Communication "on some Kinematical and Dynamical Theorems."

19th Biennial Period, 1863-65.—Principal **Forbes**, St Andrews, for his "Experimental Inquiry into the Laws of Conduction of Heat in Iron Bars," published in the Transactions of the Society.

20th Biennial Period, 1865-67.—Professor **C. Piazzi Smyth**, for his paper "on Recent Measures at the Great Pyramid," published in the Transactions of the Society.

21st Biennial Period, 1867–69.—Professor P. G. Tait, for his paper "on the Rotation of a Rigid Body about a Fixed Point," published in the Transactions of the Society.

22nd Biennial Period, 1869–71.—Professor Clerk Maxwell, for his paper "on Figures, Frames, and Diagrams of Forces," published in the Transactions of the Society.

23rd Biennial Period, 1871–73.—Professor P. G. Tait, for his paper entitled "First Approximation to a Thermo-electric Diagram," published in the Transactions of the Society.

24th Biennial Period, 1873–75.—Professor Crum Brown, for his Researches "on the Sense of Rotation, and on the Anatomical Relations of the Semicircular Canals of the Internal Ear."

25th Biennial Period, 1875–77.—Professor M. Forster Heddle, for his papers "on the Rhombohedral Carbonates," and "on the Felspars of Scotland," published in the Transactions of the Society.

26th Biennial Period, 1877–79.—Professor H. C. Fleeming Jenkin, for his paper "on the Application of Graphic Methods to the Determination of the Efficiency of Machinery," published in the Transactions of the Society; Part II having appeared in the volume for 1877–78.

27th Biennial Period, 1879–81.—Professor George Chrystal, for his paper "on the Differential Telephone," published in the Transactions of the Society.

28th Biennial Period, 1881–83.—Thomas Muir, Esq., LL.D., for his "Researches into the Theory of Determinants and Continued Fractions," published in the Proceedings of the Society.

29th Biennial Period, 1883–85.—John Aitken, Esq., for his paper "on the Formation of Small Clear Spaces in Dusty Air," and for previous papers on Atmospheric Phenomena, published in the Transactions of the Society.

30th Biennial Period, 1885–87.—John Young Buchanan, Esq., for a series of communications, extending over several years, on subjects connected with Ocean Circulation, Compressibility of Glass, etc.; two of which, viz., "On Ice and Brines," and "On the Distribution of Temperature in the Antarctic Ocean," have been published in the Proceedings of the Society.

31st Biennial Period, 1887–89.—Professor E. A. Letts, for his papers on the Organic Compounds of Phosphorus, published in the Transactions of the Society.

32nd Biennial Period, 1889–91.—R. T. Omond, Esq., for his contributions to Meteorological Science, many of which are contained in vol. xxxiv of the Society's Transactions.

33rd Biennial Period, 1891–93.—Professor Thomas R. Fraser, F.R.S., for his papers on *Strophanthus hispidus*, Strophanthin, and Strophanthidin, read to the Society in February and June 1889 and in December 1891, and printed in vols. xxxv, xxxvi, and xxxvii of the Society's Transactions.

34th Biennial Period, 1893–95.—Dr Cargill G. Knott, for his papers on the Strains produced by Magnetism in Iron and in Nickel, which have appeared in the Transactions and Proceedings of the Society.

35th Biennial Period, 1895–97.—Dr Thomas Muir, for his continued communications on Determinants and Allied Questions.

36th Biennial Period, 1897–99.—Dr James Burgess, for his paper "on the Definite Integral $\frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2}dt$, with extended Tables of Values," printed in vol. xxxix of the Transactions of the Society.

37th Biennial Period, 1899–1901.—Dr Hugh Marshall, for his discovery of the Persulphates, and for his Communications on the Properties and Reactions of these Salts, published in the Proceedings of the Society.

38th Biennial Period, 1901–03.—Sir William Turner, K.C.B., LL.D., F.R.S., etc., for his memoirs entitled "A Contribution to the Craniology of the People of Scotland," published in the Transactions of the Society, and for his "Contributions to the Craniology of the People of the Empire of India," Parts I, II, likewise published in the Transactions of the Society.

39th Biennial Period, 1903–05.—Thomas H. Bryce, M.A., M.D., for his two papers on "The Histology of the Blood of the Larva of *Lepidosiren paradoxa*," published in the Transactions of the Society within the period.

40th Biennial Period, 1905–07.—Alexander Bruce, M.A., M.D., F.R.C.P.E., for his paper entitled "Distribution of the Cells in the Intermedio-Lateral Tract of the Spinal Cord," published in the Transactions of the Society within the period.

41ST BIENNIAL PERIOD, 1907–09.—WHEELTON HIND, M.D., B.S., F.R.C.S., F.G.S., for a paper published in the Transactions of the Society, "On the Lamellibranch and Gasteropod Fauna found in the Millstone Grit of Scotland."

42ND BIENNIAL PERIOD, 1909–11.—Professor ALEXANDER SMITH, B.Sc., Ph.D., of New York, for his researches upon "Sulphur" and upon "Vapour Pressure," appearing in the Proceedings of the Society.

43RD BIENNIAL PERIOD, 1911–13.—JAMES RUSSELL, Esq., for his series of investigations relating to magnetic phenomena in metals and the molecular theory of magnetism, the results of which have been published in the Proceedings and Transactions of the Society, the last paper having been issued within the period.

44TH BIENNIAL PERIOD, 1913–15.—JAMES HARTLEY ASHWORTH, D.Sc., for his papers on "Larvæ of Lingula and Pelagodiscus," and on "Sclerocheilus," published in the Transactions of the Society, and for other papers on the Morphology and Histology of Polychæta.

45TH BIENNIAL PERIOD, 1915–17.—ROBERT C. MOSSMAN, Esq., for his work on the Meteorology of the Antarctic Regions, which originated with the important series of observations made by him during the voyage of the "Scotia" (1902–1904), and includes his paper "On a Sea-Saw of Barometric Pressure, Temperature, and Wind Velocity between the Weddell Sea and the Ross Sea," published in the Proceedings of the Society.

46TH BIENNIAL PERIOD, 1917–19.—JOHN STEPHENSON, Lt.-Col., I.M.S., for his series of papers on the Oligochæta and other Annelida, several of which have been published in the Transactions of the Society.

47TH BIENNIAL PERIOD, 1919–21.—RALPH ALLEN SAMPSON, F.R.S., for his Astronomical Researches, including the papers "Studies in Clocks and Time-keeping: No. 1, Theory of the Maintenance of Motion; No. 2, Tables of the Circular Equation," published in the Proceedings of the Society within the period of the award.

48TH BIENNIAL PERIOD, 1921–23.—JOHN WALTER GREGORY, F.R.S., for his papers published in the Transactions of the Society, and in recognition of his numerous contributions to Geology, extending over a period of thirty-six years.

49TH BIENNIAL PERIOD, 1923–25.—HERBERT WESTREN TURNBULL, M.A., for the papers on "Hyper-Algebra," "Invariant Theory," and "Algebraic Geometry," three of which have been published in the Proceedings within the period of award.

50TH BIENNIAL PERIOD, 1925–27.—THOMAS JOHN JEHU, M.A., M.D., F.G.S., and ROBERT MELDRUM CRAIG, M.A., B.Sc., F.G.S., for the joint series of papers which have recently appeared in the Transactions of the Society on the "Geology of the Outer Hebrides."

51ST BIENNIAL PERIOD, 1927–29.—CHRISTINA C. MILLER, B.Sc., Ph.D., for her papers on the "Slow Oxidation of Phosphorus Trioxide," published in the Proceedings within the period of the award, and in consideration of subsequent developments on "Slow Oxidation of Phosphorus," published elsewhere.

52ND BIENNIAL PERIOD, 1929–31.—ALAN WILLIAM GREENWOOD, M.Sc., Ph.D., for his papers on the "Biology of the Fowl," several of which have appeared in the Proceedings within the period of award.

53RD BIENNIAL PERIOD, 1931–33.—A. CRICHTON MITCHELL, D.Sc., for his paper "On the Diurnal Incidence of Disturbance in the Terrestrial Magnetic Field," published in the Transactions within the period of award.

II. MAKDOUGALL-BRISBANE PRIZE.

1ST BIENNIAL PERIOD, 1859.—Sir RODERICK IMPEY MURCHISON, on account of his Contributions to the Geology of Scotland.

2ND BIENNIAL PERIOD, 1860–62.—WILLIAM SELLER, M.D., F.R.C.P.E., for his "Memoir of the Life and Writings of Dr Robert Whytt," published in the Transactions of the Society.

3RD BIENNIAL PERIOD, 1862–64.—JOHN DENIS MACDONALD, Esq., R.N., F.R.S., Surgeon of H.M.S. "Icarus," for his paper "on the Representative Relationships of the Fixed and Free Tunicata, regarded as Two Sub-classes of equivalent value; with some General Remarks on their Morphology," published in the Transactions of the Society.

4TH BIENNIAL PERIOD, 1864–66.—Not awarded.

5TH BIENNIAL PERIOD, 1866–68.—Dr ALEXANDER CRUM BROWN and Dr THOMAS RICHARD FRASER, for their conjoint paper "on the Connection between Chemical Constitution and Physiological Action," published in the Transactions of the Society.

6TH BIENNIAL PERIOD, 1868–70.—Not awarded.

7TH BIENNIAL PERIOD, 1870–72.—GEORGE JAMES ALLMAN, M.D., F.R.S., Emeritus Professor of Natural History, for his paper "on the Homological Relations of the Cœlenterata," published in the Transactions, which forms a leading chapter of his Monograph of Gymnoblastic or Tubularian Hydroids—since published.

8TH BIENNIAL PERIOD, 1872–74.—Professor LISTER, for his paper "on the Germ Theory of Putrefaction and the Fermentive Changes," communicated to the Society, 7th April 1873.

9TH BIENNIAL PERIOD, 1874–76.—ALEXANDER BUCHAN, A.M., for his paper "on the Diurnal Oscillation of the Barometer," published in the Transactions of the Society.

10TH BIENNIAL PERIOD, 1876–78.—Professor ARCHIBALD GEIKIE, for his paper "on the Old Red Sandstone of Western Europe," published in the Transactions of the Society.

11TH BIENNIAL PERIOD, 1878–80.—Professor PIAZZI SMYTH, Astronomer-Royal for Scotland, for his paper "on the Solar Spectrum in 1877–78, with some Practical Idea of its probable Temperature of Origination," published in the Transactions of the Society.

12TH BIENNIAL PERIOD, 1880–82.—Professor JAMES GEIKIE, for his "Contributions to the Geology of the North-West of Europe," including his paper "on the Geology of the Faroes," published in the Transactions of the Society.

13TH BIENNIAL PERIOD, 1882–84.—EDWARD SANG, Esq., LL.D., for his paper "on the Need of Decimal Subdivisions in Astronomy and Navigation, and on Tables requisite therefor," and generally for his Recalculations of Logarithms both of Numbers and Trigonometrical Ratios—the former communication being published in the Proceedings of the Society.

14TH BIENNIAL PERIOD, 1884–86.—JOHN MURRAY, Esq., LL.D., for his papers "On the Drainage Areas of Continents and Ocean Deposits," "The Rainfall of the Globe, and Discharge of Rivers," "The Height of the Land and Depth of the Ocean," and "The Distribution of Temperature in the Scottish Lochs as affected by the Wind."

15TH BIENNIAL PERIOD, 1886–88.—ARCHIBALD GEIKIE, Esq., LL.D., for numerous Communications, especially that entitled "History of Volcanic Action during the Tertiary Period in the British Isles," published in the Transactions of the Society.

16TH BIENNIAL PERIOD, 1888–90.—Dr LUDWIG BECKER, for his paper on "The Solar Spectrum at Medium and Low Altitudes," printed in vol. xxxvi, Part I, of the Society's Transactions.

17TH BIENNIAL PERIOD, 1890–92.—HUGH ROBERT MILL, Esq., D.Sc., for his papers on "The Physical Conditions of the Clyde Sea Area," Part I being already published in vol. xxxvi of the Society's Transactions.

18TH BIENNIAL PERIOD, 1892–94.—Professor JAMES WALKER, D.Sc., Ph.D., for his work on Physical Chemistry, part of which has been published in the Proceedings of the Society, vol. xx, pp. 255–263. In making this award, the Council took into consideration the work done by Professor Walker along with Professor Crum Brown on the Electrolytic Synthesis of Dibasic Acids, published in the Transactions of the Society.

19TH BIENNIAL PERIOD, 1894–96.—Professor JOHN G. M'KENDRICK, for numerous Physiological papers, especially in connection with Sound, many of which have appeared in the Society's publications.

20TH BIENNIAL PERIOD, 1896–98.—Dr WILLIAM PEDDIE, for his papers on the Torsional Rigidity of Wires.

21ST BIENNIAL PERIOD, 1898–1900.—Dr RAMSAY H. TRAQUAIR, for his paper entitled "Report on Fossil Fishes collected by the Geological Survey in the Upper Silurian Rocks of Scotland," printed in vol. xxxix of the Transactions of the Society.

22ND BIENNIAL PERIOD, 1900–02.—Dr ARTHUR T. MASTERMAN, for his paper entitled "The Early Development of *Cribrella oculata* (Forbes), with remarks on Echinoderm Development," printed in vol. xl of the Transactions of the Society.

23RD BIENNIAL PERIOD, 1902–04.—JOHN DOUGALL, M.A., for his paper on "An Analytical Theory of the Equilibrium of an Isotropic Elastic Plate," published in vol. xli of the Transactions of the Society.

24TH BIENNIAL PERIOD, 1904–06.—JACOB E. HALM, Ph.D., for his two papers entitled "Spectroscopic Observations of the Rotation of the Sun," and "Some Further Results obtained with the Spectroheliometer," and for other astronomical and mathematical papers published in the Transactions and Proceedings of the Society within the period.

25TH BIENNIAL PERIOD, 1906–08.—D. T. GWYNNE-VAUGHAN, M.A., F.L.S., for his papers, 1st, "On the Fossil Osmundaceæ," and 2nd, "On the Origin of the Adaxially-curved Leaf-trace in the Filicales," communicated by him conjointly with Dr R. Kidston.

26TH BIENNIAL PERIOD, 1908–10.—ERNEST MACLAGAN WEDDERBURN, M.A., LL.B., for his series of papers bearing upon "The Temperature Distribution in Fresh-water Lochs," and especially upon "The Temperature Seiche."

27th Biennial Period, 1910–12.—John Brownlee, M.A., M.D., D.Sc., for his contributions to the Theory of Mendelian Distributions and cognate subjects, published in the Proceedings of the Society within and prior to the prescribed period.

28th Biennial Period, 1912–14.—Professor C. R. Marshall, M.A., M.D., for his studies "On the Pharmacological Action of Tetra-alkyl-ammonium Compounds."

29th Biennial Period, 1914–16.—Robert Alexander Houstoun, Ph.D., D.Sc., for his series of papers on "The Absorption of Light by Inorganic Salts," published in the Proceedings of the Society.

30th Biennial Period, 1916–18.—Professor A. Anstruther Lawson, for his Memoirs on "The Prothalli of *Tmesipteris Tannensis* and of *Psilotum*," published in the Transactions of the Society, together with previous papers on Cytology and on The Gametophytes of various Gymnosperms.

31st Biennial Period, 1918–20.—Professor J. H. Maclagan Wedderburn of Princeton University, for his Memoirs in Universal Algebra, etc., published in the Transactions and Proceedings of the Society, and elsewhere.

32nd Biennial Period, 1920–22.—Professor W. T. Gordon, M.A., D.Sc., for his paper on "Cambrian Organic Remains from a Dredging in the Weddell Sea," published in the Transactions of the Society within the period, and for his investigations on the Fossil Flora of the Pettycur Limestone, previously published in the Transactions.

33rd Biennial Period, 1922–24.—Professor H. Stanley Allen, D.Sc., for his papers on the "Quantum and Atomic Theory," published in the Society's Proceedings within the period.

34th Biennial Period, 1924–26.—Charles Morley Wenyon, C.M.G., C.B.E., F.R.S., for his distinguished work in Protozoology extending over a period of twenty-one years.

35th Biennial Period, 1926–28.—W. O. Kermack, M.A., D.Sc., for his contributions to Chemistry, published in the Society's Proceedings and elsewhere.

36th Biennial Period, 1928–30.—Nellie B. Eales, D.Sc., for her papers in the Society's Transactions on "The Anatomy of a Fœtal African Elephant."

37th Biennial Period, 1930–32.—A. C. Aitken, M.A., D.Sc., for various contributions to Mathematics, published in the Society's Proceedings and elsewhere.

38th Biennial Period, 1932–34.—A. E. Cameron, M.A., D.Sc., for his publications in Entomology, including his paper in the Transactions, "The Life-History and Structure of *Hæmatopota pluvialis* Linné (Tabanidæ)."

III. THE NEILL PRIZE.

1st Triennial Period, 1856–59.—Dr W. Lauder Lindsay, for his paper "on the Spermogones and Pycnides of Filamentous, Fruticulose, and Foliaceous Lichens," published in the Transactions of the Society.

2nd Triennial Period, 1859–62.—Robert Kaye Greville, LL.D., for his contributions to Scottish Natural History, more especially in the department of Cryptogamic Botany, including his recent papers on Diatomaceæ.

3rd Triennial Period, 1862–65.—Andrew Crombie Ramsay, F.R.S., Professor of Geology in the Government School of Mines, and Local Director of the Geological Survey of Great Britain, for his various works and memoirs published during the last five years, in which he has applied the large experience acquired by him in the Direction of the arduous work of the Geological Survey of Great Britain to the elucidation of important questions bearing on Geological Science.

4th Triennial Period, 1865–68.—Dr William Carmichael M'Intosh, for his paper "on the Structure of the British Nemerteans, and on some New British Annelids," published in the Transactions of the Society.

5th Triennial Period, 1868–71.—Professor William Turner, for his papers "on the Great Finner Whale; and on the Gravid Uterus, and the Arrangement of the Fœtal Membranes in the Cetacea," published in the Transactions of the Society.

6th Triennial Period, 1871–74.—Charles William Peach, Esq., for his Contributions to Scottish Zoology and Geology, and for his recent contributions to Fossil Botany.

7th Triennial Period, 1874–77.—Dr Ramsay H. Traquair, for his paper "on the Structure and Affinities of *Tristichopterus alatus* (Egerton)," published in the Transactions of the Society, and also for his contributions to the Knowledge of the Structure of Recent and Fossil Fishes.

8th Triennial Period, 1877–80.—John Murray, Esq., for his paper "on the Structure and Origin of Coral Reefs and Islands," published (in abstract) in the Proceedings of the Society.

9TH TRIENNIAL PERIOD, 1880–83.—Professor W. A. HERDMAN, for his papers "on the Tunicata," published in the Proceedings and Transactions of the Society.

10TH TRIENNIAL PERIOD, 1883–86.—B. N. PEACH, Esq., for his Contributions to the Geology and Palæontology of Scotland, published in the Transactions of the Society.

11TH TRIENNIAL PERIOD, 1886–89.—ROBERT KIDSTON, Esq., for his Researches in Fossil Botany, published in the Transactions of the Society.

12TH TRIENNIAL PERIOD, 1889–92.—JOHN HORNE, Esq., F.G.S., for his Investigations into the Geological Structure and Petrology of the North-West Highlands.

13TH TRIENNIAL PERIOD, 1892–95.—ROBERT IRVINE, Esq., for his papers on the Action of Organisms in the Secretion of Carbonate of Lime and Silica, and on the solution of these substances in Organic Juices. These are printed in the Society's Transactions and Proceedings.

14TH TRIENNIAL PERIOD, 1895–98.—Professor J. COSSAR EWART, for his recent Investigations connected with Telegony.

15TH TRIENNIAL PERIOD, 1898–1901.—Dr JOHN S. FLETT, for his papers entitled "The Old Red Sandstone of the Orkneys" and "The Trap Dykes of the Orkneys," printed in vol. xxxix of the Transactions of the Society.

16TH TRIENNIAL PERIOD, 1901–04.—Professor J. GRAHAM KERR, M.A., for his Researches on *Lepidosiren paradoxa*, published in the Philosophical Transactions of the Royal Society, London.

17TH TRIENNIAL PERIOD, 1904–07.—FRANK J. COLE, B.Sc., for his paper entitled "A Monograph on the General Morphology of the Myxinoid Fishes, based on a Study of Myxine," published in the Transactions of the Society, regard being also paid to Mr Cole's other valuable contributions to the Anatomy and Morphology of Fishes.

1ST BIENNIAL PERIOD, 1907–09.—FRANCIS J. LEWIS, M.Sc., F.L.S., for his papers in the Society's Transactions "On the Plant Remains of the Scottish Peat Mosses."

2ND BIENNIAL PERIOD, 1909–11.—JAMES MURRAY, Esq., for his paper on "Scottish Rotifers collected by the Lake Survey (Supplement)," and other papers on the "Rotifera" and "Tardigrada," which appeared in the Transactions of the Society—(this Prize was awarded after consideration of the papers received within the five years prior to the time of award: see Neill Prize Regulations).

3RD BIENNIAL PERIOD, 1911–13.—W. S. BRUCE, LL.D., in recognition of the scientific results of his Arctic and Antarctic explorations.

4TH BIENNIAL PERIOD, 1913–15.—ROBERT CAMPBELL, D.Sc., for his paper on "The Upper Cambrian Rocks at Craigeven Bay, Stonehaven," and "Downtonian and Old Red Sandstone Rocks of Kincardineshire," published in the Transactions of the Society.

5TH BIENNIAL PERIOD, 1915–17.—W. H. LANG, F.R.S., M.B., D.Sc., for his paper in conjunction with Dr R. KIDSTON, F.R.S., on *Rhynia Gwynne-Vaughani*, Kidston and Lang, published in the Transactions of the Society, and for his previous investigations on Pteridophytes and Cycads.

6TH BIENNIAL PERIOD, 1917–19.—JOHN TAIT, D.Sc., M.D., for his work on Crustacea, published in the Proceedings of the Society, and for his papers on the blood of Crustacea.

7TH BIENNIAL PERIOD, 1919–21.—Sir EDWARD A. SHARPEY-SCHAFER, F.R.S., for his recent contributions to our knowledge of Physiology, and in recognition of his published work extending over a period of fifty years.

8TH BIENNIAL PERIOD, 1921–23.—JOHN M'LEAN THOMPSON, M.A., D.Sc., University of Liverpool, for his series of Memoirs on Staminal Zygomorphy, and on the Anatomy of the Filicales.

9TH BIENNIAL PERIOD, 1923–25.—FREDERICK ORPEN BOWER, F.R.S., for his recent contributions to Botanical knowledge and in recognition of his published work extending over a period of forty-five years.

10TH BIENNIAL PERIOD, 1925–27.—ARTHUR ROBINSON, M.D., M.R.C.S., for his contributions to Comparative Anatomy and Embryology.

11TH BIENNIAL PERIOD, 1927–29.—EDWARD BATTERSBY BAILEY, M.C., F.R.S., in recognition of his valuable contributions to the Geology of Scotland, two of which have recently appeared in the Transactions of the Society.

12TH BIENNIAL PERIOD, 1929–31.—CHARLES HENRY O'DONOGHUE, D.Sc., for his papers on the "Blood Vascular System," and for his earlier work on the "Morphology of the *corpus luteum*."

13TH BIENNIAL PERIOD. 1931–33.—GEORGE WALTER TYRRELL, A.R.C.S., D.Sc., for his contributions to the Geology and Petrology of Sub-Arctic and Sub-Antarctic Lands.

IV. GUNNING VICTORIA JUBILEE PRIZE.

1ST TRIENNIAL PERIOD, 1884–87.—Sir WILLIAM THOMSON, Pres. R.S.E., F.R.S., for a remarkable series of papers "on Hydrokinetics," especially on Waves and Vortices which have been communicated to the Society.

2ND TRIENNIAL PERIOD, 1887–90.—Professor P. G. TAIT, Sec. R.S.E., for his work in connection with the "Challenger" Expedition, and his other Researches in Physical Science.

3RD TRIENNIAL PERIOD, 1890–93.—ALEXANDER BUCHAN, Esq., LL.D., for his varied, extensive, and extremely important Contributions to Meteorology, many of which have appeared in the Society's publications.

4TH TRIENNIAL PERIOD, 1893–96.—JOHN AITKEN, Esq., for his brilliant Investigations in Physics, especially in connection with the Formation and Condensation of Aqueous Vapour.

1ST QUADRENNIAL PERIOD, 1896–1900.—Dr T. D. ANDERSON, for his discoveries of New and Variable Stars.

2ND QUADRENNIAL PERIOD, 1900–04.—Sir JAMES DEWAR, LL.D., D.C.L., F.R.S., etc., for his researches on the Liquefaction of Gases, extending over the last quarter of a century, and on the Chemical and Physical Properties of Substances at Low Temperatures: his earliest papers being published in the Transactions and Proceedings of the Society.

3RD QUADRENNIAL PERIOD, 1904–08.—Professor GEORGE CHRYSTAL, M.A., LL.D., for a series of papers on "Seiches," including "The Hydrodynamical Theory and Experimental Investigations of the Seiche Phenomena of Certain Scottish Lakes."

4TH QUADRENNIAL PERIOD, 1908–12.—Professor J. NORMAN COLLIE, Ph.D., F.R.S., for his distinguished contributions to Chemistry, Organic and Inorganic, during twenty-seven years, including his work upon Neon and other rare gases. Professor Collie's early papers were contributed to the Transactions of the Society.

5TH QUADRENNIAL PERIOD, 1912–16.—Sir THOMAS MUIR, C.M.G., LL.D., F.R.S., for his series of Memoirs upon "The Theory and History of Determinants and Allied Forms," published in the Transactions and Proceedings of the Society between the years 1872 and 1915.

6TH QUADRENNIAL PERIOD, 1916–20.—C. T. R. WILSON, Esq., F.R.S., in recognition of his important discoveries in relation to Condensation Nuclei, Ionisation of Gases and Atmospheric Electricity.

7TH QUADRENNIAL PERIOD, 1920–24.—Sir J. J. THOMSON, O.M., F.R.S., in recognition of his great discoveries in Physical Science.

8TH QUADRENNIAL PERIOD, 1924–28.—Professor E. T. WHITTAKER, F.R.S., in recognition of his distinguished contributions to Mathematical Science, and of his promotion of Mathematical Research in Scotland.

9TH QUADRENNIAL PERIOD, 1928–32.—Emeritus Professor Sir J. WALKER, F.R.S., for numerous contributions to Physical and General Chemistry.

V. JAMES SCOTT PRIZE.

1ST AWARD, 1918–22.—Professor A. N. WHITEHEAD, F.R.S., for his lecture delivered on June 5, 1922, on "The Relatedness of Nature."

2ND AWARD, 1922–27.—Sir JOSEPH LARMOR, M.A., D.Sc., LL.D., F.R.S., for his lecture delivered on July 4, 1927, on "The Grasp of Mind on Nature."

3RD AWARD, 1927–30.—Professor NIELS BOHR, for his lecture delivered on May 26, 1930, on "Philosophical Aspects of Atomic Theory."

4TH AWARD, 1930–33.—Professor Dr ARNOLD SOMMERFELD, for his lecture delivered on May 1, 1933, on "Ways to the Knowledge of Nature."

VI. BRUCE PRIZE.

1ST AWARD, 1926.—JAMES MANN WORDIE, M.A., for his Oceanographical and Geological work in both Polar Regions.

2ND AWARD, 1928.—H. U. SVERDRUP, for his contributions to the knowledge of the Meteorology, Magnetism, and Tides of the Arctic, as an outcome of his travels with the Norwegian Expedition in the "Maud" from 1918 to 1925.

3RD AWARD, 1930.—N. A. MACKINTOSH, M.Sc., A.R.C.S., for his researches into the Biology of Whales in the Waters of the Falkland Islands Dependencies.

4TH AWARD, 1932.—HENRY GINO WATKINS, for important contributions to the topography of Spitsbergen, Labrador, and East Greenland, and investigation of the Ice Cap of Greenland.

VII. BRUCE-PRELLER LECTURE FUND.

1ST AWARD, 1931.—Professor E. T. WHITTAKER, F.R.S., for his lecture, "James Clerk Maxwell and Mechanical Descriptions of the Universe."

2ND AWARD, 1933.—Professor C. H. LANDER, C.B.E., for his lecture on October 23, 1933, on "The Liquefaction of Coal."

3RD AWARD, 1935.—Professor W. L. BRAGG, O.B.E., F.R.S., for his lecture on February 4, 1935, on "The New Crystallography."

VIII. DAVID ANDERSON-BERRY FUND.

1ST AWARD, 1935.—CHARLES MELVILLE SCOTT, M.A., M.B., D.Sc., for his essay "On the Action of X- and Gamma-Rays on Living Cells."

ABSTRACT

OF

THE ACCOUNTS

OF

THE ROYAL SOCIETY OF EDINBURGH,

SESSION—1st October 1934 to 30th September 1935.

JAMES WATT, LL.D., W.S.,
Treasurer.

I. GENERAL FUND

CHARGE.

1. Arrears of Contributions at 30th September 1934			£122 17 0
Less—Written off as irrecoverable			28 7 0
			£94 10 0
2. Contributions for present Session:—			
1. 435 Fellows at £3, 3s. each		£1370 5 0	
2. Fees of Admission and Contributions of twenty new Fellows at £6, 6s. each		126 0 0	
3. Commutation		55 13 0	
			1551 18 0
3. Extra Contributions for 1934–35 under Amended Law, No. 6:—			
1. Voluntary Contributions			44 2 0
4. Interest received—			
a. Interest on £445, 10s. 3½% War Loan, 1952 (R. M. Smith Legacy), Untaxed		£15 11 10	
b. Interest on £751, 16s. 3½% War Loan, 1952 (Special Subscription Fund), Untaxed		26 6 4	
c. General—			
Interest on £7830 3½% War Loan, 1952, Untaxed	£274 1 0		
Interest on £2100 2½% Consolidated Stock, Untaxed	52 10 0		
Interest on Deposit Receipts	7 4 4		
		333 15 4	
			375 13 6
5. Transactions and Proceedings sold			194 9 4
6. Grants—Annual Grant from Government		£600 0 0	
Grant from Royal Society, from Government Publication Grant		300 0 0	
			900 0 0
7. "Polar Year" Expenses recovered			41 15 7
Amount of the Charge			**£3202 8 5**

DISCHARGE.

1. TAXES, INSURANCE, COAL AND LIGHTING:—				
Insurance				£22 7 3
Coal, etc., to 18th May 1935				40 7 0
Gas to 10th September 1935				3 11 6
Electric Light to 23rd September 1935				15 15 11
Water, 1934–35				4 4 0
Superannuation Insurance for Staff (Federated University Scheme)—				
a. Assistant Secretary		£52 10 0		
Less paid by him		17 10 0		
			£35 0 0	
b. Assistant Librarian		£26 5 0		
Less paid by him		8 15 0		
			17 10 0	
				52 10 0
				£138 15 8
2. SALARIES:—				
General Secretary			£100 0 0	
Assistant Secretary and Librarian			350 0 0	
Assistant Librarian			175 0 0	
Office Keeper			151 12 0	
Treasurer's Clerk			35 0 0	
				811 12 0
3. EXPENSES OF TRANSACTIONS AND PROCEEDINGS:—				
a. TRANSACTIONS:—				
Neill & Co., Ltd., Printers			£632 6 8	
Hislop & Day, Ltd., Engravers			149 6 0	
The Zinco-Collotype Co.			203 5 0	
Reserve for belated Printing			200 0 0	
			£1184 17 8	
Less Receipts:—				
Grants from Carnegie Trust and others towards Papers:— Prof. E. B. Bailey and Dr W. J. M'Callien, Dr Mary G. Calder, Mr R. Carrick, Prof. W. T. Gordon, Dr R. Grant, Dr W. J. Hamilton, Dr J. A. Kitching, Mr I. Lamb, Dr Alan Mozley, Dr A. C. Stephen, Prof. J. Walton, Prof. D. Waterston, and Prof. L. J. Wills			298 18 2	
			£885 19 6	
b. PROCEEDINGS:—				
Neill & Co., Ltd., Printers		£307 12 6		
Hislop & Day, Ltd., Engravers		5 4 10		
		£312 17 4		
Less Receipts:—				
1. Revenue of Publication Fund	£70 12 0			
2. Grants from Carnegie Trust and others towards Papers: Dr R. A. R. Gresson and Dr P. Ch. Koller	13 6 10			
		83 18 10		
			228 18 6	
				1114 18 0
Carry forward				£2065 5 8

Brought forward				£2065	5	8
4. BOOKS, PERIODICALS, NEWSPAPERS, ETC.:—						
James Thin, Bookseller	£280	15	6			
R. Grant & Son, Booksellers	5	11	8			
Robertson & Scott, News Agents	7	1	8			
T. & A. Constable, Ltd.	6	2	0			
Miscellaneous Accounts and Subscriptions under £5	9	11	6			
				309	2	4
5. OTHER PAYMENTS:—						
Neill & Co., Ltd., Printers	£93	13	6			
A. Cowan & Sons, Ltd.	17	15	3			
S. Duncan & Sons, Tailors (uniform)	9	9	0			
Macvitties, Guest & Co., Ltd.	33	3	6			
Andrew H. Baird	9	14	0			
Lindsay, Jamieson & Haldane, C.A., Auditors	10	10	0			
The Window Cleaning Co., Ltd.	16	2	0			
Orrock & Son, Ltd., Bookbinders	201	0	0			
Federated Superannuation System for Universities	5	0	0			
G. Waterston & Sons, Ltd.	17	2	8			
Telephone Accounts	15	5	4			
Roneo, Ltd.—Shelving, etc.	12	7	2			
W. S. Brown & Sons, Upholsterers	18	13	6			
Geo. S. Todd, Electrician	7	14	7			
Miscellaneous Accounts under £5	28	3	9			
Charwoman	63	14	0			
Petty Expenses, Postages, Carriage, etc.	45	16	2			
				605	4	5
6. ARREARS OF CONTRIBUTIONS outstanding at 30th September 1935:—						
Present Session	£97	13	0			
Previous Sessions	34	13	0			
				132	6	0
Amount of the Discharge				£3111	18	5

Amount of the Charge	£3202	8	5
Amount of the Discharge	3111	18	5
Excess of Charge transferred to Special Subscription Fund	£90	10	0

SPECIAL SUBSCRIPTION FUND.

To 30th September 1935.

CHARGE.

Total Subscriptions towards Fund							£1128	17	9
Less—Written off War Loan Investment							7	12	0
							£1121	5	9
Less—Transfers to General Fund to meet Deficits up to 30th September 1934				£578	7	1			
Less—Surplus for year to 30th September 1935	£90	10	0						
Reserve for belated Printing	200	0	0						
				290	10	0			
							287	17	1
AMOUNT OF THE CHARGE							£833	8	8

DISCHARGE.

BALANCE OF FUND—		
£751, 16s. 3½% War Loan, 1952	£751 16 0	
Due by Treasurer	16 13 3	
Due by Union Bank of Scotland, Ltd., on Current Account	74 8 5	
		£842 17 8
Less—Contributions in Advance for 1935-36		9 9 0
AMOUNT OF THE DISCHARGE		£833 8 8

WAR LOAN SUSPENSE ACCOUNT.

To 30th September 1935.

CHARGE.

Due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1934	£110 4 6
Interest on Deposit Receipt (included in General Fund)	0 19 4
AMOUNT OF THE CHARGE	£111 3 10

DISCHARGE.

Due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1935	£110 4 6
Transferred to General Fund	0 19 4
AMOUNT OF THE DISCHARGE	£111 3 10

II. KEITH FUND

Year to 30th September 1935.

CHARGE.

1. **BALANCE** due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1934		£44 17 5
2. **INTEREST RECEIVED**:—		
On £650 3½% War Loan, 1952, Untaxed	£22 15 0	
On Deposit Receipts	0 10 7	
		23 5 7
		£68 3 0

DISCHARGE.

BALANCE due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1935	£68 3 0

III. NEILL FUND

Year to 30th September 1935.

CHARGE.

1. **BALANCE** due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1934		£17 4 8
2. **INTEREST RECEIVED**:—		
On £300 3½% War Loan, 1952, Untaxed	£10 10 0	
On Deposit Receipts	0 4 0	
		10 14 0
		£27 18 8

DISCHARGE.

BALANCE due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1935	£27 18 8

IV. MAKDOUGALL-BRISBANE FUND

Year to 30th September 1935.

CHARGE.

1. **Balance** due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1934		£67 0 9
2. **Interest Received**:—		
On £400 3½% War Loan, 1952, Untaxed	£14 0 0	
On Deposit Receipts	0 12 4	
		14 12 4
		£81 13 1

DISCHARGE.

1. Alex. Kirkwood & Son for Gold Medal	£21 10 0	
Dr A. E. Cameron 1932–34 Award	15 0 0	
		36 10 0
2. **Balance** due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1935		45 3 1
		£81 13 1

V. MAKERSTOUN MAGNETIC METEOROLOGICAL OBSERVATION FUND

Year to 30th September 1935.

CHARGE.

1. **Balance** due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1934		£116 11 1
2. **Interest Received**:—		
On £250 3½% War Loan, 1952, Untaxed	£8 15 0	
On Deposit Receipts	1 3 6	
		9 18 6
		£126 9 7

DISCHARGE.

Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1935	£126 9 7

VI. GUNNING VICTORIA JUBILEE PRIZE FUND

Year to 30th September 1935.

CHARGE.

1. **Balance** due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1934		£116 1 9
2. **Interest Received**:—		
On £599, 14s. 3½% War Loan, 1952, Untaxed	£20 19 10	
On Deposit Receipts	1 4 9	
		22 4 7
		£138 6 4

DISCHARGE.

Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1935	£138 6 4

VII. JAMES SCOTT PRIZE FUND

Year to 30th September 1935.

CHARGE.

1. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1934		£10 0 10
2. Interest Received:—		
On £247, 10s. 3½% War Loan, 1952, Untaxed	£8 13 2	
On Deposit Receipts	0 2 4	
		8 15 6
		£18 16 4

DISCHARGE.

Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1935	£18 16 4

VIII. PUBLICATION FUND

(Comprising Peter Guthrie Tait Memorial Fund and Dr John Aitken Fund)

Year to 30th September 1935.

CHARGE.

1. Peter Guthrie Tait Memorial Fund:—		
Year's Interest on £1550 3½% War Loan, 1952, Untaxed		£54 5 0
2. Dr John Aitken Fund:—		
Year's Interest on £445, 10s. 3½% War Loan, 1952, Untaxed	£15 11 10	
Interest on Deposit Receipts	0 15 2	
		16 7 0
		£70 12 0

DISCHARGE.

Transferred to General Fund to meet Cost of Publications (see General Fund Discharge, No. 3*b*)	£70 12 0

IX. DR W. S. BRUCE MEMORIAL FUND

Year to 30th September 1935.

CHARGE.

1. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1934		£30 11 2
2. Interest Received:—		
On £233 3½% Conversion Loan, 1961	£8 3 0	
On Deposit Receipts	0 6 7	
		8 9 7
		£39 0 9

DISCHARGE.

Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1935	£39 0 9

X. BRUCE-PRELLER LECTURE FUND

Year to 30th September 1935.

CHARGE.

1. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1934		£50 19 10
2. Dividend and Interest Received:—		
On £140, 9s. Royal Bank of Scotland Stock, less Tax £5, 7s. 6d.	£18 10 0	
On Deposit Receipts	0 6 0	
		18 16 0
3. Repayment of Income Tax for year to December 1934		5 7 6
		£75 3 4

DISCHARGE.

1. Prof. W. L. Bragg, 1932–34 Award	£40 0 0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1935	35 3 4
	£75 3 4

XI. DR DAVID ANDERSON-BERRY FUND

Year to 30th September 1935.

CHARGE.

1. Balance of Revenue due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1934		£56 3 7
2. Interest Received:—		
On £1528, 0s. 4d. Local Loans 3% Stock	£45 16 8	
On Deposit Receipts	0 15 3	
		46 11 11
3. Income Tax repaid for quarter to 5th July 1934		2 11 6
		£105 7 0

DISCHARGE.

Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1935	£105 7 0

STATE OF THE FUNDS BELONGING TO THE ROYAL SOCIETY OF EDINBURGH

As at 30th September 1935.

1. GENERAL FUND—

1. £7830 3½% War Loan, 1952		£7830 0 0
2. £2100 2½% Consolidated Stock at 53%		1113 0 0
3. £445, 10s. 3½% War Loan, 1952. Robert Mackay Smith, Legacy		445 10 0
4. Arrears of Contributions, as per preceding Abstract of Accounts		122 17 0
5. Balance of Special Subscription Fund—		
£751, 16s. 3½% War Loan, 1952	£751 16 0	
Cash due by Treasurer	16 13 3	
Cash due by Union Bank of Scotland, Ltd., on Current Account	74 8 5	
	842 17 8	
Less—Contributions in Advance for 1935–36	9 9 0	
		833 8 8
6. War Loan Suspense Account— Due by Union Bank of Scotland, Ltd., on Deposit Receipt		110 4 6
7. Contributions in Advance for 1935–36		9 9 0
Amount		£10,464 9 2

Exclusive of Library, Museum, Pictures, etc., and Furniture in the Society's Rooms at George Street, Edinburgh.

2. KEITH FUND—

1. £650 3½% War Loan, 1952	£650 0 0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	68 3 0
AMOUNT	£718 3 0

3. NEILL FUND—

1. £300 3½% War Loan, 1952	£300 0 0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	27 18 8
AMOUNT	£327 18 8

4. MAKDOUGALL-BRISBANE FUND—

1. £400 3½% War Loan, 1952	£400 0 0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	45 3 1
AMOUNT	£445 3 1

5. MAKERSTOUN MAGNETIC METEOROLOGICAL OBSERVATION FUND—

1. £250 3½% War Loan, 1952	£250 0 0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	126 9 7
AMOUNT	£376 9 7

6. GUNNING VICTORIA JUBILEE PRIZE FUND—Instituted by Dr Gunning of Edinburgh and Rio de Janeiro—

1. £599, 14s. 3½% War Loan, 1952	£599 14 0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	138 6 4
AMOUNT	£738 0 4

7. JAMES SCOTT PRIZE FUND—

1. £247, 10s. 3½% War Loan, 1952	£247 10 0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	18 16 4
AMOUNT	£266 6 4

8. PUBLICATION FUND—

(COMPRISING PETER GUTHRIE TAIT MEMORIAL FUND AND DR JOHN AITKEN FUND)

1. PETER GUTHRIE TAIT MEMORIAL FUND:—		
£1550 3½% War Loan, 1952		£1550 0 0
2. DR JOHN AITKEN FUND:—		
£445, 10s. 3½% War Loan, 1952	£445 10 0	
Deposit Receipt with Union Bank of Scotland, Ltd.	71 6 1	
		516 16 1
AMOUNT		£2066 16 1

9. DR W. S. BRUCE MEMORIAL FUND—

1. £233 3½% Conversion Loan, at 72⅞% (cost price)	£169 15 11
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	39 0 9
AMOUNT	£208 16 8

10. BRUCE-PRELLER LECTURE FUND—

1. £140, 9s. Royal Bank of Scotland Stock, taken over at 350%	£491 11 6
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	35 3 4
AMOUNT	£526 14 10

11. DR DAVID ANDERSON-BERRY FUND—

1. £1528, 0s. 4d. Local Loans 3% Stock at 65 7/16% (cost price)	£1000 0 0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	105 7 0
AMOUNT	£1105 7 0

Note 1.—As previously, 3½% War Loan, 1952, has been uniformly valued at par in the above State of Funds.

2.—Under the Will of the late Prof. Charles Piazzi Smyth and his wife, the Society will, on the expiry of certain liferents, become entitled to payment of the residue to be applied as set out in the will.

EDINBURGH, 18*th October* 1935.—We have examined the preceding Accounts of the Treasurer of the Royal Society of Edinburgh for the Session 1934–1935, and have found them to be correct. The securities for the various Investments, as noted in the foregoing Statement of Funds, have been verified by us at 30th September 1935.

LINDSAY, JAMIESON & HALDANE, C.A.,
Auditors.

VOLUNTARY CONTRIBUTORS under Law VI (end of para. 3), to 30th September 1935.

Col. Arthur F. Appleton,	£1	1	0
Sir James Barr, C.B.E.,	1	1	0
Professor F. O. Bower, F.R.S.,	1	1	0
Principal O. Charnock Bradley, M.D.,	1	1	0
Dr G. Sandison Brock,	1	1	0
John W. Butters, Esq.,	1	1	0
W. L. Calderwood, Esq.,	1	1	0
Emeritus Professor E. W. W. Carlier,	1	1	0
Col. David Carnegie,	1	1	0
Professor Ernest G. Coker, F.R.S.,	1	1	0
Professor John N. Collie, F.R.S.,	1	1	0
Sir James Crichton-Browne, F.R.S.,	1	1	0
M. McCallum Fairgrieve, Esq.,	1	1	0
Edward Oswald Fergus, Esq.,	1	1	0
Dr R. A. Fleming,	1	1	0
John S. Ford, Esq.,	1	1	0
Alexander Fraser, Esq.,	1	1	0
Dr W. Dawson Henderson,	1	1	0
Dr E. M. Horsburgh,	1	1	0
Dr William F. Hume,	1	1	0
	£21	0	0
Carried forward,	£21	0	0
Professor T. J. Jehu,	1	1	0
Professor J. Graham Kerr, F.R.S.,	1	1	0
Forrest H. Lightbody, Esq.,	1	1	0
Dr Peter M'Bride,	1	1	0
James A. Macdonald, Esq.,	1	1	0
Dr George M'Gowan,	1	1	0
Sir W. Leslie Mackenzie,	1	1	0
Emeritus Professor James MacKinnon,	1	1	0
Dr Hugh Robert Mill,	1	1	0
Professor John Miller,	1	1	0
Dr Alexander Morgan,	1	1	0
Robert C. Mossman, Esq.,	1	1	0
John T. Pearce, Esq.,	1	1	0
Professor William Peddie,	1	1	0
Professor A. G. Perkin, F.R.S.,	1	1	0
Alexander G. Ramage, Esq.,	1	1	0
Charles A. Stevenson, Esq.,	1	1	0
Dr H. F. Stockdale,	1	1	0
G. Sutherland Thomson, Esq.,	1	1	0
R. Tatlock Thomson, Esq.,	1	1	0
William Williamson, Esq.,	1	1	0
J. Christie Wright, Esq.,	1	1	0
Total,	£44	2	0

THE COUNCIL OF THE SOCIETY.

28th October 1935.

PRESIDENT.

PROFESSOR D'ARCY WENTWORTH THOMPSON, C.B., D.Litt., D.Sc., LL.D., F.R.S.

VICE-PRESIDENTS.

PROFESSOR C. G. DARWIN, M.A., F.R.S.
PROFESSOR R. A. SAMPSON, M.A., D.Sc., LL.D., F.R.S.
PRINCIPAL O. CHARNOCK BRADLEY, M.D., D.Sc.
PROFESSOR P. T. HERRING, M.D., F.R.C.P.E.
THE MOST HON. THE MARQUIS OF LINLITHGOW, P.C., K.T., G.C.I.E., D.L.
PROFESSOR E. B. BAILEY, M.C., M.A., F.R.S.

GENERAL SECRETARY.

PROFESSOR J. H. ASHWORTH, D.Sc., F.R.S.

SECRETARIES TO ORDINARY MEETINGS.

PROFESSOR F. A. E. CREW, M.D., D.Sc., Ph.D.
PROFESSOR J. P. KENDALL, M.A., D.Sc., F.R.S.

TREASURER.

JAMES WATT, W.S., LL.D.

CURATOR OF LIBRARY AND MUSEUM.

LEONARD DOBBIN, Ph.D.

COUNCILLORS.

LT.-COL. A. G. M'KENDRICK, M.B., D.Sc., F.R.C.P.E.

EMERITUS PROFESSOR JAMES MACKINNON, M.A., Ph.D., LL.D.

PROFESSOR WILLIAM PEDDIE, D.Sc.

ALEXANDER C. AITKEN, M.A., D.Sc.

PRINCIPAL J. C. SMAIL, O.B.E., Companion Inst.E.E.

SIR HAROLD J. STILES, K.B.E., M.B., F.R.C.S.E., LL.D.

PROFESSOR JOHN WALTON, M.A., D.Sc.

PROFESSOR EDWIN BRAMWELL, M.D., F.R.C.P. Edin. and Lond.

EMERITUS PROFESSOR T. H. BRYCE, M.A., M.D., F.R.S.

PROFESSOR I. DE BURGH DALY, M.A., M.D., B.Ch.

JOHN ALEXANDER INGLIS, K.C., M.A., LL.B.

PROFESSOR A. D. PEACOCK, D.Sc.

OFFICE STAFF.

Assistant Secretary and Librarian, G. A. STEWART.
Assistant Librarian, R. J. B. MUNRO.
Housekeeper, SAMUEL HEDDLE.

PATRON.

HIS MOST EXCELLENT MAJESTY THE KING.

FELLOWS OF THE SOCIETY,

Corrected to 28th October 1935.

N.B.—*Those marked * are Annual Contributors.*
,, ,, † *have commuted Voluntary Contribution (see 3rd Paragraph, Law VI).*

M-B. prefixed to a name indicates that the Fellow has received a Makdougall-Brisbane Medal.
K. ,, ,, ,, ,, Keith Medal.
N. ,, ,, ,, ,, Neill Medal.
V. J. ,, ,, ,, ,, the Gunning Victoria Jubilee Prize.
B. ,, ,, ,, ,, Bruce Medal.
B-P. ,, ,, ,, ,, Bruce-Preller Lectureship.
C. ,, ,, ,, has contributed one or more Communications to the Society's TRANSACTIONS or PROCEEDINGS.

Date of Election			Service on Council, etc.
1925	M-B. C.	* Aitken, Alexander Craig, M.A., D.Sc., Lecturer in Actuarial Science, University of Edinburgh (16 Chambers Street). 54 Braid Road, Edinburgh 10	1934–
1889		† Alison, John, M.A., LL.D., formerly Head Master, George Watson's College. 126 Craiglea Drive, Edinburgh 10	
1927	C.	* Allan, Douglas Alexander, D.Sc., Director, City of Liverpool Public Museums, William Brown Street, Liverpool	
1920	C. M-B.	* Allen, Herbert Stanley, M.A. (Cantab.), D.Sc. (London), F.R.S., Professor of Natural Philosophy, University of St Andrews	1921–24.
1920	C.	* Anderson, Ernest Masson, M.A., D.Sc., F.G.S., 50 Greenbank Crescent, Edinburgh	
1905		Anderson, William, M.A., Head Science Master, George Watson's College, Edinburgh. 6 Lockharton Crescent, Edinburgh 11	
1905		Andrew, George, M.A., B.A., H.M.I.S., Royal Technical College, George Street, Glasgow. Hamewith, Kilmacolm, Renfrewshire	
1930		* Annan, William, M.A., C.A., Professor of Accounting and Business Method, University of Edinburgh (South Bridge). Tofthill, Ferry Road West, Edinburgh 5	
1915		Anthony, Charles, M.Inst.C.E., M.Am.Soc.C.E., F.R.A.S., F.C.S., Spring Croft, Les Croutes, St Peter Port, Guernsey, Channel Islands	
1906		Appleton, Colonel Arthur Frederick, F.R.C.V.S., Nutwell, 34 Shortlands Road, Shortlands, Kent	
1910	C.	Archibald, E. H., B.Sc., Professor of Chemistry, University of British Columbia, Vancouver, Canada	
1933		* Arnot, Frederick Latham, B.Sc. (Sydney), Ph.D. (Cantab.), Lecturer in Natural Philosophy, University of St Andrews. Lindatham, Melville Grove, Largo Road, St Andrews	
1921		* Arthur, William, M.A., Lecturer in Mathematics, University of Glasgow. 148 Carmunnock Road, Cathcart, Glasgow	
1911	C. K.	Ashworth, James Hartley, D.Sc., F.R.S. (GENERAL SECRETARY), Professor of Natural History, University of Edinburgh (West Mains Road). Hillbank, Grange Loan, Edinburgh 9	1912–14, 1915–18, 1927–30. Sec. 1918–23. V.P 1923–26, 1930–33. Gen. Sec. 1933–
1920		* Bagnall, Richard Siddoway, Hon. D.Sc., F.R.E.S., 13 Learmonth Terrace, Edinburgh 4	
1920	C. N.	* Bailey, Edward Battersby, M.C., M.A., F.R.S., F.G.S. (VICE-PRESIDENT), Professor of Geology, University of Glasgow	1932–35 V.P 1935–

Date of Election			Service on Council, etc.
1896	C.	† Baily, Francis Gibson, M.A., M.Inst.E.E., Emeritus Professor of Electrical Engineering, Heriot-Watt College, Edinburgh. Newbury, Juniper Green, Midlothian	1909–12, 1920–23. V.P 1929–32.
1934		* Bain, David, M.Sc. (Manch.), D.Sc. (Edin.), Lecturer in Technical Chemistry, University of Edinburgh (West Mains Road). 87 Cluny Gardens, Edinburgh 10	
1931	C.	* Bain, William Alexander, Ph.D., Reader in Pharmacology, School of Medicine, University of Leeds. 26 Weetwood Road, Headingley, Leeds 6	
1931		* Baird, William Macdonald, Fellow and Past President of the Faculty of Surveyors of Scotland, F.S.A.Scot., J.P. Dalveen, Barnton Avenue, Davidson's Mains, Edinburgh 4	
1921		* Baker, Bevan Braithwaite, M.A., D.Sc., Professor of Mathematics, Royal Holloway College, Englefield Green, London	
1928	C.	* Baker, Edwin Arthur, D.Sc. (Edin.), Assistant at the Royal Observatory, Edinburgh. 17 Ladysmith Road, Edinburgh 9	
1905	C.	Balfour-Browne, William Alexander Francis, M.A., F.Z.S., F.L.S., F.R.E.S., F.R.M.S., Barrister-at-Law, formerly Professor of Entomology, Imperial College of Science, London. Winscombe Court, Winscombe, Somerset, and 17 Langham Mansions, Earls Court Square, London, S.W. 5	
1933		* Banerjee, Prabodh Chandra, L.R.C.P.E., L.R.C.S.E., F.R.F.P.S.G., F.A.C.S., Major, I.M.S. C/o Lloyds Bank, Ltd., 101–1 Clive Street, Calcutta, India	
1928		Barbour, George Brown, M.A. (Edin.), M.A. (Cantab.), Ph.D., F.G.S., c/o Professor B. B. Baker, Pentlands, Englefield Green, Surrey	
1886		Barclay, A. J. Gunion, M.A., 3 Chandos Avenue, Oakleigh Park, London, N.	
1903		Bardswell, Noël Dean, M.V.O., M.D., M.R.C.P. (Ed. and Lond.), New County Hall, Westminster Bridge Road, London, S.E. 1	
1922		* Barger, George, M.A., D.Sc., Dr *h. c.* (Padua), Hon. D.Sc. (Liverp.), Hon. M.D. (Heidelberg), LL.D. (Michigan), F.R.S., Professor of Chemistry (Medical), University of Edinburgh (Teviot Place). 48 St Alban's Road, Edinburgh 9	1925–28.
1929		* Barker, Sydney George, O.B.E., D.I.C., F.Inst.P., formerly Director of Research, Wool Industries Research Association, Torridon. 4 Purcell Mansions, Queen's Club Gardens, Barons Court, London, W. 14	
1914	C.	* Barkla, Charles Glover, M.A., D.Sc., F.R.S., Professor of Natural Philosophy, University of Edinburgh (Drummond Street), Nobel Laureate, Physics, 1917. The Hermitage of Braid, Edinburgh	1915–18. 1924–27.
1927		* Barnett, John, F.F.A., C.A., Scottish Widows' Fund Life Assurance Society, 9 St Andrew Square, Edinburgh 2	
1904		Barr, Sir James, C.B.E., M.D., LL.D., F.R.C.P., Hindhead Brae, Hindhead, Surrey	
1921		* Bartholomew, John, M.C., M.A., F.R.G.S., Geographical Institute, Duncan Street, Edinburgh. 19 George Square, Edinburgh 8	1925–28.
1932	C.	* Barton-Wright, Eustace Cecil, M.Sc. (Lond.), Research Assistant of British Flour Millers, Old London Road, St Albans, Herts	
1927		* Bastow, Stephen Everard, M.Inst.E.E., M.Inst.Min.E., Managing Director, Bruce Peebles & Co., Ltd., Edinburgh. Northwood, Russell Place, Trinity, Edinburgh 5	
1929		* Bath, Frederick, Ph.D., Lecturer in Mathematics, University of St Andrews, Assistant to the Professor of Mathematics, University College, Dundee	
1913		† Beard, Joseph, F.R.C.S.E., M.R.C.S., L.R.C.P., D.P.H. (Cantab.), formerly Medical Officer of Health, City of Carlisle. 8 Carlton Gardens, Carlisle	
1888		Beare, Sir Thomas Hudson, Kt., B.A., B.Sc., M.Inst.C.E., Hon. M.I.Mech.E., J.P., D.L., Professor of Engineering, University of Edinburgh (Sanderson Engineering Laboratory, Mayfield Road). 10 Regent Terrace, Edinburgh 7	1907–09. V.P 1909–15, 1923–26.
1897	C.	Beattie, Sir John Carruthers, K.B., D.Sc., LL.D., Vice-Chancellor and Principal, The University, Cape Town	
1893	C. M-B.	Becker, Ludwig, Ph.D., formerly Regius Professor of Astronomy, University of Glasgow. The Observatory, Dowanhill, Glasgow	
1933		* Begg, James Livingstone, F.G.S. (Treasurer, Geological Society of Glasgow). Elms, Mount Vernon, Glasgow	
1916		* Bell, Robert John Tainsh, M.A., D.Sc., LL.D. (Glas.), Professor of Mathematics, University of Otago, Dunedin, New Zealand	

Date of Election			Service on Council, etc.
1929		* Bennet, George, A.H.-W.C., B.Sc., A.M.I.Mech.E., Lecturer in Mechanical Engineering, Heriot-Watt College. 68 Arden Street, Edinburgh 10	
1893	C.	Berry, Sir George A., M.B., C.M., LL.D., F.R.C.S.E., King's Knoll, North Berwick	1916–19. V-P 1919–22.
1897	C.	Berry, Richard J. A., M.D., F.R.C.S.E., Director of Medical Services, Stoke Park Colony, Stapleton, Bristol. Rufford, Canford Lane, Westbury-on-Trym, Bristol	
1932		* Bhatia, Sohan Lal, M.C., M.A., M.D. (Cantab.), M.R.C.P., Major, I.M.S., Professor of Physiology and Dean, Grant Medical College, Bombay. Two Gables, Mount Pleasant Road, Malabar Hill, Bombay, India	
1880	C.	Birch, de Burgh, C.B., M.D., Emeritus Professor of Physiology, University of Leeds	
1931	C.	* Black, Thomas Purves, M.A. (Edin.), B.Sc. (Lond.), Ph.D. (Edin.), Headmaster, Bellevue Technical and Commercial School, Edinburgh. 32 Eastfield, Joppa 6	
1933		* Bolam, Thomas Robert, M.Sc. (Bristol), D.Sc. (Edin.), Lecturer in Chemistry, University of Edinburgh (West Mains Road). 8 Wilton Road, Edinburgh 9	
1894		Bolton, Herbert, D.Sc., F.G.S., F.Z.S., formerly Director, Bristol Museum and Art Gallery, Bristol. 318 Tilehurst Road, Reading, Berks	
1915		* Boon, Alfred Archibald, D.Sc., B.A., F.I.C., Emeritus Professor of Chemistry, Heriot-Watt College, Edinburgh	
1925		* Borthwick, Albert William, O.B.E., D.Sc., Professor of Forestry, University of Aberdeen	
1925		* Bose, Sahay Ram, M.A., D.Sc., F.L.S., Professor of Botany, Carmichael Medical College, Belgachia, Calcutta, India	
1886	C. N.	Bower, Frederick Orpen, M.A., D.Sc., LL.D., F.R.S., F.L.S., Emeritus Regius Professor of Botany, University of Glasgow. 2 The Crescent, Ripon, Yorks	1887–90, 1893–96, 1907–09, 1917–19. V-P 1910–16. P 1919–24.
1924		* Bowman, Alexander, D.Sc., formerly Scientific Superintendent, Marine Laboratory, Fishery Board for Scotland, Torry, Aberdeen	
1903	C.	Bradley, O. Charnock, M.D., D.Sc. (VICE-PRESIDENT), Principal, Royal (Dick) Veterinary College, Edinburgh 9	1907–10, 1915–17. V-P 1934–
1926		* Braid, Kenneth William, M.A. (Cantab.), B.Sc., Professor of Botany, West of Scotland Agricultural College, 6 Blythswood Square, Glasgow	
1907		† Bramwell, Edwin, M.D., F.R.C.P. Edin. and Lond., formerly Professor of Clinical Medicine, University of Edinburgh (Royal Infirmary). 23 Drumsheugh Gardens, Edinburgh 3	1935–
1932		* Brash, James Couper, M.C., M.A., M.B., Ch.B. (Edin.), M.D. (Birm.), Professor of Anatomy, University of Edinburgh (Teviot Place)	1932–35.
1918		* Bremner, Alexander, M.A., D.Sc., Headmaster, Demonstration School, Training Centre, Aberdeen. 13 Belgrave Terrace, Aberdeen	
1895		Bright, Sir Charles, M.Inst.C.E., M.Inst.E.E., F.R.Ae.S., F.Inst.Radio E., Little Brewers', Hatfield Heath, Harlow, Essex	
1893		Brock, G. Sandison, M.D., F.R.C.P.E., 53 Cheniston Gardens, Kensington, London, W. 8	
1901	C.	Brodie, W. Brodie, M.D., Camden House, Bletchingley, Surrey	
1934		* Brough, Patrick, M.A., B.Sc. (Glas.), D.Sc. (Sydney), Lecturer in Botany, University of Sydney, N.S.W.	
1907		Brown, Alexander, M.A., B.Sc., Professor of Applied Mathematics, University, Cape Town	
1928		* Brown, Hugh Wylie, F.I.A., F.F.A., 1 Cobden Crescent, Edinburgh 9	
1885		Brown, J. Macdonald, M.D., F.R.C.S., Oriental Club, Hanover Square, London, W. 1	
1924	C.	* Brown, Thomas Arnold, M.A., B.Sc., Professor of Mathematics, University College, Exeter	
1923		* Brown, Walter, M.A., B.Sc., Professor of Mathematics, University, Hong Kong, China	

Date of Election			Service on Council, etc.
1935		* Brownlie, James Law, M.D. (Glas.), D.P.H., M.R.C.P.E., Chief Medical Officer, Department of Health for Scotland. Grange Park, 38 Dick Place, Edinburgh 9	
1921		* Bruce, Alexander, B.Sc. (Edin.), Government Agricultural Chemist and City Analyst, The Laboratory, Turret Road S., Colombo, Ceylon	
1912		* Bruce, Alexander Ninian, D.Sc., M.D., 8 Ainslie Place, Edinburgh 3	
1898	C. K.	† Bryce, Thomas Hastie, M.A., M.D. (Edin.), F.R.S., Emeritus Professor of Anatomy, University of Glasgow. The Loaning, Peebles	1911–14, 1922–25, 1935– V.P 1925–28.
1887		† Burnet, Sir John James, R.A., R.S.A., LL.D., Killermont, Rowledge, Farnham, Surrey	
1888		Burns, Rev. Thomas, C.B.E., D.D., J.P., F.S.A.Scot., Minister of Lady Glenorchy's Parish Church. Croston Lodge, Chalmers Crescent, Edinburgh 9	
1917		* Burnside, George Barnhill, M.Inst.Mech.E., Fairhill, Dullatur	
1930	C.	* Burt, David Raitt Robertson, B.Sc. (St Andrews), F.L.S., Lecturer in Zoology, Ceylon University College, Colombo	
1896		Butters, John W., M.A., B.Sc., formerly Rector of Ardrossan Academy. 116 Comiston Drive, Edinburgh 10	
1929	C.	* Calder, Alexander, Ph.D., Chief Marketing Officer, Pig Marketing Board, Thames House, Millbank, London, S.W. 1	
1910		Calderwood, Rev. Robert Sibbald, D.D., formerly Minister of Cambuslang. 84 Findhorn Place, Edinburgh 9	
1893	C.	Calderwood, W. L., I.S.O., formerly Inspector of Salmon Fisheries of Scotland. New Club, Princes Street, Edinburgh	1923–26.
1926	C. M-B.	* Cameron, Alfred Ernest, M.A., D.Sc. (Aberd.), Lecturer in Entomology, Department of Zoology, University of Edinburgh (West Mains Road). 8 West Savile Road, Edinburgh 9	
1933		* Cameron, Finlay James, F.F.A., F.I.A., General Manager, Caledonian Insurance Company. Beech Knowe, Barnton, near Edinburgh	
1905	C.	Cameron, John, M.D., D.Sc., M.R.C.S., formerly Professor of Anatomy, Dalhousie University, Halifax, Nova Scotia. Balmashanner, Grove Road, East Cliff, Bournemouth	
1921		* Campbell, Andrew, Advisory Chemist, c/o Burmah Oil Co., Ltd., Research Laboratory, Fairlawn, Honor Oak Road, Forrest Hill, London, S.E. 68 Albemarle Road, Beckenham, Kent	
1918		* Campbell, John Menzies, D.D.S. (Toronto), L.D.S. (Glas.), L.D.S. (Ontario), F.I.C.D., 14 Buckingham Terrace, Glasgow, W.	
1915	C. N.	* Campbell, Robert, M.A., D.Sc., F.G.S., Reader in Petrology, University of Edinburgh (Grant Institute of Geology, West Mains Road). Maryton, Colinton	1920–23.
1927	C.	* Cannon, Herbert Graham, M.A., Sc.D. (Cantab.), D.Sc. (Lond.), M.Sc. (Manc.), F.R.S., F.L.S., Beyer Professor of Zoology, University of Manchester	
1899	C.	Carlier, Edmund W. W., B. ès Sc., M.Sc., M.D., F.R.E.S., Emeritus Professor of Physiology, University of Birmingham. Morningside, Dorridge, near Birmingham	
1910		Carnegie, Col. David, C.B.E., M.Inst.C.E., J.P., The Haven, Seasalter, Whitstable	
1931		* Carroll, John Anthony, M.A., Ph.D. (Cantab.), Professor of Natural Philosophy, University of Aberdeen. Marischal College, Aberdeen	
1920	C.	* Carruthers, R. G., F.G.S., District Geologist, H.M. Geological Survey, High Barn, Stocksfield-on-Tyne	
1905	C.	Carse, George Alexander, M.A., D.Sc., Reader in Natural Philosophy, University of Edinburgh (Drummond Street). 3 Middleby Street, Edinburgh 9	
1901		Carslaw, Horatio Scott, M.A., Sc.D. (Cantab.), D.Sc., LL.D. (Glas.), Emeritus Professor of Mathematics, University of Sydney, New South Wales. Emmanuel College, Cambridge	
1933		* Carswell, John Irvine, B.Sc., A.M.Inst.C.E., A.M.Inst.Mech.E., Lecturer in Engineering, University of Edinburgh (Sanderson Engineering Laboratory, Mayfield Road). 43 Mansionhouse Road, Edinburgh 9	
1925		* Carter, George Stuart, M.A., Ph.D., Corpus Christi College, Cambridge	
1932		* Cathcart, Edward Provan, C.B.E., M.D., D.Sc., LL.D., F.R.S., Professor of Physiology, University of Glasgow. 28 Hillhead Street, Glasgow	

Date of Election			Service on Council, etc.
1932	C.	* Childe, Vere Gordon, B.A., B.Litt., F.R.A.I., F.S.A., Professor of Prehistoric Archæology, University of Edinburgh (14 Chambers Street)	
1925	C.	* Chumley, James, M.A., Ph.D., Lecturer in Oceanography, Department of Zoology, University of Glasgow. Thalassa, Thorn Drive, Bearsden, Dumbartonshire	
1928	C.	* Clark, Alfred Joseph, M.C., B.A., M.D., F.R.S., Professor of Materia Medica, University of Edinburgh (Teviot Place). 67 Braid Avenue, Edinburgh 10	1932–35
1933		* Clark, Arthur Melville, M.A. (Edin.), D. Phil. (Oxon.), Lecturer in English Literature, University of Edinburgh. 34 Bruntsfield Gardens, Edinburgh 10	
1891		Clark, John Brown, C.B.E., M.A., LL.D., J.P., formerly Head Master of George Heriot's School. Garleffin, 146 Craiglea Drive, Edinburgh 10	1928–1931. V-P 1931–34.
1935		* Clark, Robert Selbie, M.A., D.Sc. (Aberd.), Scientific Superintendent, Fishery Board for Scotland. The Cottage, Murtle, Aberdeenshire	
1932		* Clark, Sir Thomas, Bart., Publisher, Head of T. & T. Clark, Ltd. 6 Wester Coates Road, Edinburgh 12	
1903		Clarke, William Eagle, I.S.O., LL.D., F.L.S., Honorary Supervisor of the Bird Collection and formerly Keeper of the Natural History Collections, Royal Scottish Museum, Edinburgh	
1909		Clayton, Thomas Morrison, M.D., D.Hy., B.Sc., D.P.H., Medical Officer of Health, Greenesfield House, Gateshead-on-Tyne	
1932		* Clouston, David, M.A., B.Sc. (Agric.), D.Sc., C.I.E., formerly Director, Imperial Agricultural Research Institute, Pusa. Forthview, Boswall Road, Edinburgh 5	
1904	C.	Coker, Ernest George, M.A. (Cantab.), D.Sc. (Edin.), Hon. D.Sc. (Sydney and Louvain), M.Sc. (McGill), F.R.S., M.Inst.C.E., M.I.Mech.E., Emeritus Professor of Civil and Mechanical Engineering, University of London. Engineering Laboratories, 14 Connaught Avenue, Chingford, London, E. 4	
1904		Coles, Alfred Charles, M.D., D.Sc., York House, Poole Road, Bournemouth, W.	
1888	V. J. C.	Collie, John Norman, Ph.D., D.Sc., LL.D., F.R.S., F.C.S., F.I.C., Emeritus Professor of Organic Chemistry, University College, Gower Street, London. 20 Gower Street, London, W.C. 1	
1909	C.	Comrie, Peter, M.A., B.Sc., LL.D., formerly Rector, Leith Academy. 19 Craighouse Terrace, Edinburgh 10	
1924	C.	* Copson, Edward Thomas, M.A. (Oxon.), D.Sc. (Edin.), Professor of Mathematics, University College, Dundee (University of St Andrews)	
1928		* Coutie, Rev. Alexander, Ph.D., 13 Mayfield Gardens, Edinburgh 9	
1914		* Coutts, William Barron, M.A., B.Sc., Senior Lecturer in Range Finding and Optics, Military College of Science, Red Barracks, Woolwich, S.E. 18. 11 Coleraine Road, Blackheath, S.E. 3	
1911		Cowan, Alexander, Papermaker, Valleyfield, Penicuik, Midlothian	
1931		* Cowan, John Macqueen, M.A., D.Sc. (Edin.), B.A. (Oxon.), F.L.S., Assistant Keeper, Royal Botanic Garden, Edinburgh. 17 Inverleith Place, Edinburgh 4	
1935		* Cowan, Samuel Hunter, Lt.-Col. R. E., Lecturer in Forestry Engineering, University of Edinburgh (George Square). New Club, Edinburgh 2	
1916	C.	† Craig, E. H. Cunningham, B.A. (Cantab.), Geologist and Mining Engineer, The Dutch House, Beaconsfield	
1908		Craig, James Ireland, M.A., B.A., Woolwich House, The Drive, Sydenham, London, S.E. 26. (At present—Turf Club, Cairo)	
1925	C. K.	* Craig, Robert Meldrum, M.A., D.Sc., F.G.S., Lecturer in Economic Geology, University of Edinburgh (Grant Institute of Geology, West Mains Road)	
1933		* Craig-Bennett, Arthur Lancelot, M.A., Ph.D. (Cantab.), Lecturer in Zoology, University of Edinburgh (West Mains Road). 27 Grange Loan, Edinburgh 9	
1903		Crawford, Lawrence, M.A., D.Sc., Professor of Pure Mathematics, University, Cape Town	
1922	C.	* Crew, Francis Albert Eley, M.D., D.Sc., Ph.D. (SECRETARY TO ORDINARY MEETINGS), Professor of Animal Genetics and Director of the Institute of Animal Genetics, University of Edinburgh (West Mains Road). 41 Mansionhouse Road, Edinburgh 9	1928–31. Sec. 1931–
1931		* Crichton, John, M.A., B.Sc. (Edin.), Rayleigh House, The Observatory, Eskdalemuir, Langholm	

Date of Election			Service on Council, etc.
1870		Crichton-Browne, Sir James, Kt., M.D., LL.D., D.Sc., F.R.S., Vice-President and Treasurer of the Royal Institution of Great Britain. 45 Hans Place, London, S.W. 1	
1929		* Cruickshank, Ernest William Henderson, M.D., D.Sc., Ph.D., Regius Professor of Physiology, University of Aberdeen	
1914		* Cumming, Alexander Charles, O.B.E., D.Sc., Roselands, Crescent Road, Blundell Sands, Liverpool	
1928		* Cumming, William Murdoch, D.Sc. (Glas.), F.I.C., M.Inst.Chem.E., "Young" Professor of Technical Chemistry, Royal Technical College, Glasgow. Bonnieblink, 4 Newlands Road, Newlands, Glasgow, C. 1	
1917		* Cunningham, Brysson, D.Sc., B.E., M.Inst.C.E., Lecturer in Waterways, Harbours, and Docks, University College, London. 141 Copers Cope Road, Beckenham, Kent	
1930		* Cunningham, John, C.I.E., B.A., M.D., Lt.-Colonel, I.M.S. (retired). South Bank, Grange Loan, Edinburgh 10	
1934		* Daly, Ivan de Burgh, M.A., M.D., B.Ch., Professor of Physiology, University of Edinburgh (Teviot Place). Cooliney, Spylaw Avenue, Edinburgh	1935–
1885		Daniell, Alfred, M.A., LL.B., D.Sc., Advocate, The Athenæum Club, London	
1934		* Darling, Frank Fraser, Ph.D. (Edin.), N.D.A., Dundonnell, by Garve, Wester Ross	
1924		* Darwin, Charles Galton, M.A., F.R.S. (VICE-PRESIDENT). Tait Professor of *Natural Philosophy, University of Edinburgh (Drummond Street).* 4 Church Hill, Edinburgh 10	1925–28. Sec. 1928–33. V-P 1933–
1935	C.	* Davidson, Charles Findlay, B.Sc. (St Andrews), Geologist, H.M. Geological Survey of Great Britain, Geological Survey and Museum, Exhibition Road, South Kensington, London, S.W. 7	
1932		* Davidson, Leybourne Stanley Patrick, B.A. (Cantab.), M.D. (Edin.), F.R.C.P.E., Regius Professor of Medicine, University of Aberdeen. 55 Queen's Road, Aberdeen	
1935		* Davidson, Maxwell, B.Sc. (Eng.), M.I.Mech.E., Lecturer in Heat Engines and Thermodynamics, University of Edinburgh (Sanderson Engineering Laboratory, Mayfield Road). 47 Liberton Drive, Edinburgh 9	
1930	C.	* Davies, Lewis Merson, M.A., F.G.S., F.R.A.I., Lt.-Colonel, Royal Artillery (retired). 8 Garscube Terrace, Murrayfield, Edinburgh 12	
1928		Dawson, Warren Royal, F.R.S.L., F.S.A.Scot., Honorary Librarian of Lloyd's. 28 Grange Road, Barnes, London, S.W. 13	
1923		* Deane, Arthur, M.R.I.A., Curator, Public Art Gallery and Museum, Belfast. Threave, 57 Cranmore Park, Belfast	
1894		† Denny, Sir Archibald, Bart., LL.D., 5 St Helen's Place, London, E.C. 4	
1935		* Desai, Bhimbhai Nichhabhai, B.A., M.Sc. (Bombay), Ph.D. (Edin.), Assistant Meteorologist, Government of India, Meteorological Department, Poona 5, India	
1925		Dey, Alexander John, Managing Director of T. & H. Smith, Ltd., Manufacturing Chemists, Edinburgh. Rothiemay, Corstorphine, Edinburgh 12	
1924		* Dinham, C. H., B.A., H.M. Geological Survey. Edgemoor, 19 Highfield Road, Northwood, Middlesex	
1923		* Dixon, Ronald Audley Martineau, of Thearne, F.G.S., F.S.A.Scot., F.R.G.S., Thearne Hall, near Beverley	
1881	C.	Dobbin, Leonard, Ph.D. (CURATOR OF LIBRARY AND MUSEUM), formerly Reader in Chemistry, University of Edinburgh. Faladam, Blackshiels, Midlothian	1904–07, 1913–16. Curator 1934–
1918		* Dodd, Alexander Scott, Ph.D., F.I.C., F.C.S., City Analyst for Edinburgh. 20 Stafford Street, Edinburgh 3	
1925		* Donald, Alexander Graham, M.A., F.F.A., F.S.A.Scot., Secretary of the *Scottish Provident Institution, Edinburgh.* 18 Carlton Terrace, Edinburgh 7	
1882	C.	Dott, David B., F.I.C., Memb. Pharm. Soc., Ravenslea, Musselburgh	
1921	M-B. C.	* Dougall, John, M.A., D.Sc., 47 Airthrey Avenue, Glasgow, W. 4	
1901 & 1918		Douglas, Carstairs Cumming, M.D., D.Sc., Professor of Medical Jurisprudence and Hygiene, Anderson's College, Glasgow. 110 South Brae Drive, Jordanhill, Glasgow	

Date of Election			Service on Council, etc.
1910		Douglas, Loudon MacQueen, Newpark, Mid-Calder, Midlothian	
1934		* Dow, David Rutherford, M.D., D.P.H., F.R.C.P.E., Professor of Anatomy, University of St Andrews (University College, Dundee). 16 Windsor Street, Dundee	
1932		* Drennan, Alexander Murray, M.D. (Edin.), F.R.C.P.E., Professor of Pathology, University of Edinburgh (Teviot Place)	
1923	C.	* Drever, James, M.A., B.Sc., D.Phil., Professor of Psychology, University of Edinburgh (South Bridge). Ivybank, Wardie Road, Edinburgh 5	1929–32.
1901		Drinkwater, Thomas W., L.R.C.P.E., L.R.C.S.E., Chemical Laboratory, Surgeons' Hall, Edinburgh	
1923		* Drummond, J. Montagu F., M.A. (Cantab.), Harrison Professor of Botany, University of Manchester	1928–31.
1925		* Dryerre, Henry, M.R.C.S., Ph.D., Professor of Physiology, Royal (Dick) Veterinary College; Physiological Biochemist, Animal Diseases Research Association. Kenmore, Lasswade	
1921		* Drysdale, Charles Vickery, C.B., O.B.E., D.Sc. (Lond.), M.I.E.E., F.Inst.P., formerly Director of Scientific Research to the Admiralty. Derek Corner, Kingsway, Hove, Sussex	
1904		Dunlop, William Brown, M.A., 4A St Andrew Square, Edinburgh 2. Seton Castle, Longniddry, E. Lothian	
1892	C.	Dunstan, Malcolm James Rowley, M.A., F.I.C., F.C.S., formerly Principal, Royal Agricultural College, Cirencester. Windyacres, Wrotham, Kent	
1933		* Dymond, Edmund Gilbert, M.A., Lecturer in Natural Philosophy, University of Edinburgh (Drummond Street). 8 Cobden Crescent, Edinburgh 9	
1906	C.	Dyson, Sir Frank Watson, K.B.E., M.A., D.Sc., LL.D., F.R.S., formerly Astronomer Royal, Royal Observatory, Greenwich. 27 Westcombe Park Road, Blackheath, S.E. 3	
1925		* Eastwood, George Samuel, B.Sc., Principal Teacher of Mathematics, Beath Secondary School, Cowdenbeath, Fife. Craigie Lea, Cowdenbeath, Fife	
1934		* Edge, William Leonard, M.A. (Cantab.), Lecturer in Mathematics, University of Edinburgh (16 Chambers Street)	
1904		Edwards, John, LL.D., 4 Great Western Terrace, Kelvinside, Glasgow	
1931		* Eggleton, Philip, D.Sc., Lecturer in Biochemistry, Department of Physiology, University of Edinburgh (Teviot Place). 36 Gillespie Crescent, Edinburgh 10	
1924		* Elliot, Right Hon. Walter Elliot, P.C., M.C., M.B., Ch.B., D.Sc., LL.D., M.P., F.R.S., Minister of Agriculture. 17 North Street, London, S.W. 1	
1906	C.	Ellis, David, D.Sc., Professor of Botany and Bacteriology, Royal Technical College, Glasgow	
1933		* Erskine, John Maxwell, General Manager of the Commercial Bank of Scotland, Ltd. 10 Spylaw Road, Edinburgh 10	
1934		* Etherington, Ivor Malcolm Haddon, B.A. (Oxon.), Ph.D. (Edin.), Lecturer in Mathematics, University of Edinburgh (16 Chambers Street). 41 Scotland Street, Edinburgh 3	
1924		* Evans, Arthur Humble, M.A., Sc.D., Lecturer in English History. Cheviot House, Crowthorne, Berks	
1924		Evans, William Edgar, B.Sc., Assistant in charge of Herbarium, Royal Botanic Garden, Edinburgh 4	
1902		Ewen, John Taylor, O.B.E., B.Sc., M.I.Mech.E., J.P., H.M. Inspector of Schools (Emeritus), Pitscandly, Forfar	
1900	C.	Eyre, John W. H., M.D., M.S. (Dunelm), D.P.H. (Cantab.), Emeritus Professor of Bacteriology, Guy's Hospital. 51 Portland Place, London, W. 1	
1931		* Fairbairn, William Ronald Dodds, M.A., M.D., D.Psych. (Edin.), F.R.A.I. 18 Lansdowne Crescent, Edinburgh 12	
1910	C.	Fairgrieve, Mungo M'Callum, M.A. (Cantab. and Glas.), Senior Science Master, Edinburgh Academy. 37 Queen's Crescent, Edinburgh 9	
1907	C.	Falconer, John Downie, M.A., D.Sc., F.G.S., formerly Director of the Geological Survey of Nigeria. The Cedars, Hatton Road, Harlington, Hayes, Middlesex	
1923		* Feldman, William Moses, M.D., B.S., F.R.C.P., F.R.A.S., Senior Physician, St Mary's Hospital for Women and Children, Plaistow. 851 Finchley Road, London, N.W. 11	

Date of Election			Service on Council, etc.
1928		* Fenton, Edward Wyllie, M.A., B.Sc. (Aberd.), Head of Botany Department, Edinburgh and East of Scotland College of Agriculture, 13 George Square, Edinburgh 8	
1907		Fergus, Edward Oswald, c/o Messrs M'Kay & Boyd, Solicitors, 50 Wellington Street, Glasgow	
1933		* Ferguson, Thomas, M.D., D.P.H. (Edin.), F.R.C.P.E., D.Sc., Medical Officer, Department of Health for Scotland. 4 Park Street, Falkirk	
1925	C.	* Ferrar, William Leonard, M.A., Fellow and Tutor of Hertford College, Oxford	
1932		* Findlay, Sir John Edmund Ritchie, Bart., B.A. (Oxon.), Proprietor of the *Scotsman*. 18 Lauder Road, Edinburgh 9	
1927	C.	* Finlay, Thomas Matthew, M.A., D.Sc. (Edin.), Lecturer in Palæontology, University of Edinburgh. 11 Dudley Terrace, Leith 6	
1911		† Fleming, John Arnold, F.C.S., Pottery Manufacturer, Locksley, Helensburgh, Dumbartonshire	
1906		Fleming, Robert Alexander, M.A., M.D., LL.D., F.R.C.P.E., Consulting Physician, Royal Infirmary. 10 Chester Street, Edinburgh 3	
1900	C. N.	Flett, Sir John S., K.B.E., M.A., D.Sc., LL.D., F.R.S., formerly Director of the Geological Survey of Great Britain and Museum of Practical Geology. 98 Prince of Wales Mansions, Battersea Park, London, S.W. 11	1916–19.
1872		Forbes, George, M.A., LL.D., F.R.S., M.Inst.C.E., M.Inst.E.E., F.R.A.S., formerly Professor of Natural Philosophy, Anderson's College, Glasgow. 11 Little College Street, Westminster, S.W.	
1892		Ford, John Simpson, F.I.C., 7 Corrennie Drive, Edinburgh 10	
1921		* Forrest, George Topham, F.R.I.B.A., F.G.S., formerly Architect to the London County Council. Calder Lodge, Maidenhead.	
1928	C.	* Forrest, James, M.A., B.Sc. (Glas.), D.Sc. (St Andrews), Lecturer in Physics, University College, Dundee. Cumbrae, Oxford Street, Blackness, Dundee	
1933		* Forrester, Charles, A.H.-W.C., F.I.C., F.Inst.F., Vice-Principal and Professor of Chemistry, Indian School of Mines, Dhanbad, India	
1920	C.	* Franklin, Thomas Bedford, B.A. (Cantab.), 28 Kingshill Drive, Kenton, Middlesex	
1910		Fraser, Alexander, Actuary, 15 S. Learmonth Gardens, Edinburgh 4	
1929		* Fraser, David Kennedy, M.A., B.Sc., Psychologist to Glasgow Education Authority. Edge o' the Moor, Milngavie, Dumbartonshire	
1934		* Fraser, George, Chartered Civil Engineer, M.Inst.C.E., M.I.Struct.E. 25 Murrayfield Gardens, Edinburgh 12	
1928		* Fraser, John, M.C., M.D., Ch.M., F.R.C.S.E., Regius Professor of Clinical Surgery, University of Edinburgh (Royal Infirmary). 20 Moray Place, Edinburgh 3	
1928		* Fraser, Kenneth, M.D. (Edin.), D.P.H. (Cantab.), D.T.M. (Edin.), County Medical Officer of Health, Cumberland. The Croft, Scotby, near Carlisle	
1914		* Fraser, William, Managing Director, Neill & Co., Ltd., Printers, 212 Causewayside, Edinburgh 9	
1896	C.	Fraser-Harris, David Fraser, B.Sc. (Lond.), D.Sc. (Birm.), M.D., formerly Professor of Physiology, Dalhousie University, Halifax, Nova Scotia. Grove Park Lodge (3), Chiswick, London, W. 4	
1907		Galbraith, Alexander, "Ravenswood," Dalmuir, Glasgow	
1933		* Galbraith, Augustus William de Rohan, M.Inst.C.E., M.Inst.C.E.I., F.S.E., City Engineer, Christchurch, New Zealand. The Spur, Sumner, New Zealand	
1888	C.	Galt, Alexander, D.Sc., formerly Keeper of the Department of Technology, Royal Scottish Museum, Edinburgh. C/o Clydesdale Bank, 1 Melville Place, Edinburgh 3	
1901		Ganguli, Sanjiban, M.A., Principal, Maharaja's College, and Director of Public Instruction, Jaipur State, Jaipur, India	
1933		* Gardner, Alfred Charles, M.Inst.C.E., M.Inst.Mech.E., M.Inst.E.E., F.G.S., Chief Engineer, Clyde Navigation Trust, 16 Robertson Street, Glasgow. 324 Albert Drive, Pollokshields, Glasgow	
1926		* Gardner, John Davidson, B.Sc., A.M.Inst.C.E., Chief Assistant to Messrs D. & C. Stevenson, Civil Engineers, Edinburgh. 23 Ivy Terrace, Edinburgh 11	
1930	C.	* Geddes, Alexander Ebenezer M'Lean, O.B.E., M.A., D.Sc., Lecturer in Natural Philosophy, University of Aberdeen. 12 Louisville Avenue, Aberdeen	

Date of Election			Service on Council, etc.
1909	C.	† Geddes, Rt. Hon. Sir Auckland C., P.C., G.C.M.G., K.C.B., M.D., LL.D. Frensham, The Layne, Rolvenden, Kent	
1909		† Gentle, William, B.Sc., Head Master, George Heriot's School. 10 West Savile Road, Edinburgh 9	
1920	C.	* Ghosh, Sudhamoy, M.Sc. (Cal.), D.Sc. (Edin.), F.C.S., Professor of Chemistry, School of Tropical Medicine and Hygiene, Central Avenue, Calcutta, India	
1914		* Gibb, Sir Alexander, G.B.E., C.B., M.Inst.C.E., Queen Anne's Lodge, Westminster, London, S.W. 1	
1916		* Gibb, Alfred William, M.A., D.Sc., Professor of Geology, University of Aberdeen. 1 Belvidere Street, Aberdeen	
1910	C.	Gibb, David, M.A., B.Sc., Reader in Mathematics, University of Edinburgh (16 Chambers Street). 45 Fountainhall Road, Edinburgh 9	
1917	C.	* Gibson, Alexander, M.B., Ch.B., F.R.C.S., 620 Medical Arts Building, Winnipeg, Canada	
1921		* Gibson, Walcot, D.Sc., F.R.S., F.G.S., formerly Assistant Director, H.M. Geological Survey (Scotland). Pathways, Fairlight Road, Hythe, Kent	
1911		Gidney, Sir Henry A. J., Kt., M.L.A., J.P., F.R.C.S.E., Lt.-Col., I.M.S. (retired), c/o The Allahabad Bank, Ltd., Calcutta, India	
1933		* Gillespie, Robert Pollock, M.A., B.Sc., Ph.D. (Cantab.), Lecturer in Mathematics, University of Glasgow. Ashcot, Kilbarchan Road, Johnstone, Renfrewshire	
1933		* Gillespie, Thomas Haining, Director-Secretary, Zoological Society of Scotland. Corstorphine Hill House, Murrayfield, Edinburgh 12	
1925		* Gillies, William King, M.A., B.A., F.E.I.S., LL.D. (Glas.), Rector of the Royal High School, Edinburgh. Davaar, 12 Suffolk Road, Edinburgh 9	
1907		Gilruth, John Anderson, M.R.C.V.S., D.V.Sc. (Melb.), Clowes Street, South Yarra, Melbourne, Australia	
1909		Gladstone, Hugh Steuart, M.A., M.B.O.U., F.Z.S., Capenoch, Penpoint, Dumfries	
1911		Gladstone, Reginald John, M.D., F.R.C.S., Lecturer and Senior Demonstrator of Anatomy, King's College, University of London. 22 Court Lane Gardens, London, S.E. 21	
1934		* Glaister, John, M.D., D.Sc. (Glas.), Professor of Forensic Medicine, University of Glasgow. 5 Kew Terrace, Glasgow, W. 2	
1925	C.	* Goldie, Archibald Hayman Robertson, M.A., B.A., Superintendent, Meteorological Office, Air Ministry, Edinburgh, 6 Drumsheugh Gardens, Edinburgh 3	1929-32.
1910		Goodall, Joseph Strickland, M.B. (Lond.), M.R.C.P., F.R.C.S.E., M.S.A. (Eng.), Professor of Physiology and Biology, City of London Hospital. 136 Harley Street, London, W. 1	
1901		Goodwillie, James, M.A., B.Sc., 239 Clifton Road, Aberdeen	
1913	C. M-B.	* Gordon, William Thomas, M.A., D.Sc. (Edin.), M.A. (Cantab.), Professor of Geology, University of London, King's College, Strand, W.C.	
1897		Gordon-Munn, John Gordon, M.D., Croys, Dalbeattie	
1934		* Gorrie, Robert Maclagan, D.Sc. (Edin.), Working Plan Officer for Mandi State, Punjab. C/o Chief Conservator of Forests, Lahore, Punjab, India	
1923		* Grabham, George Walter, O.B.E., M.A. (Cantab.), F.G.S., Government Geologist, Anglo-Egyptian Sudan. Box 178, Khartoum	
1924		Graham, Robert James Douglas, M.A., D.Sc., Professor of Botany, University of St Andrews	
1931		* Grant, Robert, Publisher (Oliver & Boyd), Edinburgh. 6 Kilgraston Road, Edinburgh 10	
1935	C.	* Grant, Ronald, Ph.D. (Edin.), Department of Zoology, University of Chicago, Ill., U.S.A.	
1898	C.	Gray, Albert A., M.D., 5 Hammersmith Terrace, London, W. 6	
1918		* Gray, William Forbes, F.S.A.Scot., 8 Mansionhouse Road, Edinburgh 9	
1927	C. K.	* Greenwood, Alan William, D.Sc. (Melb.), Ph.D. (Edin.), Lecturer in the Institute of Animal Genetics, University of Edinburgh (West Mains Road)	
1922		* Greenwood, William Osborne, M.D. (Leeds), B.S. (Lond.), L.S.A., Clerk in Holy Orders, Woodroyd, 19 Ripon Road, Harrogate, Yorks	
1925		* Greig, David Middleton, M.B., C.M., F.R.C.S.E., LL.D., Conservator of the Museum of the Royal College of Surgeons of Edinburgh. 12 Abbotsford Crescent, Edinburgh 10	
1906		Greig, Edward David Wilson, C.I.E., M.D., D.Sc., Lt.-Col., I.M.S. (retired). 38 Coates Gardens, Edinburgh 12	

Date of Election			Service on Council, etc.
1931		* Greig, John Russell, Ph.D. (Edin.), Director, Moredun Institute, Animal Diseases Research Association. Wedderlie, Kirkbrae, Liberton 9	
1905		† Greig, Sir Robert Blyth, M.C., LL.D., formerly Secretary to the Department of Agriculture for Scotland. The Shaws, Barnton, Midlothian	1921–24. V-P 1924–27.
1935		* Grierson, Alexander Millar Meek, M.D. (Edin.), D.P.H. (Edin. and Glas.), Senior Assistant to the Medical Officer of Health, Manchester	
1910		Grimshaw, Percy Hall, I.S.O., F.R.E.S., formerly Keeper, Natural History Department, Royal Scottish Museum. 133 Liberton Brae, Edinburgh 9	
1899		† Guest, Edward Graham, M.A., B.Sc., J.P., 5 Newbattle Terrace, Edinburgh 10	
1927		* Gulland, John Masson, M.A. (Oxon.), D.Sc. (Edin.), Ph.D. (St Andrews), Reader in Biochemistry, University of London, Lister Institute, Chelsea Bridge Road, London, S.W. 1	
1907		Gulliver, Gilbert Henry, D.Sc., A.M.I.Mech.E., 99 Southwark Street, London, S.E.	
1930		* Guthrie, Douglas, M.D., F.R.C.S., Lecturer in Diseases of the Ear, Nose, and Throat, School of Medicine of the Royal Colleges, Edinburgh. 4 Rothesay Place, Edinburgh 3	
1933	C.	* Guthrie, William Gilmour, M.A. (Edin.), B.A. (Cantab.), Ph.D., Professor of Mathematics and Natural Philosophy, Magee College, Londonderry, Northern Ireland	
1911		Guy, William, F.R.C.S., L.R.C.P., L.D.S.Ed., LL.D. (Penn.), Consulting Dental Surgeon, Edinburgh Royal Infirmary; Lecturer on Human and Comparative Dental Anatomy and Physiology. 11 Wemyss Place, Edinburgh 3	
1934		* Haldane, David, B.Sc. (Edin.), Senior Geologist, H.M. Geological Survey (Scotland), 19 Grange Terrace, Edinburgh 9. 6 Kilmaurs Road, Edinburgh 9	
1922		* Hannay, Robert Kerr, M.A., LL.D., H.R.S.A., Fraser Professor of Scottish History and Palæography, University of Edinburgh (South Bridge). Historiographer-Royal for Scotland. 5 Royal Terrace, Edinburgh 7	
1923		* Hanneford-Smith, William, A.M.Inst.C.E., Hon. A.R.I.B.A. 1 The Avenue, Gravesend, Kent	
1918		* Hardie, Patrick Sinclair, M.A., B.Sc., formerly Head of the Physics Department, Medical School, Cairo. 19 Ardmillan Terrace, Edinburgh 11	
1928		* Harding, William Gerald, F.R.Hist.S., F.S.A.Scot., F.R.E.S., Peckwater House, Charing, Kent	
1923	C.	* Harris, Robert Graham, M.A., D.Sc. (Edin.), Lorraine, Manor Road, Farnborough, Hants	
1914		Harrison, Edward Philip, Ph.D., F.Inst.P., Chief Scientist, H.M.S. "Vernon," Portsmouth	
1934		* Harrison, John Vernon, D.Sc.(Glas.), F.G.S., 34 Rowallan Gardens, Glasgow, W.1	
1921		* Harrison, John William Heslop, D.Sc. (Durham), F.R.S., Professor of Botany, Armstrong College, Newcastle. The Avenue, Birtley, Co. Durham	
1926	C.	* Harrower, John Gordon, M.B., Ch.M. (Glas.), F.R.C.S.E., D.Sc. (Edin.), Professor of Anatomy, King Edward VII Medical College, and Surgeon, General Hospital, Singapore	
1926		* Harvey, William Frederick, C.I.E., M.A., M.B., C.M., D.P.H., Lieut.-Col., I.M.S. (retired), Histologist, Research Laboratory, Royal College of Physicians, Edinburgh. 56 Garscube Terrace, Edinburgh 12	
1893		Hehir, Sir Patrick, K.C.I.E., C.B., C.M.G., F.R.C.P.E., F.R.C.S.E., D.P.H., (Cantab.), D.T.M. (Liverpool), Maj.-General, I.M.S. (retired). 2A Brunswick Place, Hove, Sussex	
1931		* Henderson, John, F.C.I.I., Manager and Secretary, Edinburgh Assurance Co., Ltd. Seaforth Cottage, York Road, Trinity, Edinburgh 5	
1929		* Henderson, Thomas, C.B.E., J.P., F.S.A.Scot., Actuary of the Savings Bank of Glasgow. 5 Belmont Crescent, Glasgow, W. 2	
1908		Henderson, William Dawson, M.A., B.Sc., Ph.D., Lecturer, Zoological Laboratories, University, Bristol. 77 Coldharbour Road, Bristol 6	
1925		* Heron, Alexander Macmillan, D.Sc. (Edin.), F.G.S., Director, Geological Survey of India, Calcutta, India	
1916		* Herring, Percy Theodore, M.D., F.R.C.P.E. (Vice-President), Professor of Physiology, University of St Andrews. Linton, St Andrews	1917–20, 1931–34. V-P 1934–

Date of Election			Service on Council, etc.
1922		Hindle, Edward, M.A., Sc.D. (Cantab.), Ph.D., A.R.C.S., Regius Professor of Zoology in the University of Glasgow	
1902		Hinxman, Lionel W., B.A., formerly of the Geological Survey of Scotland. 4 Morant Gardens, Ringwood, Hants	
1904		Hobday, Sir Frederick T. G., Kt., C.M.G., Dr. Med. Vet. (Zurich), F.R.C.V.S., Principal, Royal Veterinary College, Camden Town, London, N.W. 1	
1928	C.	* Hobson, Alfred Dennis, M.A. (Cantab.), Professor of Zoology, Armstrong College, Newcastle-upon-Tyne	
1928		* Hodge, William Vallance Douglas, M.A. (Edin.), M.A. (Cantab.), Fellow and Lecturer and Director of Mathematical Studies, Pembroke College, Cambridge. 28 Barrow Road, Cambridge	
1923	C.	* Hogben, Lancelot Thomas, M.A., D.Sc., Professor of Social Biology (London School of Economics), University of London, Houghton Street, W.C. 2	
1927		Holden, Henry Smith, D.Sc., F.L.S., Professor of Botany, University College, Nottingham	
1930		* Holland, Sir Thomas Henry, K.C.S.I., K.C.I.E., D.L., Hon. D.Sc., LL.D., F.R.S., Vice-Chancellor and Principal of the University of Edinburgh. Blackford Brae, Edinburgh 9	1931–32. V-P 1932–35.
1929	C.	* Hora, Sunder Lal, D.Sc. (Punjab et Edin.), F.L.S., F.Z.S., F.A.S.B., Senior Assistant Superintendent, Zoological Survey of India. Indian Museum, Calcutta	
1920	C.	* Horne, Alexander Robert, O.B.E., B.Sc., M.I.Mech.E., A.M.Inst.C.E., Professor of Mechanical Engineering, Heriot-Watt College, Edinburgh. 31 Queen's Crescent, Edinburgh 9	
1896		Horne, J. Fletcher, M.D., F.R.C.S.E., Shelley Hall, Huddersfield	
1904	C.	Horsburgh, Ellice Martin, M.A., D.Sc., Reader in Technical Mathematics, University of Edinburgh (16 Chambers Street). 11 Granville Terrace, Edinburgh 10	
1912	C. M-B.	* Houstoun, Robert Alexander, M.A., Ph.D., D.Sc., F.Inst.P., Lecturer in Physical Optics, University of Glasgow. 45 Kirklee Road, Glasgow	1929–32.
1893		Howden, Robert, M.A., M.B., C.M., D.Sc., LL.D., Emeritus Professor of Anatomy, University of Durham. Rockearn, Perth Road, Crieff	
1933		* Hume, Edgar Erskine, D.S.M., B.A., M.A., LL.D., Lieut.-Col., U.S. Army, Librarian of the Army Medical Library, Washington. The Magnolias, Frankfort, Kentucky	
1910		Hume, William Fraser, D.Sc. (Lond.), Director, Geological Survey of Egypt, Helwân, Egypt. The Laurels, Rustington, Sussex	
1927		* Hunt, Owen Duke, B.Sc. (Manch.), Corrofell, Newton Ferrers, South Devon	
1923		* Hunter, Rev. Adam Mitchell, M.A., D.Litt., Librarian of New College, Edinburgh. 3 Suffolk Road, Edinburgh 9	
1932		* Hunter, Andrew, M.A., B.Sc., M.D., F.R.S.C., F.R.F.P.S.G., Professor of Pathological Chemistry, University of Toronto, Canada	
1928		* Hunter, Arthur, F.F.A., LL.D. (Edin.), Vice-President and Chief Actuary of the New York Life Insurance Co. 124 Lloyd Road, Montclair, N.J., U.S.A.	
1916		* Hunter, Charles Stewart, L.R.C.P.E., L.R.C.S.E., D.P.H., Cotswold, 36 Streatham Hill, London, S.W. 2	
1911		Hunter, Gilbert Macintyre, M.Inst.C.E., M.Inst.E.S., M.Inst.M.E., 27 Kilmaurs Road, Edinburgh 9	
1887	C.	Hunter, William, C.B., M.D., M.R.C.P.L. and E., M.R.C.S., LL.D., 103 Harley Street, London	
1935		* Hutchinson, Arthur Cyril William, M.D.S. (Manch.), D.D.S. (Witwatersrand), Dean of the Edinburgh Dental Hospital and School. 12 Glencairn Crescent, Edinburgh 12	
1923	C.	* Ince, Edward Lindsay, M.A. (Cantab.), D.Sc. (Edin.), Lecturer in Technical Mathematics, University of Edinburgh (16 Chambers Street)	
1920		* Inglis, James Gall, Publisher. 36 Blacket Place, Edinburgh 9	
1927		* Inglis, John Alexander, of Auchindinny and Redhall, K.C., M.A. (Oxon.), LL.B. (Edin.), King's and Lord Treasurer's Remembrancer. 13 Randolph Crescent, Edinburgh 3	1935–
1912		* Inglis, Robert John Mathieson, M.Inst.C.E., Assistant Chief Engineer, L.N.E.R. Dixton, Hadley Common, Monken Hadley, Herts.	

Date of Election			Service on Council, etc.
1917		* Irvine, Sir James Colquhoun, Kt., C.B.E., Ph.D. (Leipzig), D.Sc. (St Andrews), Hon. D.Sc. (Liverpool, Princeton), Hon. Sc.D. (Cantab., Yale, Pennsylvania), Hon. LL.D. (Glas., Aberd., Edin., and Toronto), Hon. D.C.L. (Durham), D.L., F.R.S., Hon. F.E.I.S., Vice-Chancellor and Principal of the University of St Andrews	1920–22. V-P 1922–25.
1930	C.	* Jack, David, M.A., B.Sc. (Edin.), Ph.D. (St Andrews), Lecturer in Natural Philosophy, United College, University of St Andrews. 22 Grange Road, St Andrews	
1923		* Jack, John Louttit, Solicitor, Assistant Secretary, Department of Health for Scotland, 121A Princes Street, Edinburgh 2	
1912	C.	* Jeffrey, George Rutherford, M.D. (Glas.), F.R.C.P.E., Bootham Park Private Mental Hospital, York	
1934		* Jeffrey, Sir John, K.C.B., C.B.E., Under-Secretary of State for Scotland. 9 Cluny Gardens, Edinburgh 10	
1906	C. K.	Jehu, Thomas John, M.A., M.D., F.G.S., Professor of Geology, University of Edinburgh (Grant Institute of Geology, West Mains Road). 35 Great King Street, Edinburgh 3	1917–20, 1923–26. V-P 1929–32.
1900		† Jerdan, David Smiles, M.A., D.Sc., Ph.D., 26 Avenue du Château d'Eau, Saventhem, Belgium	
1931		* Johnson, Thomas, D.Sc. (Lond.), Emeritus Professor of Botany, Royal College of Science for Ireland. Tomeg, Hillview Drive, Corstorphine 12	
1895		Johnston, Col. Henry Halcro, C.B., C.B.E., D.Sc., M.D., C.M. (Edin.), F.L.S., late Administrative Staff, Army. C/o Miss J. Grant, 32 Dundas Crescent, Kirkwall, Orkney	
1934		* Johnston, Sir William Campbell, W.S., LL.D., J.P., formerly Deputy Keeper of the Signet. 43 Castle Street, Edinburgh 2	
1928		* Johnston-Saint, Percy Johnston, M.A. (Cantab.), Conservator, Wellcome Historical Medical Museum, 183–193 Euston Road, London, N.W. 1. 4 Wyndham Place, Bryanston Square, London, W. 1	
1928		* Johnstone, Robert William, C.B.E., M.A., M.D. (Edin.), F.R.C.S.E., M.R.C.P.E., Professor of Midwifery and Diseases of Women, University of Edinburgh. 26 Palmerston Place, Edinburgh 12	
1927		* Jones, Edward Taylor, D.Sc. (Lond.), Professor of Natural Philosophy, University of Glasgow	1927–30.
1930	C.	* Jones, Samuel Griffith, D.Sc. (Univ. Wales), Lecturer in Botany, University of Glasgow. Broomfield, Kilmacolm, Renfrewshire	
1928	C.	* Jones, Tudor Jenkyn, D.Sc., M.B., Ch.B. (Glas.), Lecturer in Anatomy (Embryology), University of Liverpool	
1922		* Juritz, Charles Frederick, M.A., D.Sc., F.I.C., Chief of the Union Department of Chemistry. Villa Marina, Three Anchor Bay, Cape Town, South Africa	
1925	C.	* Kemp, Charles Norman, B.Sc., Technical Radiologist, Secretary of the Royal Scottish Society of Arts. Ivy Lodge, Laverockbank Road, Edinburgh 5	
1929		* Kendall, James Pickering, M.A., D.Sc., F.R.S. (SECRETARY TO ORDINARY MEETINGS), Professor of Chemistry, University of Edinburgh (West Mains Road). 14 Mayfield Gardens, Edinburgh 9	1931–33. Sec. 1933–
1912		† Kennedy, Robert Foster, M.D. (Belfast), M.B., B.Ch. (R.U.I.), Associate Professor of Neurology, Cornell University, New York. 410 East 57th Street, New York, U.S.A.	
1927		* Kennedy, Walter Phillips, Ph.D. (Edin.), L.R.C.P. and S.E., A.I.C., Professor of Biology, British Medical College, Baghdad	
1935		* Kenneth, John Henry, M.A., Ph.D. (Edin.), Assistant, Imperial Bureau of Animal Genetics, University of Edinburgh (West Mains Road). University Union, Edinburgh 8	
1909		Kenwood, Henry Richard, C.M.G., M.B., C.M., Emeritus Chadwick Professor of Hygiene, University of London. Wadhurst, Queen's Road, Finsbury Park, London, N.	
1925	C. M-B.	* Kermack, William Ogilvy, M.A., D.Sc., Chemist, Research Laboratory of the Royal College of Physicians, 2 Forrest Road, Edinburgh 1	
1903 & 1923	C. N.	Kerr, John Graham, M.A., LL.D., F.R.S., F.L.S., F.Z.S., M.P., formerly Regius Professor of Zoology, University of Glasgow. The University, Glasgow	1904–07, 1913–16, 1924–27. V-P 1928–31.

Date of Election			Service on Council, etc.
1891		Kerr, Joshua Law, M.D., J.P., Bundarra, via Armidale, N.S.W., Australia	
1913		* Kerr, Walter Hume, M.A., B.Sc., formerly Lecturer in Engineering Drawing and Structural Design, University of Edinburgh. Glenfriars, Jedburgh	
1926		* Khastgir, Satis Ranjan, M.Sc. (Calcutta), D.Sc. (Edin.), Physics Department, University, Dacca, India	
1907		King, Archibald, M.A., B.Sc., H.M. Inspector of Schools, 10 Leslie Road, Pollokshields, Glasgow	
1925		* King, Leonard Augustus Lucas, M.A., Professor of Zoology, West of Scotland Agricultural College, Glasgow. 14 Bank Street, Glasgow, W. 2	
1918		* Kingon, Rev. John Robert Lewis, M.A., D.Sc., The Manse, Dundee, Natal Province, South Africa	
1901		Knight, Rev. G. A. Frank, M.A., D.D., F.S.A.Scot., 10 Hillhead Street, Glasgow	
1907		Knight, James, M.A., D.Sc., F.C.S., F.G.S., J.P., Rector, Queen's Park High School, Langside, Glasgow	
1927		* Lambie, Charles George, M.C., M.D., F.R.C.P.E., Bosch Professor of Medicine, University of Sydney, Australia	
1920	C.	* Lamont, John Charles, Lieut.-Col., I.M.S. (retired), C.I.E., M.B., C.M. (Edin.), M.R.C.S., formerly Professor of Anatomy, Medical College, Lahore, India. 7 Merchiston Park, Edinburgh 10	
1925	C. N.	* Lang, William Henry, M.B., C.M., D.Sc., LL.D. (Glas.), F.R.S., Barker Professor of Cryptogamic Botany, University of Manchester	
1931		* Langrishe, John du Plessis, D.S.O., M.B., B.Ch. (Dub.), D.P.H., Lt.-Col. R.A.M.C. (retired), Lecturer in Public Health, University of Edinburgh (Usher Institute of Public Health, Warrender Park Road). 2 South Gillsland Road, Edinburgh 10	
1910	C.	Lauder, Alexander, D.Sc., Head of Chemistry Department, Edinburgh and East of Scotland College of Agriculture, 13 George Square, Edinburgh 8. Lecturer in Agricultural Chemistry, University of Edinburgh. 78 Dalkeith Road, Edinburgh 9	1917–20. Sec. 1923–28.
1885	C.	Laurie, Arthur Pillans, M.A., D.Sc., LL.D., formerly Principal, Heriot-Watt College, Edinburgh. 38 Springfield Road, St John's Wood, London, N.W. 8	1908–11, 1913–16.
1921		* Laurie, The Rev. Albert Ernest, M.C., C.F., D.D., Rector of Old St Paul's, and Canon of St Mary's Cathedral, Edinburgh. Lauder House, Jeffrey Street, Edinburgh 1	
1905		Lawson, David, M.A., M.D., L.R.C.P. and S.E., Druimdarroch, Banchory, Kincardineshire	
1903		Leighton, Gerald Rowley, O.B.E., M.D., D.Sc., formerly Medical Officer (Foods), Department of Health for Scotland. Sharston, near Ramsey, Isle of Man	
1930		* Lelean, Percy Samuel, C.B., C.M.G., F.R.C.S., L.R.C.P., D.P.H., Professor of Public Health, University of Edinburgh (Usher Institute of Public Health, Warrender Park Road). 4 South Lauder Road, Edinburgh 9	
1910		Levie, Alexander, F.R.C.V.S., D.V.S.M., Balmae, Manor Road, Littleover, Derby	
1916	C.	* Levy, Hyman, M.A., D.Sc., Professor of Mathematics, Imperial College of Science and Technology, London. Kitelands, Micheldever Station, Hants	
1914	C. N.	Lewis, Francis John, D.Sc., F.L.S., Professor of Botany, University of Alberta, Edmonton, Alberta, Canada	
1918		* Lidstone, George James, F.F.A., F.I.A., LL.D., formerly Manager and Actuary, Scottish Widows' Fund Life Assurance Society. Hermiston House, Hermiston, Currie, Midlothian	1919–22.
1905		Lightbody, Forrest Hay, 53 Queen Street, Edinburgh 2	
1931		* Lightfoot, Nicholas Morpeth Hutchinson, M.A. (Cantab.), Lecturer in Mathematics, Heriot-Watt College, Edinburgh. 3 Park Gardens, Liberton, Edinburgh 9	
1923		* Lim, Robert Kho Seng, M.B., Ch.B., D.Sc., Peking Union Medical College, Department of Physiology, Peking, China	
1912		* Lindsay, John George, M.A., B.Sc. (Edin.), Rector of Dunfermline High School	
1920	C.	* Lindsay, Thomas A., M.A., B.Sc., Head Master, Higher Grade School, Bucksburn, Aberdeenshire	
1912		* Linlithgow, The Most Honourable the Marquis of, P.C., K.T., G.C.I.E., D.L. (VICE-PRESIDENT), Hopetoun House, South Queensferry	V-P 1934–

Date of Election			Service on Council, etc.
1903		† Liston, William Glen, C.I.E., M.D., Lt.-Col., I.M.S. (retired), Milburn Tower, Gogar, Corstorphine, Edinburgh 12	
1929		* Little, John Robert, F.C.I.I., F.C.I.S., formerly Manager and Secretary, Century Insurance Co., Ltd. 5 Dalrymple Crescent, Edinburgh 9	
1932		* Lockhart, James Balfour, M.A., B.Sc. (Edin.), Mathematical Master, Edinburgh Academy. Dundas House, Kinnear Road, Edinburgh 4	
1926		* Lorraine, Norman Stanley Rees, M.D., D.P.H. (Edin. and Glas.), Medical Officer of Health, Benfleet Urban District. 1 Burlescoombe Leas, Burlescoombe Road, Thorpe Bay, Southend-on-Sea	
1930		* Low, James Wotherspoon, B.Sc., Ph.D., 1 Hamilton Park Avenue, Glasgow W. 2	
1934		* Low, R. Cranston, M.D., F.R.C.P.E., formerly Lecturer in Dermatology, University of Edinburgh. 1 Randolph Crescent, Edinburgh 3	
1923	C.	* Ludlam, Ernest Bowman, M.A., D.Sc., Lecturer in Chemistry, University of Edinburgh (West Mains Road)	
1923		* Lyford-Pike, James, M.A., B.Sc., Lecturer in Forestry, University of Edinburgh. Rosetta, Liberton, Edinburgh 9	
1924		* Lyon, David Murray, M.D., F.R.C.P.E., D.P.H., D.Sc., Professor of Therapeutics and Clinical Medicine, University of Edinburgh (Royal Infirmary). Druim, Colinton, Edinburgh 11	
1894		Mabbott, Walter John, M.A., formerly Rector of County High School, Duns, Berwickshire. The Hawthorn, Farnham Lane, Haslemere, Surrey	
1929		* M'Arthur, Donald Neil, D.Sc., F.I.C., Professor of Agricultural Chemistry, West of Scotland Agricultural College, Glasgow, C. 2. 35 Kersland Street, Glasgow, W. 2	
1921		* M'Arthur, Neil, M.A., B.Sc., Lecturer in Mathematics, University of Glasgow. The Whins, Heathfield Drive, Milngavie	
1926		* M'Bride, James Alexander, B.A. (Roy. Univ., Ireland), B.Sc. (Lond.), formerly Rector of Queen's Park Secondary School, Glasgow. Scottish Liberal Club, Princes Street, Edinburgh 2	
1883		M'Bride, Peter, M.D., F.R.C.P.E., 3 St Peter's Grove, York	
1931	C.	* McCallien, William John, D.Sc. (Glas.), Lecturer in Geology, University of Glasgow. Glenorchy, Tarbert, Argyll	
1935		* MacCallum, Peter, M.Sc., M.A. (New Zealand), M.B., Ch.B., D.P.H. (Edin.), Professor of Pathology, University of Melbourne. Blackrock House, Blackrock, Victoria, Australia	
1923		* M'Cracken, William, J.P., F.S.I., Englesea House, Crewe	
1931	C.	* M'Crea, William Hunter, M.A., Ph.D. (Cantab.), B.Sc. (Lond.), F.R.A.S., Asst. Professor of Mathematics, University of London. Imperial College of Science and Technology, South Kensington, S.W. 7	
1918		* M'Culloch, Rev. James David, 3 Ardgowan Street, Greenock	
1897	C.	Macdonald, James A., M.A., B.Sc., formerly H.M. Inspector of Schools. Rothes, Frankscroft, Peebles	
1920		* M'Donald, Stuart, M.A., M.D., F.R.C.P.E., Professor of Pathology, University of Durham. College of Medicine, Newcastle-on-Tyne	
1928		* MacDonald, Thomas Logie, M.A., B.Sc. (Glas.), F.R.A.S., 9 Colebrooke Terrace, Glasgow, W. 2	
1904		Macdonald, William, M.S.Agr., Sc.D., D.Sc., Editor, *Agricultural Journal* of South Africa. Rand Club, Johannesburg, Transvaal	
1886		Macdonald, William J., M.A., 15 Comiston Drive, Edinburgh 10	
1931		* M'Dougall, John Bowes, M.D. (Glas.), F.R.F.P.S.G., F.R.C.P.E., Medical Director, British Legion Village, Preston Hall, Kent. Preston Hall, Aylesford, Kent	
1901	C.	MacDougall, R. Stewart, M.A., D.Sc., LL.D. (Edin.), Emeritus Professor of Biology, Royal (Dick) Veterinary College, Edinburgh. Ivy Lodge, Gullane, East Lothian	1914–17.
1910		Macewen, Hugh Allen, O.B.E., M.B., Ch.B., D.P.H. (Lond. and Cantab.), Local Government Board, Ministry of Health, Whitehall, London, S.W.	
1888	C.	M'Fadyean, Sir John, Kt., M.B., B.Sc., LL.D., formerly Principal and Professor of Comparative Pathology, Royal Veterinary College, Camden Town, London. Highlands House, Leatherhead	
1885	C.	† Macfarlane, John M., D.Sc., LL.D., Emeritus Professor of Botany. 427 West Hansberry Street, Germantown, Pa., U.S.A.	
1897		MacGillivray, Angus, M.D., C.M., D.Sc., 23 South Tay Street, Dundee	

Date of Election			Service on Council, etc.
1878		M'Gowan, George, F.I.C., Ph.D., 21 Montpelier Road, Ealing, London, W. 5	
1932		* MacGregor, Archibald Gordon, M.C., B.Sc., Geologist, H.M. Geological Survey (Scotland). 1 Greenbank Terrace, Edinburgh 10	
1922		* Macgregor, Murray, M.A., D.Sc., Assistant Director (Scotland), H.M. Geological Survey. 19 Grange Terrace, Edinburgh 9	1930–33.
1903		M'Intosh, Donald C., M.A., D.Sc., formerly Education Officer, Elgin. The Cottage, Boat of Garten	
1911		M'Intosh, John William, M.R.C.V.S., Dollis Hill Farm, Cricklewood, London, N.W. 2	
1927	C.	* M'Intyre, Donald, M.B.E., M.D. (Glas.), F.R.C.S.E., Royal Samaritan Lecturer in Gynæcology, University of Glasgow. 9 Park Circus, Glasgow, C. 3	
1912	C.	M'Kendrick, Anderson Gray, M.B., D.Sc., F.R.C.P.E., Lt.-Col., I.M.S. (retired), Superintendent, Research Laboratory, Royal College of Physicians, 2 Forrest Road, Edinburgh 1	1924–27. 1933–
1914		* M'Kendrick, Archibald, F.R.C.S.E., D.P.H., L.D.S., J.P., 12 Rothesay Place, Edinburgh 3	
1900	C.	M'Kendrick, John Soutar, M.D., F.R.F.P.S. (Glas.), 2 Buckingham Terrace, Hillhead, Glasgow	
1910	C.	Mackenzie, Alister, M.A., M.D., D.P.H., Principal Medical Officer and Lecturer in Hygiene, Training Centre, Jordanhill, Glasgow. 22 Queen's Gate, Dowanhill, Glasgow	
1916	C.	* Mackenzie, John E., D.Sc., Reader in Chemistry, University of Edinburgh (West Mains Road). 2 Ramsay Garden, Edinburgh 1	
1905		Mackenzie, Sir William Colin, K.B., M.D., F.R.C.S., Director, Australian Institute of Anatomy, Canberra, F.C.T., Australia	
1929	C.	* Mackie, John, M.A., D.Sc., Rector, Leith Academy. 7 York Road, Trinity, Leith 5	
1928		* Mackie, Thomas Jones, M.D., M.R.C.P.E., Professor of Bacteriology, University of Edinburgh (Teviot Place). 22 Mortonhall Road, Edinburgh 9	
1910		MacKinnon, James, M.A., Ph.D., LL.D., Emeritus Professor of Ecclesiastical History, University of Edinburgh. 12 Lygon Road, Edinburgh 9	1933–
1904		Mackintosh, Donald James, C.B., M.V.O., D.L., M.B., C.M., LL.D., Superintendent, Western Infirmary, Glasgow	
1899		Maclean, Sir Ewan John, M.D., D.Sc. (Hon.), LL.D., F.R.C.P. (Lond.), D.L., J.P., Emeritus Professor of Obstetrics and Gynæcology, Welsh National Medical School. 12 Park Place, Cardiff	
1888	C.	Maclean, Magnus, M.A., D.Sc., LL.D., M.Inst.C.E., M.I.E.E., formerly Professor of Electrical Engineering, Royal Technical College. 108 University Avenue, Glasgow, W. 2	1916–19.
1933		* Macleod, James, F.I.C., Manager, Glasgow Corporation Chemical Works Department. 16 Colebrooke Street, Glasgow, W. 2	
1916	C.	* M'Lintock, William Francis Porter, D.Sc. (Edin.), Geological Survey and Museum, Exhibition Road, South Kensington, London, S.W. 7	
1923		* Macmillan, Rt. Hon. Lord, P.C., LL.D., 44 Millbank, Westminster, S.W. 1	
1932		* M'Neil, Charles, M.A., M.D. (Edin.), F.R.C.P.E., Professor of Child Life and Health, University of Edinburgh (Royal Edinburgh Hospital for Sick Children). 44 Heriot Row, Edinburgh 3	
1917		* Macpherson, Rev. Hector Copland, M.A., Ph.D., F.R.A.S., Guthrie Memorial U.F. Church. 7 Wardie Crescent, Edinburgh 5	
1921		* M'Quistan, Dougald Black, M.A., B.Sc., Associate-Professor of Natural Philosophy, Royal Technical College, Glasgow. 29 Viewpark Drive, Rutherglen	
1921	C.	* MacRobert, Thomas Murray, M.A., D.Sc., Professor of Mathematics, University of Glasgow. 10 The University, Glasgow	1931–34.
1921	C.	* M'Whan, John, M.A. (Glasgow), Ph.D. (Gött.), Lecturer in Mathematics, University of Glasgow. 84 Munro Road, Jordanhill, Glasgow, W. 3	
1927		* Madwar, Mohamed Reda, Ph.D. (Edin.), A.M.Inst.C.E., Director, Helwân Observatory, Egypt	
1898	C.	† Mahâlanobis, S. C., B.Sc. (Edin.), Professor of Physiology, University of Calcutta. P. 45 New Park Street, Calcutta	
1913		Majumdar, Tarak Nath, D.P.H. (Cal.), L.M.S., F.C.S., Health Officer, IV, Calcutta. P. 235 Russa Road, P.O. Tollygunge	
1933		* Malcolm, John, M.D. (Edin.), Professor of Physiology, University of Dunedin, New Zealand, Medical School, King Street, Dunedin	

Date of Election			Service on Council, etc.
1917		* Malcolm, L. W. Gordon, M.Sc. (Cantab.), Ph.D., F.L.S., formerly Conservator, Wellcome Historical Medical Museum. Robins Herne, Northwood, Middlesex	
1908		Mallik, Devendranath, Sc.D., B.A., Principal, Carmichael College, Rungpur, Bengal, India	
1912		† Maloney, William Joseph, M.B.E., M.C., M.D. (Edin.), LL.D., formerly Professor of Neurology, Fordham University. Casa del Sale, Newport, Rhode Island, U.S.A.	
1913		Marchant, Rev. Sir James, K.B.E., LL.D., F.R.A.S., F.L.S., Director, National Council for Promotion of Race-Regeneration. Pinegarth, Buccleuch Road, Bournemouth	
1901	C.	Marshall, Francis Hugh Adam, C.B.E., Sc.D., F.R.S., Reader in Agricultural Physiology, University of Cambridge. Christ's College, Cambridge	
1920	C.	* Marshall, John, M.A., D.Sc. (St Andrews), B.A. (Cantab.), University Reader in Mathematics, Bedford College, London. Logan House, 123 Torrington Park, London, N. 12	
1931		* Mason, John Huxley, F.R.C.V.S., Government Veterinary Laboratory, Onderstepoort, Pretoria, South Africa	
1885	C.	Masson, Sir David Orme, K.B.E., M.A., D.Sc., LL.D., F.R.S., Emeritus Professor of Chemistry, University of Melbourne	
1913		Masson, George Henry, M.D., D.Sc., F.R.C.P.E., Port of Spain, Trinidad, British West Indies	
1898	C. M-B.	Masterman, Arthur Thomas, M.A., D.Sc., F.R.S., formerly Superintending Inspector, H.M. Board of Agriculture and Fisheries. 3 Kedale Road, Seaford	1902–04.
1911		† Mathews, Gregory Macalister, M.B.O.U., Meadway, St Cross, Winchester, Hants	
1921		* Mathieson, John, F.R.S.G.S., Division Superintendent, Ordnance Survey (retired), 42 East Claremont Street, Edinburgh 7	
1906		Mathieson, Robert, F.C.S., St Serf's, Innerleithen	
1928		* Matthai, George, M.A. (Cantab.), F.Z.S., F.L.S., formerly Professor of Zoology, Government College, Lahore, India. C/o Messrs Thomas Cook & Sons, Lucerne, Switzerland	
1924		* Matthews, James Robert, M.A., F.L.S., Regius Professor of Botany, University of Aberdeen, and Keeper of the Cruickshank Botanic Garden	
1932		* Maxwell, William, Managing Director of R. & R. Clark, Ltd. 14 South Inverleith Avenue, Edinburgh 4	
1917		* Maylard, A. Ernest, M.B., B.Sc. (Lond.), F.R.F.P.S. (Glasgow), Kingsmuir, Peebles	
1922		* Meakins, Jonathan Campbell, M.D., LL.D., F.R.C.P.E., Professor of Medicine and Director of the Department of Medicine, McGill University, Montreal, Canada	
1931		* Mears, Frank Charles, F.R.I.B.A., 44 Queen Street, Edinburgh 2	
1901	C.	Menzies, Alan W. C., M.A., Ph.D., F.C.S., Professor of Chemistry, Princeton University, Princeton, New Jersey, U.S.A.	
1927		* Menzies, Sir Frederick Norton Kay, K.B.E., M.D., LL.D. (Edin.), F.R.C.P.E., D.P.H. (Lond.), Medical Officer of Health and School Medical Officer, Administrative County of London, County Hall, London, S.E.	
1933	C.	* Menzies, William John Milne, Inspector of Salmon Fisheries of Scotland, Fishery Board for Scotland. Tighbeag, Whitehouse Road, Cramond Bridge, near Edinburgh	
1929		* Mercer, Walter, M.B., Ch.B., F.R.C.S.E., Lecturer in Clinical Surgery, University of Edinburgh (Royal Infirmary). 12 Rothesay Terrace, Edinburgh 3	
1917		* Merson, George Fowlie, Manufacturing Technical Chemist, St John's Hill Works, Edinburgh 8	
1902	C.	† Metzler, William H., A.B., D.Sc., Ph.D., formerly Dean of the New York State College for Teachers, Albany, N.Y., U.S.A. 4 Glenwood Street, Albany, N.Y., U.S.A.	
1885	C. M-B.	Mill, Hugh Robert, D.Sc., LL.D., Hill Crest, Dormans Park, E. Grinstead	
1910		Miller, John, M.A., D.Sc., formerly Professor of Mathematics, Royal Technical College. 212 Wilton Street, Glasgow, N.W.	
1930		* Miller, William Christopher, M.R.C.V.S., Courtauld Professor of Animal Husbandry, Royal Veterinary College, Camden Town, London, N.W. 1. Blair Hyrne, Hadley Common, Barnet, Herts	

Date of Election			Service on Council, etc.
1905		Milne, Archibald, M.A., D.Sc., Deputy Director of Studies, Edinburgh Provincial Training College. 38 Morningside Grove, Edinburgh 10	
1905		Milne, C. H., M.A., D.Litt., formerly Head Master, Daniel Stewart's College. 19 Merchiston Gardens, Edinburgh 10	
1904	C.	Milne, James Robert, D.Sc., Lecturer in Natural Philosophy, University of Edinburgh (Drummond Street). 7 Grosvenor Crescent, Edinburgh 12	
1886		Milne, William, M.A., B.Sc., 70 Beechgrove Terrace, Aberdeen	
1933	C.	Milne-Thomson, Louis Melville, M.A. (Cantab.), F.R.A.S., Professor of Mathematics, Royal Naval College, Greenwich. Gothic House, Maze Hill, London, S.E. 10	
1899		Milroy, Thomas Hugh, M.D., B.Sc., LL.D., formerly Professor of Physiology, Queen's University, Belfast. Woodville, North Berwick	
1889	C. K.	Mitchell, A. Crichton, D.Sc., Hon. Doc. Sc. (Genève), formerly Director of Public Instruction in Travancore, India. 246 Ferry Road, Edinburgh 5. (Society's Representative on Governing Body of Heriot-Watt College)	1915–16, 1930–33. Cur. 1916–26. V-P 1926–29.
1897		† Mitchell, Sir George Arthur, Kt., M.A., M.I.M.E., J.P., 9 Lowther Terrace, Kelvinside, Glasgow	
1900		Mitchell, James, M.A., B.Sc., Islay Lodge, Lochgilphead, Argyll	
1911		Modi, Edalji Manekji, D.Sc., LL.D., Litt.D., F.C.S., Arthur Road Chemical Works, Meher Buildings, Tardeo, Bombay, India	
1906		† Moffat, Rev. Alexander, M.A., LL.D., formerly Professor of Physics, Christian College, Madras. C/o National Bank of India, Post Box No. 64, Madras, India	
1929		Moir, Henry, F.F.A., F.I.A., President, United States Life Insurance Co., 101 Fifth Avenue, New York City. Upper Montclair, New Jersey	
1890	C.	† Mond, Sir Robert Ludwig, Kt., M.A. (Cantab.), LL.D., F.C.S., 9 Cavendish Square, London, W. 1	
1887	C.	Moos, N. A. F., D.Sc., J.P., Director of Bombay and Alibag Observatories (retired). Red Leaf, Pedder Road, Bombay, India	
1896		Morgan, Alexander, O.B.E., M.A., D.Sc., formerly Principal, Edinburgh Provincial Training College. 1 Midmar Gardens, Edinburgh 10	
1930		* Morison, John Miller Woodburn, M.D., F.R.C.P.E., D.M.R. and E., Professor of Radiology, University of London, and Director of the Radiological Department, Cancer Hospital, Fulham Road, London, S.W. 3	
1926		* Morris, James Archibald, R.S.A., F.S.A.Scot., Savoy Croft, Ayr	
1919		* Morris, Robert Owen, O.B.E., M.A., M.D., C.M. (Edin.), D.P.H. (Liverpool). King Edward VII Welsh National Memorial Association (Tuberculosis). Hafod-ar-For, Aberdovey, N. Wales	
1892	C.	Morrison, J. T., M.A., D.Sc., Emeritus Professor of Mathematical Physics, University, Stellenbosch, South Africa	
1930		* Morton, James, LL.D., Governing Director, Scottish Dyes, Ltd. Tuethur, Carlisle	
1901		Moses, O. St John, M.D., D.Sc., F.R.C.S., Lt.-Col., I.M.S. (retired), formerly Professor of Medical Jurisprudence, Medical College, Calcutta. 18 Manstone Road, Cricklewood, London, N.W. 2	
1892	C. K.	Mossman, Robert Cockburn, Lacar 4332, Villa Devoto F.C.P., Buenos Aires, Argentina	
1934		* Mowat, Magnus, C.B.E., M.Inst.C.E., M.I.Mech.E., Brigadier-General, Secretary of the Institution of Mechanical Engineers, Storey's Gate, London, S.W. 1	
1935	C.	* Mozley, Walter Alan, B.Sc. (Manitoba), Ph.D. (Edin.), Lecturer in Zoology, Johns Hopkins University, Baltimore, Mo., U.S.A.	
1916		* Muir, Sir Robert, Kt., M.A., M.D., Sc.D., LL.D., F.R.S., F.R.C.P.E., Professor of Pathology, University of Glasgow. 30 Victoria Crescent Road, Glasgow, W. 2	
1907		Muirhead, James M. P., J.P., F.R.S.L., F.S.S., Carmel, Rouwkoop Road, Rondebosch, South Africa	
1935		* Munnoch, James, F.S.A.Scot., formerly Controller, General Post Office, Edinburgh. 15 Liberton Drive, Edinburgh 9	
1933		* Murray, John, M.A. (Aberd.), Ph.D. (Edin.), Rector, The Academy, Annan. 9 Seaforth Avenue, Annan	

Date of Election			Service on Council, etc.
1932		* Murray, John Murdoch, B.Sc. (Edin.), Assistant Commissioner for Scotland, Forestry Commission. 76 Hillview Terrace, Corstorphine, Edinburgh 12	
1934		* Murray, Walter George Robertson, A.I.C., Technical Assistant, Department of Chemistry, University of Edinburgh (West Mains Road). 20 Montpelier Park, Edinburgh 10	
1935		* Narayana, Basudeva, M.Sc., M.B. (Calcutta), Ph.D. (Edin.), Lecturer in Physiology, University of Patna. Department of Physiology, Medical College, Patna, India	
1931		* Nelson, Alexander, B.Sc. (Glas.), Ph.D. (Edin.), N.D.A., Lecturer in Plant Physiology and Agricultural Botany, University of Edinburgh. 14 Netherby Road, Edinburgh 5	
1924		* Nelson, Philip, M.A., M.D., Ph.D., F.S.A., Beechwood, Calderstones, Liverpool	
1898		Newman, Sir George, G.B.E., K.C.B., M.D., D.C.L., LL.D., F.R.C.P., formerly Chief Medical Officer, Ministry of Health and Board of Education, Whitehall, London, S.W. 1	
1928		* Nichols, James Edward, M.Sc. (Dunelm), Ph.D. (Edin.), Professor of Agriculture, University of Western Australia, Perth, W.A.	
1933	C.	* Nicol, Thomas, M.D., Ch.B., F.R.C.S.E., D.Sc., Lecturer in Anatomy, University of Glasgow. 77 Fotheringay Road, Glasgow, S. 1	
1927		* Noble, Thomas Paterson, M.D. (Edin.), F.R.C.S., Phya Thai Palace, Bangkok, Siam	
1934	C.	* Normand, Alexander Robert, M.A., B.Sc., Ph.D. (Edin.), formerly Professor of Chemistry, Wilson College, Bombay. 7 India Street, Edinburgh 3	
1928	C. N.	* O'Donoghue, Charles Henry, D.Sc. (Lond.), Reader in Zoology, University of Edinburgh (West Mains Road). Appin Lodge, Eskbank, Midlothian	
1925		* Ogg, William Gammie, M.A., Ph.D., Director, Macaulay Institute for Soil Research, Craigiebuckler, Aberdeen; Research Lecturer in Soil Science in the University of Aberdeen	
1923	C.	* Ogilvie, Alan G., O.B.E., M.A., B.Sc. (Oxon.), Professor of Geography, University of Edinburgh (High School Yards). 40 Fountainhall Road, Edinburgh 9	1932-35.
1929		* Ogilvie, Frederick Wolff, M.A., Vice-Chancellor and President of Queen's University, Belfast	
1886		Oliver, James, M.D., F.L.S., Physician to the London Hospital for Women. 123 Harley Street, London, W.	
1895	C.	† Oliver, Sir Thomas, Kt., D.L., M.D., LL.D., F.R.C.P., formerly Vice-Chancellor of the University of Durham. 7 Ellison Place, Newcastle-upon-Tyne	
1930		* Oliver, William, B.Sc., A.M.Inst.C.E., Professor of Organisation of Industry and Commerce, University of Edinburgh (South Bridge). 70 Netherby Road, Trinity, Edinburgh 5	
1930		* O'Riordan, George Francis, B.Sc. (Eng.), M.I.Mech.E., Principal of Battersea Polytechnic, London, S.W. 7. Hesslewood, 3 Hazlewell Road, Putney, London, S.W. 15	
1924		* Orr, Sir John Boyd, Kt., D.S.O., M.C., M.A., D.Sc., M.D., LL.D., F.R.S., Director of the Rowett Research Institute for Research in Animal Nutrition, Aberdeen	
1915		* Orr, Lewis P., F.F.A., formerly General Manager of the Scottish Life Assurance Co., 3 Belgrave Place, Edinburgh 4	
1932		* Orr, Matthew Young, 38 Lennox Row, Edinburgh 5	
1927		* Owen, William John, Memb. Roy. Soc. Victoria, Director, Department of Histology, Australian Institute of Anatomy, Canberra, F.C.T., Australia	
1908		Page, William Davidge, 13 Yarrell Mansions, Queen's Club Gardens, London, W. 14	
1934		* Pal, Rudrendra Kumar, D.Sc. (Edin.), M.R.C.P.E., L.R.C.S.E., Professor of Physiology, Prince of Wales Medical College, Patna, India	
1905		Pallin, Lt.-Col. William Alfred, C.B.E., D.S.O., F.R.C.V.S., 5 Tower Gardens, Hythe, Kent	
1933		Parsons, Charles Wynford, M.A., Lecturer in Zoology, University of Glasgow. Westbank, 1 Chapelton Terrace, Bearsden, Dumbartonshire	
1901		Paterson, David, F.C.S., Leewood, Roslyn Castle, Midlothian	
1918		* Paterson, Rev. William Paterson, D.D., LL.D., Emeritus Professor of Divinity, University of Edinburgh (South Bridge). 17 South Oswald Road, Edinburgh 9	

Date of Election			Service on Council, etc.
1927		* Patterson, Charles, A.M.I.Mar.E., Lecturer in Mechanical Engineering Design, University of Edinburgh (Sanderson Engineering Laboratories, Mayfield Road). 22 Dudley Terrace, Trinity, Edinburgh 6	
1926		* Patton, Donald, M.A., Ph.D., Lecturer in Botany, Glasgow Provincial College for the Training of Teachers. 15 Jordanhill Drive, Glasgow, W. 3	
1923	C.	* Peacock, Alexander David, D.Sc., Professor of Zoology, University College, Dundee	1935–
1907		Pearce, John Thomson, B.A., B.Sc., Bolton Manse, Haddington, East Lothian	
1914		Pearson, Joseph, D.Sc., F.L.S., formerly Director of the Colombo Museum, and Marine Biologist to the Ceylon Government. Director of the Tasmanian Museum, Hobart, Tasmania	
1904		Peck, James Wallace, C.B., M.A., Second Secretary, Scottish Education Department, Dover House, Whitehall, London, S.W. 1	1926–28.
1887	C. M-B.	Peddie, William, D.Sc., Professor of Natural Philosophy, University College, Dundee. The Weisha, Ninewells, Dundee	1904–07, 1908–11, 1933– V-P 1919–22.
1925		* Penman, David, D.Sc., M.Inst.M.E., Principal, Indian School of Mines, Dhanbad, India	
1893		Perkin, Arthur George, Hon. D.Sc. (Leeds), F.R.S., F.I.C., Emeritus Professor in the University of Leeds. Grosvenor Lodge, Grosvenor Road, Headingley, Leeds	
1931	C.	* Phemister, James, M.A., D.Sc. (Glas.), Petrographer, H.M. Geological Survey and Museum, Exhibition Road, London, S.W. 7	
1889		† Philip, Sir Robert William, Kt., M.A., M.D., LL.D., F.R.C.P.E., Professor of Tuberculosis, University of Edinburgh. 45 Charlotte Square, Edinburgh 2	V-P 1927–30.
1907	C.	† Phillips, Charles E. S., O.B.E., Castle House, Shooters Hill, Woolwich, S.E. 18	
1929	C.	* Phillips, John Frederick Vicars, D.Sc., F.L.S., Professor of Botany, University of the Witwatersrand, Johannesburg, Union of South Africa	
1935		* Pichamuthu, Charles Solomon, B.Sc. (Mysore), Assistant Professor of Geology, University of Mysore. Central College, Bangalore, India	
1932		* Pickard, James Nichol, B.A. (Cantab.), Ph.D. (Edin.), Little Craufurd, Lasswade, Midlothian	
1928		* Pilcher, Robert Stuart, General Manager, Manchester Corporation Tramways. 55 Piccadilly, Manchester	
1933		* Pillers, Alfred William Noel, F.R.C.V.S., D.V.S.M., Chief Veterinary Officer to the Corporation of the City of Liverpool; Lecturer in Veterinary Parasitology, University of Liverpool. Hawkstone, Queen's Drive, Walton, Liverpool 4	
1908	C.	† Pirie, James Hunter Harvey, B.Sc., M.D., F.R.C.P.E., Research Pathologist and Bacteriologist, South African Institute for Medical Research. P.O. Box 1038, Johannesburg, South Africa	
1911		Pirie, James Simpson, M.Inst.C.E., 25 Grange Road, Edinburgh 9	
1906		Pitchford, Herbert Watkins, C.M.G., F.R.C.V.S., Victoria Club, Pietermaritzburg, South Africa	
1934		* Plenderleith, Harold James, M.C., B.Sc., Ph.D. (St Andrews), F.C.S., Assistant Keeper, 1st Class, in Research Laboratory of British Museum, London. 134 The Vale, Golder's Green, N.W. 11	
1924		* Ponder, Eric, M.D., D.Sc., Professor of General Physiology, Washington Square College, New York University, New York, U.S.A.	
1919		* Porritt, B. D., M.Sc. (Lond.), F.I.C., Director of Research, Research Association of British Rubber Manufacturers, 105 Lansdowne Road, Croydon, Surrey	
1888		† Prain, Sir David, Kt., Lt.-Col., I.M.S. (retired), C.M.G., C.I.E., M.A., M.B., LL.D., F.R.S., F.L.S., formerly Director, Royal Botanic Gardens, Kew, Surrey. The Well Farm, Warlingham, Surrey	
1932		* Prasad, Gorakh, D.Sc. (Edin.), Reader in Mathematics, University of Allahabad. Beli Road, Allahabad, India	
1926	C.	* Prashad, Baini, D.Sc., Superintendent, Zoological Survey of India, Indian Museum, Calcutta	

Date of Election			Service on Council, etc.
1933		* Preston, Frank Anderson Baillie, L.R.I.B.A., F.S.A.Scot., Lecturer in Municipal Engineering, Royal Technical College, Glasgow. Craigrownie, Briarwell Road, Milngavie	
1915		† Price, Frederick William, M.D. (Edin.), Consulting Physician to the Royal Northern Hospital, London; Senior Physician to the National Hospital for Diseases of the Heart. 133 Harley Street, London, W.	
1932		* Price, Thomas Slater, O.B.E., D.Sc., F.R.S., Professor of Chemistry, Heriot-Watt College, Edinburgh. 2 Cluny Drive, Edinburgh 10	
1932		* Pringle, John, Hon. D.Sc., F.G.S., Palæontologist, Geological Survey of Great Britain, Exhibition Road, South Kensington, London, S.W. 7	
1911		Purdy, John Smith, D.S.O., M.D., C.M. (Aberd.), D.P.H. (Cantab.), F.R.G.S., Town Hall, Sydney, N.S.W., Australia	
1920	C.	* Purser, George Leslie, M.A. (Cantab.), F.Z.S., Lecturer in Embryology, University of Aberdeen	
1898		Purves, John Archibald, D.Sc., Chilliswood, Trull, Taunton	
1899	C.	Ramage, Alexander G., Lochcote, Linlithgowshire	
1904		Ratcliffe, Joseph Riley, M.B., C.M., c/o The Librarian, University, Birmingham	
1900		Raw, Nathan, C.M.G., M.D., 22 Ashworth Road, Maida Vale, London, W. 9	
1927	C.	* Read, Herbert Harold, D.Sc. (Lond.), A.R.C.S., F.G.S., George Herdman Professor of Geology, University of Liverpool	
1929		* Read, Selwyn, B.A., Schoolmaster, Edinburgh Academy. 2 Oxford Terrace, Edinburgh 4	
1902		Rees-Roberts, John Vernon, M.D., D.Sc., D.P.H., 90 Fitzjohns Avenue, Hampstead, London, N.W. 3	
1913		Reid, Harry Avery, O.B.E., F.R.C.V.S., D.V.H., Bacteriologist and Pathologist, Department of Agriculture, Wellington, New Zealand. C/o Bank of New Zealand, 1 Queen Victoria Street, London, E.C.	
1924		* Reid, William Carstairs, Civil Engineer, 72A George Street, Edinburgh 2	
1914		Renshaw, Graham, M.D., M.R.C.S., L.R.C.P., L.S.A., Editor of the *Avicultural Magazine*, Sale Bridge House, Sale, Manchester	
1913		* Richardson, Harry, M.Inst.E.E., M.Inst.M.E., Rockwood, Station Avenue, Walton-on-Thames	
1908		Richardson, Linsdall, F.G.S., 104 Greenfield Road, Harbourne, Birmingham	
1927		* Richey, James Ernest, M.C., B.A., B.A.I. (T.C.D.), Sc.D., F.G.S., District Geologist, H.M. Geological Survey (Scotland), 19 Grange Terrace, Edinburgh 9	
1930		* Ritchie, Allan Watt, Chief Sanitary Inspector, City of Edinburgh. 2 Queensferry Terrace, Edinburgh 4	
1916	C.	* Ritchie, James, M.A., D.Sc., Regius Professor of Natural History, University of Aberdeen	1921–24, 1926–28. Sec. 1928–31. V-P 1931–34.
1914	C.	* Ritchie, James Bonnyman, D.Sc., Rector, The Academy, Ayr. 28 Carrick Road, Ayr	
1906	C.	Ritchie, William Thomas, M.D., F.R.C.P. Edin. and Lond., Professor of Medicine, University of Edinburgh (Teviot Place). 10 Douglas Crescent, Edinburgh 12	
1929	C.	* Robb, Richard Alexander, M.A., B.Sc., M.Sc., Lecturer in Mathematics, University of Glasgow. 27 Moor Road, Eaglesham, Renfrewshire	
1931		* Robb, William, N.D.A., Director of Research, Scottish Society for Research in Plant Breeding. Craigs House, Corstorphine, Edinburgh 12.	
1898	C.	Roberts, Hon. Alexander William, D.Sc., F.R.A.S., Lovedale, South Africa	
1919		* Roberts, Alfred Henry, O.B.E., M.Inst.C.E., formerly Superintendent and Engineer, Leith Docks. 40 Buckingham Terrace, Edinburgh 4	
1926		* Roberts, John Alexander Fraser, M.A. (Cantab.), D.Sc., Stoke Park Colony, Stapleton, Bristol	
1928	C.	* Roberts, Owen Fiennes Temple, M.C., M.A. (Cantab.), Lecturer in Astronomy and Meteorology, University of Aberdeen. 20 Belgrave Terrace, Aberdeen	
1919		* Robertson, William Alexander, F.F.A., Century Insurance Co., Ltd., 18 Charlotte Square. Mardale, 3 Buckstone Park, Edinburgh 10	
1896	C.	Robertson, W. G. Aitchison, D.Sc., M.D., F.R.C.P.E., Barrister-at-Law, Lincoln's Inn. St Margarets, St Valerie Road, Bournemouth	

Date of Election			Service on Council, etc.
1932	C.	* Robson, John Michael, M.D., B.Sc., Beit Fellow, Institute of Animal Genetics, University of Edinburgh (West Mains Road). Tayinloan, Loanhead	
1926		* Romanis, William Hugh Cowie, M.A., M.B., M.C. (Cantab.), F.R.C.S., Surgeon to St Thomas's Hospital, London. 120 Harley Street, London, W. 1	
1916		* Ronald, David, M.Inst.C.E., Chief Engineer, Scottish Board of Health, 125 George Street, Edinburgh 2	
1909	C.	Ross, Alexander David, M.A., D.Sc., F.Inst.P., F.R.A.S., Professor of Physics, University of Western Australia, Perth, Western Australia	
1921		* Ross, Edward Burns, M.A., formerly Professor of Mathematics, Madras Christian College, Madras. 41 Liberton Brae, Edinburgh 9	
1935		* Rowatt, Thomas, M.I.Mech.E., F.S.A.Scot., Director, Royal Scottish Museum. Spottiswoode, Spylaw Bank Road, Colinton, Edinburgh	
1931	C.	* Ruse, Harold Stanley, M.A. (Oxon.), D.Sc., Lecturer in Mathematics, University of Edinburgh (16 Chambers Street)	
1906		Russell, Alexander Durie, B.Sc., Mathematical Master, Falkirk High School. 14 Heugh Street, Falkirk	
1930		Russell, David, LL.D., Paper Manufacturer. Silverburn, Leven, Fife	
1902	C. K.	Russell, James, 22 Glenorchy Terrace, Edinburgh 9	
1934		* Rutherford, Daniel Edwin, M.A., B.Sc. (St Andrews), D.Math. (Amsterdam), Lecturer in Applied Mathematics, United College, University of St Andrews. 8 Moredun Terrace, Perth	
1925	C.	* Saddler, William, M.A., B.A., Professor of Mathematics, Canterbury College, Christchurch, N.Z.	
1906		Saleeby, Caleb Williams, M.D., 13 Greville Place, Hampstead, London, N.W. 6	
1916	C.	* Salvesen, The Rt. Hon. Lord, P.C., K.C., LL.D., Judge of the Court of Session (retired), Dean Park House, Edinburgh 4	1920–22. V-P 1922–25.
1934		* Salvesen, Harold Keith, M.A. (Oxon. and Harvard), Captain (retired, I.A.), Shipowner, Inveralmond, Cramond, Midlothian	
1914		* Salvesen, Theodore Émile, of Culrain, F.R.S.A., F.S.A.Scot., Chevalier de la Légion d'Honneur. 37 Inverleith Place, Edinburgh 4	
1912	C. K.	* Sampson, Ralph Allen, M.A., D.Sc., LL.D., F.R.S. (VICE-PRESIDENT), Astronomer Royal for Scotland, Professor of Astronomy, University of Edinburgh (South Bridge). Royal Observatory, Edinburgh	1912–15, 1919–21. V-P 1915–18, 1933– Sec. 1922–23. Gen. Sec. 1923–33.
1927	C.	* Sandeman, Ian, M.A., B.Sc., Ph.D. (St Andrews), Acting Chief Inspector of Schools, Education Department, Colombo, Ceylon	
1930		* Sansome, Frederick Whalley, Ph.D., F.L.S., Senior Lecturer in Horticulture, Botany Department, University of Manchester	
1922		* Sarkar, Bijali Behari, M.Sc., D.Sc. (Edin.), Lecturer in Physiology, University, Calcutta. 33/3 Lansdowne Road, Calcutta	
1903		Sarolea, Charles, Ph.D., D.Litt., formerly Professor of French, University of Edinburgh. 21 Royal Terrace, Edinburgh 7	
1935		* Say, Maurice George, Ph.D., M.Sc. (Lond.), A.M.I.E.E., Professor of Electrical Engineering, Heriot-Watt College, Edinburgh. Dreghorn Loan, Colinton, Edinburgh	
1927	C.	* Schlapp, Robert, M.A. (Edin.), Ph.D. (Cantab.), Lecturer in Applied Mathematics, University of Edinburgh (Drummond Street). 1 Peel Terrace, Edinburgh 9	
1885	C.	† Scott, Alexander, M.A., D.Sc., F.R.S., Director of Scientific Research at the British Museum. 34 Upper Hamilton Terrace, London, N.W. 8	
1919		* Scott, Alexander, M.A., D.Sc., 3 Winton Terrace, Stoke-on-Trent	
1919		* Scott, Alexander Ritchie, B.Sc. (Edin.), D.Sc. (Lond.), Principal, London County Council, Beaufoy Institute, Prince's Road, London, S.E. 11	
1917		* Scott, Henry Harold, M.D., F.R.C.P., M.R.C.S., D.P.H., Director, Bureau of Hygiene and Tropical Diseases, Keppel Street, Gower Street, London, W.C. 1. 18 Ridgmount Gardens, Gower Street, London, W.C. 1	
1928		* Senior-White, Ronald, F.R.E.S., Malariologist, Bengal-Nagpur Railway, Kidderpore, P.O., Calcutta, India	

Date of Election			Service on Council, etc.
1930		* Shankland, Ernest Claud, F.R.Met.S., River Superintendent, Port of London Authority. Mariners, Balfour Gardens, Folkestone	
1927		* Sharpley, Forbes Wilmot, B.Sc. (Eng.) (Lond.), M.Inst.E.E., Professor of Electrical and Mechanical Engineering, Indian School of Mines, Dhanbad, India	
1931		* Shaw, John James M'Intosh, M.A., M.D., F.R.C.S., Lecturer in Surgery and Clinical Surgery, University of Edinburgh. Greenaway, Kinnear Road, Edinburgh 4	
1927		* Shearer, Ernest, M.A., B.Sc. (Edin.), Professor of Agriculture and Rural Economy, University of Edinburgh, and Principal, Edinburgh and East of Scotland College of Agriculture, 13 George Square, Edinburgh 8	
1931		* Shearer, James Fleming, M.A., B.Sc., Ph.D., Lecturer in Natural Philosophy, University of Glasgow. 149 Queen's Drive, Wavertree, Liverpool 15	
1932		* Sheppard, William Fleetwood, Sc.D., LL.M., formerly a Senior Examiner to the Board of Education. Cardrona, Berkhamsted, Herts	
1932		* Simpson, Alexander Rudolf Barbour, B.Sc. (Edin.), M.A. (Cantab.), F.R.G.S., Senior Geography Master, Canford School. The Brook, Canford Magna, Dorset	
1908		Simpson, George Freeland Barbour, M.D., F.R.C.P.E., F.R.C.S.E., J.P., 43 Manor Place, Edinburgh 3	
1932	C.	* Simpson, John Baird, D.Sc. (Aberd.), Senior Geologist, H.M. Geological Survey (Scotland), 19 Grange Terrace, Edinburgh 9	
1900		Sinhjee, Sir Bhagvat, G.C.I.E., M.D., LL.D. (Edin.), H.H. the Thakur Sahib of Gondal, Kathiawar, Bombay, India	
1903		† Skinner, Robert Taylor, M.A., J.P., formerly House Governor, Donaldson's Hospital. 35 Campbell Road, Edinburgh 12	
1930	C.	* Slater, Robert Henry, D.Sc., Ph.D. (Edin.), F.I.C., Department of Chemical Pathology, St Mary's Hospital, London, W. 2	
1929		* Smail, James Cameron, O.B.E., Companion Inst.E.E., Principal, Heriot-Watt College, Edinburgh. 1 Grange Terrace, Edinburgh 9	1934–
1926		* Small, James, D.Sc., Professor of Botany, Queen's University, Belfast. Orkla, 50 Myrtlefield Park, Belfast	
1901		Smart, Edward, B.A., B.Sc., Tillyloss, Tullylumb Terrace, Perth	
1920		* Smellie, William Robert, M.A., D.Sc., Geologist on the Staff of the Anglo-Persian Oil Company. Baron Cliff, Cove, Dumbartonshire	
1928		Smith, Alick Drummond Buchanan, M.A., B.Sc. (Agric.) (Aberd.), M.S.A. (Iowa), Lecturer, Institute of Animal Genetics, University of Edinburgh (West Mains Road)	
1921		* Smith, Norman Kemp, M.A., D.Phil., D.Litt., LL.D., Professor of Logic and Metaphysics, University of Edinburgh (South Bridge). Ellerton, Grange Loan, Edinburgh 9	
1923		* Smith, Percy James Lancelot, M.A. (Oxon.), F.I.C., F.C.S., Science Master, Loretto School. 47 Dalrymple Loan, Musselburgh	
1911		* Smith, Stephen, B.Sc., 34 Craigmillar Park, Edinburgh 9	
1929		* Smith, Sydney, M.D., F.R.C.P., D.P.H., Professor of Forensic Medicine, University of Edinburgh (Teviot Place). 10 Oswald Road, Edinburgh 9	
1907	C.	† Smith, William Ramsay, D.Sc., M.D., C.M., Permanent Head of the Health Department, South Australia. Belair, South Australia	
1919		* Smith, Sir William Wright, Kt., M.A., D. ès Sc., Regius Professor of Botany, University of Edinburgh, Regius Keeper of the Royal Botanic Garden, and King's Botanist in Scotland. Inverleith House, Edinburgh 4	Sec. 1923–28. V-P 1928–31.
1932		* Sneeden, Jean-Baptiste Octave, B.Sc., Ph.D. (Glas.), Lecturer on Heat Engines, Royal Technical College, Glasgow. 39 Kingshouse Avenue, Glasgow, S. 4	
1899		Snell, Ernest Hugh, M.D., B.Sc., D.P.H. (Cantab.), Barrister-at-Law, late Medical Officer of Health, Coventry. 3 Eaton Road, Coventry	
1880		Sollas, William Johnson, M.A., D.Sc., LL.D., F.R.S., Professor of Geology and Palæontology, University of Oxford. 104 Banbury Road, Oxford	
1933		* Somerville, John Livingston, C.A., Auditor to the University of Edinburgh. Beechcroft, Ravelston Dykes, Edinburgh 4	
1929		* Southwell, Thomas, D.Sc., A.R.C.S., Lecturer in Parasitology, School of Tropical Medicine, Liverpool. 53 Greenhill Road, Mossley Hill, Liverpool 18	

Date of Election			Service on Council, etc.
1925		* Staig, Robert Arnot, M.A., Ph.D., Lecturer in Zoology, University of Glasgow. Glenlea, Lasswade, Midlothian	
1891		Stanfield, Richard, A.R.S.M., M.Inst.C.E., Emeritus Professor of Mechanics and Engineering, Heriot-Watt College, Edinburgh. 24 Mayfield Gardens, Edinburgh 9	1926–29.
1923		* Stebbing, Edward Percy, M.A., Professor of Forestry, University of Edinburgh (George Square)	
1885 & 1915		Steggall, John Edward Aloysius, M.A., Hon. A.R.I.B.A., LL.D., Emeritus Professor of Mathematics, University College, Dundee (St Andrews University). Woodend, Perth Road, Dundee	
1923		* Stenhouse, Andrew G., F.G.S., 191 Newhaven Road, Edinburgh 6	
1929	C.	* Stephen, Alexander Charles, D.Sc., Keeper, Natural History Department, Royal Scottish Museum, Edinburgh. Eastcroft, Cramond Bridge, Edinburgh 4	
1910		Stephenson, Thomas, D.Sc., F.C.S., 65 Castle Street, Edinburgh 2	
1931		* Steven, George Alexander, B.Sc. (Edin.), Assistant Naturalist, Marine Laboratory, Plymouth. 1 Lipson Terrace, Plymouth, Devon	
1925		* Stevens, Alexander, M.A., B.Sc., Lecturer in Geography, University of Glasgow	
1886	C.	Stevenson, Charles A., B.Sc., M.Inst.C.E., Radella, North Berwick	
1884		† Stevenson, David Alan, B.Sc., M.Inst.C.E., Troqueer, Kingsknowe, Colinton, Midlothian	1928–31.
1919		* Stevenson, David Alan, B.Sc., M.Inst.C.E., 22 Glencairn Crescent, Edinburgh 12	
1935		* Stevenson, Eric, B.Sc. (Edin.), A.M.I.Mech.E., Lecturer in Engineering, University of Edinburgh (Sanderson Engineering Laboratory, Mayfield Road). 7 Beauchamp Road, Edinburgh 9	
1931		* Stewart, Corbet Page, Ph.D. (Edin.), Lecturer in General Biochemistry, University of Edinburgh (Teviot Place). 17 Orchard Road South, Edinburgh 4	
1925		* Stewart, David Smith, Ph.D., M.Inst.C.E., Lecturer on Structural Engineering Drawing, University of Edinburgh (Sanderson Engineering Laboratories, Mayfield Road). 82 Lasswade Road, Edinburgh 9	
1924		* Stiles, Sir Harold Jalland, K.B.E., M.B., F.R.C.S.E., LL.D., Emeritus Professor of Clinical Surgery, University of Edinburgh. Whatton Lodge, Gullane, E. Lothian	1934–
1902		Stockdale, Herbert Fitton, LL.D., formerly Director of the Royal Technical College, Glasgow. Clairinch, Upper Helensburgh, Dumbartonshire	
1889	C.	Stockman, Ralph, M.D., LL.D., F.R.C.P.E., F.F.P.S.G., Professor of Materia Medica and Therapeutics, University of Glasgow	1903–05.
1926		* Stokoe, William Norman, B.Sc., Ph.D. (Lond.), Chief Chemist, Craigmillar Creamery Co., Ltd. 8 Cobden Road, Edinburgh 9	
1906		Story, Fraser, O.B.E., formerly Professor of Forestry, University College, Bangor, North Wales. 4K Artillery Mansions, Victoria Street, London, S.W. 1	
1907		Strong, John, C.B.E., M.A., LL.D., Emeritus Professor of Education, University of Leeds. C/o The Librarian, The University, Leeds	
1930	C.	* Stump, Claude Witherington, M.D., D.Sc., Professor of Embryology and Histology, University of Sydney	
1903		Sutherland, David W., C.I.E., M.D., M.R.C.P., Lt.-Col., I.M.S. (retired), Braeside, Belhaven, Dunbar	
1935		* Sutherland, John Derg, B.Sc., B.Ed., Ph.D. (Edin.), Lecturer in Psychology, University of Edinburgh. 2 Windsor Street, Edinburgh 7	
1930		* Sutherland, Sir John Donald, C.B.E., LL.D., Legion of Honour (France), Order of Leopold (Belgium), formerly Forestry Commissioner, Scotland. 11 Inverleith Row, Edinburgh 4	
1925		Sutton, Richard L., M.D., D.Sc., LL.D., 1308 Bryant Building, 1102 Grand Avenue, Kansas City, Mo., U.S.A.	
1932		* Swinton, William Elgin, B.Sc., Ph.D. (Glas.), F.L.S., F.Z.S., F.G.S., Curator of Fossil Reptiles and Amphibia, British Museum (Natural History), South Kensington, London, S.W. 7	
1917	C. N.	* Tait, John, D.Sc., M.D., Professor of Physiology, McGill University, Montreal, Canada	
1933		* Tait, John Barclay, B.Sc. (Edin.), Ph.D., A.H.-W.C., Senior Naturalist (Hydrographer), Marine Laboratory, Fishery Board for Scotland, Aberdeen. 23 Cromwell Road, Aberdeen	
1890	C.	† Tanakadate, Aikitu, Hon. Professor of Natural Philosophy, Imperial University of Japan. Koisikawa, Zôsigayamati, 144, Tokyo, Japan	

Date of Election			Service on Council, etc.
1899		Taylor, James, M.A., formerly Mathematical Master, Edinburgh Academy. 18 Hillview, Blackhall, Edinburgh 4	
1933		* Taylor, George, D.Sc. (Edin.), F.L.S., Assistant Keeper, Department of Botany, British Museum. Ballochmyle, Loudwater, Rickmansworth, Herts	
1885	C.	Thompson, D'Arcy Wentworth, C.B., M.A., LL.D., D.Sc. (Witwatersrand), D.Litt., F.R.S. (PRESIDENT), Professor of Natural History, University, St Andrews. 44 South Street, St Andrews	1892–95, 1896–99, 1907–10, 1912–15, 1922–25. V-P 1916–19. Curator 1926–34. P 1934–
1932		Thompson, Harold William, D.Litt. (Edin.), A.M., Ph.D. (Harvard), F.S.A. Scot., Professor of English, N.Y. State College, Albany, N.Y., U.S.A.	
1917	C. N.	* Thompson, John M'Lean, M.A., D.Sc., F.L.S., Professor of Botany, University of Liverpool	
1896		Thomson, George Ritchie, C.M.G., M.B., C.M., formerly Professor of Surgery, University of the Witwatersrand, Johannesburg, Transvaal. Hordle Grange, Hordle, Hants	
1903		Thomson, George S., Kenmore Farm, Whelpley Hill, Chesham, Bucks	
1906		Thomson, Gilbert, M.A., M.Inst.C.E., 164 Bath Street, Glasgow, C. 2	
1926		* Thomson, Godfrey Hilton, D.Sc., Ph.D., Professor of Education, University of Edinburgh (Moray House)	1931–34.
1926	C.	* Thomson, John, M.A., Ph.D. (Glas.), Lecturer in Plant Physiology, University of Glasgow. 2 Chartwell Terrace, Bearsden, Glasgow	
1934		* Thomson, Matthew Sydney, M.A., M.D., B.Ch. (Cantab.), F.R.C.P., M.R.C.S., Physician for Diseases of the Skin, King's College Hospital, Belgrave Hospital for Children. 106 Harley Street, London, W. 1	
1899		Thomson, R. Tatlock, F.C.S., 156 Bath Street, Glasgow	
1912	C.	Thomson, Robert Black, M.B. (Edin.), Aliwal North, Cape Province, S.A.	
1882		Thomson, Sir William, Kt., M.A., B.Sc., LL.D., formerly Principal, University of the Witwatersrand. Dunedin, Glencairn, Simonstown, South Africa	
1917		* Thorneycroft, Wallace, J.P., Strete Ralegh, Whimple, Exeter, Devon	
1933		* Timms, Geoffrey, Ph.D. (Cantab.), Lecturer in Mathematics, University of St Andrews. Deanscourt, St Andrews	
1920		* Todd, John Barber, B.Sc., Ph.D., M.I.Mech.E., Reader in Engineering, University of Edinburgh. 4 Bright's Crescent, Edinburgh 9	
1935		Touche, John Edward, M.I.Mech.E., 11 Blackford Road, Edinburgh 10	
1917		* Tovey, Sir Donald Francis, Kt., B.A. (Oxon.), M.Mus. (Hon.), Birmingham, Professor of Music, University of Edinburgh (Reid School of Music). 39 Royal Terrace, Edinburgh 7	
1914		† Tredgold, Alfred Frank, M.D. (Durham), F.R.C.P. (Lond.), Lecturer on Mental Deficiency, University of London. St Martins, Guildford	
1915		* Trotter, George Clark, M.D. (Edin.), D.P.H. (Aberdeen), F.S.A.Scot., Medical Officer of Health, Metropolitan Borough, Islington. Braemar, 17 Haslemere Road, Crouch End, London, N. 8	
1922	C. K.	* Turnbull, Herbert Westren, M.A., F.R.S., Professor of Mathematics, University of St Andrews. 2 Queens Terrace, St Andrews	1928–31.
1905		Turner, Arthur Logan, M.D., LL.D., F.R.C.S.E., 27 Walker Street, Edinburgh 3	1926–29. V-P 1930–33.
1925		* Turner, Harry Moreton Stanley, M.B.E., M.D., M.R.C.S., L.R.C.P., D.T.M. and H., Chevalier de l'Ordre Royale du Sauveur de Grèce. Wing Commander, R.A.F. (retired). Meerbrook House, Overdale, Ashtead, Surrey	
1924		* Turner, Richard, O.B.E., M.B., C.M., Hotel Hydropathic, Peebles	
1895		Turton, Albert H., M.Inst.M.M., 233 George Road, Erdington, Birmingham	
1918	C. N.	* Tyrrell, George Walter, A.R.C.S., D.Sc., F.G.S., Lecturer in Petrology, Geological Department, University of Glasgow	1926–29.
1930	C.	* Voge, Cecil Innes Bothwell, Ph.D. (Edin.), 2 Gray's Inn Road, London, W.C. 1	
1932		* Wade, Henry, C.M.G., D.S.O., M.D., Senior Lecturer in Clinical Surgery, University of Edinburgh (Royal Infirmary). 6 Manor Place, Edinburgh 3	

Date of Election			Service on Council, etc.
1926		* Wakeley, Cecil Pembrey Grey, F.R.C.S., Surgeon to King's College Hospital, London, Lecturer in Anatomy, King's College, London. 24 Queen Anne Street, Cavendish Square, London, W. 1	
1925	C.	* Walker, Frederick, M.A., Ph.D., D.Sc., Lecturer in Geology, University of St Andrews	
1931		* Walker, William James Stirling, Ph.D. (Edin.), A.H.-W.C., F.I.C., Scientific Officer, H.M. Fuel Research Station, East Greenwich, London, S.E. 10. C/o Harrison, 64 Sandtoft Road, Charlton, London, S.E. 7	
1902		Wallace, Alexander G., M.A., 56 Fonthill Road, Aberdeen	
1886	C.	Wallace, Robert, M.A., LL.D., F.L.S., Emeritus Professor of Agriculture and Rural Economy, University of Edinburgh. Mid Park House, Kincardine-on-Forth, Fife	
1898		Wallace, William, M.A., Campsie, Alta, Canada	
1920		* Walmsley, Thomas, M.D. (Glas.), Professor of Anatomy, Queen's University, Belfast	
1931	C.	* Walton, John, M.A. (Cantab.), D.Sc. (Manchester), Regius Professor of Botany, University of Glasgow. 23 Lilybank Gardens, Glasgow, W. 2	1934–
1927	C.	* Wardlaw, Claude Wilson, D.Sc. (Glas.), Imperial College of Tropical Agriculture, Trinidad, B.W.I.	
1923		* Warren, John Alexander, M.Inst.C.E. 74 Balshagray Avenue, Partick	
1901	C.	Waterston, David, M.A., M.D., F.R.C.S.E., Professor of Anatomy, University of St Andrews	1916–19, 1925–28.
1927		* Watson, Charles Brodie Boog, F.S.A.Scot. 24 Garscube Terrace, Edinburgh 12	
1923		* Watson, H. Ferguson, M.D., F.R.F.P.S., Ph.D., D.P.H. (Glas.), H.M. Senior Deputy Commissioner, General Board of Control for Scotland. 5 Hillside Street, Edinburgh 7	
1923	C.	* Watson, William, M.A., B.Sc. (Edin.), Lecturer in Physics, Heriot-Watt College, Edinburgh. 17 Braidburn Crescent, Edinburgh 10	
1911		† Watt, James, W.S., F.F.A., LL.D. (TREASURER), 7 Blackford Road, Edinburgh 10	1924–26. Treasurer 1926–
1933		* Watt, John Mitchell, M.B., Ch.B., M.R.C.P. (Edin.), F.R.S.S.A., Professor of Pharmacology, University of the Witwatersrand, Johannesburg, South Africa	
1911		Watt, Right Rev. Lauchlan MacLean, D.D., LL.D., Kinloch, Lochcarron, Ross-shire	
1928		* Watters, Alexander Marshall, M.A., B.Sc. (Glas.), Rector of Hawick High School. High School House, Hawick	
1896		† Webster, John Clarence, C.M.G., M.D., D.Sc., LL.D., F.R.C.P.E., *formerly* Professor of Obstetrics and Gynæcology, Rush Medical College, Shediac, N.B., Canada	
1907	M-B. C.	† Wedderburn, Ernest Maclagan, O.B.E., M.A., D.Sc., LL.B., W.S., Deputy Keeper of the Signet. 6 Succoth Gardens, Edinburgh 12	1913–16, 1921–24, 1932–35.
1903	M-B. C.	† Wedderburn, J. H. Maclagan, M.A., D.Sc., F.R.S., Professor of Mathematics, Princeton University. Fine Hall, Princeton, N.J., U.S.A.	
1904		Wedderspoon, William Gibson, M.A., LL.D.	
1934	C.	* Weir, John, M.A., Ph.D., D.Sc. (Glas.), Lecturer in Palæontology, University of Glasgow. 18 Botanic Crescent, Glasgow, N.W.	
1930		* White, Adam Cairns, M.B., Ch.B., Ph.D., Assistant Pharmacologist, Wellcome Physiological Research Laboratory, Beckenham, Kent	
1933		* Whitley, William Frederic James, M.D. (Edin.), D.P.H. (Oxon.), Medical Officer of Health, Northumberland County Council. Westfield, Cramlington, Northumberland	
1931		* Whitson, Sir Thomas Barnby, D.L., LL.D., C.A., Lord Provost of the City of Edinburgh (1929–32). 27 Eglinton Crescent, Edinburgh 12	
1911		Whittaker, Charles Richard, F.R.C.S.E., F.S.A.Scot., Lynwood, Hatton Place, Edinburgh 9	
1912	C. V. J. B-P.	* Whittaker, Edmund Taylor, M.A., Hon. Sc.D. (Dubl.), Hon. LL.D. (St Andrews and California), F.R.S., Foreign Member of the R. Accademia dei Lincei, Rome, Professor of Mathematics, University of Edinburgh (16 Chambers Street). 48 George Square, Edinburgh 8	1912–15, 1922–25. Sec. 1916–22. V-P 1925–28.

Date of Election			Service on Council, etc.
1928	C.	* Whittaker, John Macnaghten, M.A. (Edin.), M.A. (Cantab.), D.Sc., Professor of Pure Mathematics, University of Liverpool	
1918		* Whyte, Rev. Charles, M.A., LL.D., F.R.A.S., U.F. Church Manse, Kingswells, Aberdeen	
1935		* Whyte, Sir William Edward, Kt., O.B.E., J.P., Solicitor. Balgay, Uddingston	
1934		* Whyte, William, Cashier and General Manager of the Royal Bank of Scotland. Baberton House, Juniper Green, Edinburgh	
1929	C.	* Wiesner, Bertold Paul, D.Sc., formerly Macaulay Lecturer, Institute of Animal Genetics, University of Edinburgh. 3 Holly Terrace, Highgate Village, London, N. 6	
1918		* Wight, John Thomas, M.I.Mech.E., M.I.Mar.E., Joint Managing Director, Messrs MacTaggart, Scott & Co., Ltd., Loanhead. Calderwood Villa, Lasswade	
1934		* Wightman, William Persehouse Delisle, Ph.D., M.Sc. (Lond.), Science Master, Edinburgh Academy. 36 Coltbridge Terrace, Edinburgh 12	
1925		* Wilkie, David Percival Dalbreck, O.B.E., M.D., Ch.M., F.R.C.S., Professor of Surgery, University of Edinburgh (Royal Infirmary). 9 Ainslie Place, Edinburgh 3	
1926	C.	* Williams, Samuel, Ph.D., Lecturer in Plant Morphology, University of Glasgow. 27 Lindsay Place, Kelvindale, Glasgow	
1908		Williamson, Henry Charles, M.A., D.Sc., formerly Naturalist to the Fishery Board for Scotland. 13 Windsor Street, Dundee	
1928	C.	* Williamson, John, M.A. (Edin.), Ph.D. (Chicago), Associate Professor of Mathematics, Johns Hopkins University, Baltimore, U.S.A.	
1910	C.	Williamson, William, F.L.S., 47 St Alban's Road, Edinburgh 9	
1927	C.	* Williamson, William Turner Horace, B.Sc. (Aberd.), Ph.D. (Edin.), Chief Chemist, Egyptian Ministry of Agriculture, Cotton Research Board, Giza, Egypt	
1911		Wilson, Andrew, O.B.E., D.L., M.Inst.C.E., 66 Netherby Road, Edinburgh 5	
1902	V. J.	† Wilson, Charles Thomson Rees, M.A., LL.D., D.Sc., F.R.S., Nobel Prize, Physics, 1927, Emeritus Jacksonian Professor of Natural Philosophy, University of Cambridge. Glencorse, Storey's Way, Cambridge	
1922		* Wilson, John, F.R.I.B.A., Chief Architect, Scottish Department of Health. 20 Lomond Road, Edinburgh 5	
1920	C.	* Wilson, Malcolm, D.Sc. (London), A.R.C.S., F.L.S., Reader in Mycology and Bacteriology, University of Edinburgh (Royal Botanic Garden). Brent Knoll, Kinnear Road, Edinburgh 4	1931-34.
1924		* Wilson, William, M.A., LL.B., Advocate, Regius Professor of Public Law, University of Edinburgh (South Bridge). 38 Moray Place, Edinburgh 3	
1895		Wilson-Barker, Sir David, Kt., R.D., R.N.R., F.R.G.S., formerly Captain-Superintendent, Thames Nautical Training College, H.M.S. "Worcester." 12 Bolan Street, London, S.W. 11	
1934		* Winstanley, Arthur, M.B.E., D.Sc. (Eng.) (Lond.), M.I.Min.E., Mining Engineer to Safety in Mines Research Board. 18 St John's Road, Edinburgh 12	
1931	C.	* Wishart, John, M.A., B.Sc. (Edin.), M.A. (Cantab.), D.Sc. (Lond.), Reader in Statistics, University of Cambridge. Croft Lodge, Barton Road, Cambridge	
1922	C. B.	* Wordie, James Mann, M.A. (Cantab.), B.Sc. (Glas.), 52 Montgomery Drive, Glasgow	
1933		* Wright, James, F.G.S., Balado, 212 Colinton Road, Edinburgh 11	
1896		† Wright, Sir Robert Patrick, LL.D., formerly Chairman of the Board of Agriculture for Scotland. The Heugh, North Berwick, East Lothian	
1911	C.	Wrigley, Ruric Whitehead, M.A. (Cantab.), Assistant Astronomer, Royal Observatory, Edinburgh	
1882		Young, Frank W., C.B.E., F.C.S., H.M. Inspector of Schools (Emeritus). 31 Oxgangs Road, Fairmilehead, Edinburgh 10	
1904		Young, Robert B., M.A., D.Sc., F.G.S., Professor of Geology, University of the Witwatersrand (South African School of Mines and Technology), Johannesburg, Transvaal	

Number of Fellows, 718.

HONORARY FELLOWS OF THE SOCIETY.

(At 28th October 1935.)

1920 HIS ROYAL HIGHNESS THE PRINCE OF WALES.
1934 HIS ROYAL HIGHNESS THE DUKE OF YORK.

FOREIGNERS (LIMITED TO FORTY-FOUR BY LAW I).

Elected
1933 John Jacob Abel, Professor of Pharmacology, Johns Hopkins University, Baltimore.
1935 Richard Anschütz, Emeritus Professor of Chemistry, University of Bonn. Darmstadt, den Hermannstrasse 16.
1916 Charles Eugène Barrois, formerly Professor of Geology and Mineralogy, Université, Lille, France. 41, Rue Pascal, Lille.
1923 Henri Bergson, Honorary Professor, College of France, Paris.
1930 Vilhelm Frimann Koren Bjerknes, Professor of Physics, Geophysical Institute, Bergen.
1927 Niels Bohr, Nobel Laureate, Physics, 1922, Professor of Physics, University of Copenhagen.
1927 Jules Bordet, Nobel Laureate, Medicine, 1919, Professor of Bacteriology, University of Brussels.
1933 Filippo Bottazzi, Professor of Experimental Physiology, Royal Institute of Physiology, S. Andrea delle Dame, 21, Naples.
1923 Marcellin Boule, Professor at the National Museum of Natural History, Laboratory of Palæontology, 3 Place Valhubert, Paris 5^e.
1905 Waldemar Christofer Brögger, Professor of Mineralogy and Geology, K. Frederiks Universitet, Oslo.
1916 Douglas Houghton Campbell, Emeritus Professor of Botany, Stanford University, California.
1920 William Wallace Campbell, President-Emeritus of the University of California, Berkeley, Director-Emeritus of the Lick Observatory, Mt. Hamilton, California, and President of the National Academy of Sciences.
1930 Walter Bradford Cannon, Professor of Physiology, Harvard University, Cambridge, Mass.
1930 Maurice Caullery, Professor of Zoology in the University of Paris. Laboratoire d'Évolution des Êtres organisés, 105 Bould. Raspail, Paris, VI^e.
1933 Edwin Grant Conklin, formerly Professor of Biology, Princeton University, N.J.
1921 Reginald Aldworth Daly, Professor of Geology, Harvard University, Cambridge, Mass.
1927 Albert Einstein, Nobel Laureate, Physics, 1921, Princeton University, N.J.
1913 George Ellery Hale, Honorary Director of Mount Wilson Observatory (Carnegie Institution of Washington), Pasadena, California.
1934 Björn Helland-Hansen, Geophysical Institute, Bergen.
1921 Johan Hjort, Professor of Marine Biology, University, Oslo.
1923 Arnold Frederik Holleman, Emeritus Professor of Organic Chemistry, University, Amsterdam. Boekensteijn Parkweg 7, Bloemendaal.
1934 Bernardo Houssay, Professor of Physiology, National University of Buenos Aires.
1933 Nikolaj Konstantinovic Koltzoff, formerly Professor of Zoology, State University, Moscow; Director of the Research Institute of Experimental Biology. Moscow 64, Voronzovo Pole 6.
1923 Tullio Levi-Civita, Professor of Mathematics, Regia Università, Rome.
1934 Frank Rattray Lillie, Professor of Zoology and Embryology, Whitman Laboratory, University of Chicago.
1927 Hans Horst Meyer, Emeritus Professor of Pharmacology, University of Vienna.
1934 Thomas Hunt Morgan, Nobel Laureate, Medicine, Professor of Biology, California Institute of Technology, Pasadena.
1923 Arthur Amos Noyes, Institute of Technology, Pasadena, California.
1908 Henry Fairfield Osborn, Research Professor of Zoology, Columbia University, Honorary President, American Museum of Natural History, New York, and Senior Geologist, U.S. Geological Survey.
1908 Ivan Petrovitch Pavlov, Emeritus Professor of Physiology, Academy of Sciences, Inst. Exper. Med., Nobel Laureate, Physiology and Medicine, 1904. V.O., 7 ligne No. 2, Leningrad.
1933 Albrecht Penck, Geheimrat Emeritus Professor of Geography, Friedrich-Wilhelms-Universität, Berlin.

Elected
1920 Charles Émile Picard, Perpetual Secretary, Academy of Sciences, Paris.
1921 Salvatore Pincherle, Professor of Mathematics, University of Bologna.
1920 Charles Richet, Nobel Laureate, Medicine, 1913, Professor of Physiology, Faculty of Medicine, Paris.
1934 Paul Sabatier, Nobel Laureate, Chemistry, Professor of Chemistry, University of Toulouse.
1930 Erik Helge Oswald Stensiö, Professor of Palæontology and Historical Geology, Royal University of Upsala.
1913 Vito Volterra, formerly Professor of Mathematical Physics, Regia Università. 17 Via in Lucina, Rome.
1927 Richard Willstätter, Nobel Laureate, Chemistry, 1915, Professor of Chemistry, University of Munich. Moehlstrasse 29, Munich 27.
1923 Edmund Beecher Wilson, formerly Professor of Zoology, Columbia University, New York.
1933 Pieter Zeeman, Nobel Laureate, Physics, 1902, Professor of Physics, University, Amsterdam.

Total, 40.

BRITISH SUBJECTS (LIMITED TO TWENTY-TWO BY LAW I).

1934 Henry Edward Armstrong, Ph.D., LL.D., F.R.S., Emeritus Professor of Chemistry, Imperial College of Science and Technology, City and Guilds (Engineering) College, London. 55 Granville Park, London, S.E. 13.
1927 Sir William Henry Bragg, O.M., K.B.E., M.A., D.Sc., LL.D., F.R.S., Nobel Laureate, Physics, 1915, Fullerian Professor of Chemistry, Royal Institution, London.
1930 Sir Arthur Stanley Eddington, Kt., M.A., Hon. D.Sc., LL.D., F.R.S., Plumian Professor of Astronomy and Experimental Philosophy, University of Cambridge.
1927 Sir John Bretland Farmer, Kt., M.A., D.Sc., LL.D., F.R.S., Emeritus Professor of Botany, Imperial College of Science and Technology, London.
1900 Andrew Russell Forsyth, M.A., Sc.D., LL.D., Hon. Math.D., F.R.S., Emeritus Professor of Mathematics, Imperial College of Science and Technology, London; formerly Sadleirian Professor of Pure Mathematics, University of Cambridge.
1910 Sir James George Frazer, Kt., O.M., D.C.L., LL.D., Litt.D., F.R.S., Commandeur de la Légion d'Honneur. Trinity College, Cambridge.
1934 John Scott Haldane, C.H., M.A., M.D., LL.D., F.R.S., Director of the Mining Research Laboratory, and Honorary Professor, University of Birmingham. Cherwell, Oxford.
1927 Sir Frederick Gowland Hopkins, O.M., M.A., M.B., D.Sc., LL.D., President R.S., Joint Nobel Laureate, Medicine, 1929, Professor of Biochemistry, University of Cambridge.
1930 Sir Arthur Keith, Kt., M.D., LL.D., F.R.S., Master, Buckston Browne Research Farm, Downe, Farnborough, Kent.
1910 Sir Joseph Larmor, Kt., M.A., D.Sc., LL.D., D.C.L., F.R.S., formerly Lucasian Professor of Mathematics, University of Cambridge. St John's College, Cambridge.
1933 Sir George Macdonald, K.C.B., LL.D., formerly Secretary, Scottish Education Department. 17 Learmonth Gardens, Edinburgh 4.
1934 Karl Pearson, M.A., LL.B., LL.D., F.R.S., Emeritus Galton Professor of Eugenics, University of London. 7 Well Road, London, N.W. 3.
1934 Sir Edward Bagnall Poulton, Kt., M.A., D.Sc., LL.D., F.R.S., formerly Hope Professor of Zoology, University of Oxford. Wykeham House, Banbury Road, Oxford.
1930 Robert Robinson, D.Sc., LL.D., F.R.S., Waynflete Professor of Chemistry, University of Oxford.
1921 Rt. Hon. Lord Rutherford of Nelson, O.M., M.A., D.Sc., B.A., LL.D., Past President R.S., Nobel Laureate, Chemistry, 1908, Cavendish Professor of Experimental Physics, University of Cambridge.
1933 Sir William Napier Shaw, Kt., Sc.D.(Cantab.), LL.D., F.R.S., formerly Director Meteorological Office. 10 Moreton Gardens, London, S.W. 5.
1908 Sir Charles Scott Sherrington, O.M., G.B.E., M.A., D.Sc., M.D., LL.D., Past President R.S., Joint Laureate, Nobel Prize, Medicine, 1932, formerly Waynflete Professor of Physiology, University of Oxford.
1934 Sir Grafton Elliot Smith, Kt., M.A., M.D., Litt.D., F.R.S., Professor of Anatomy, University College, London. 62 Albert Road, London, N.W. 8.
1905 Sir Joseph John Thomson, Kt., O.M., D.Sc., LL.D., Past President R.S., Nobel Laureate, Physics, 1906, formerly Cavendish Professor of Experimental Physics, now Professor of Physics, University of Cambridge, Master of Trinity College, Cambridge.
1934 William Whitehead Watts, Sc.D., LL.D., F.R.S., Emeritus Professor of Geology, Imperial College of Science and Technology, London. Hillside, Langley Park, Sutton, Surrey.

Total, 20.

CHANGES IN FELLOWSHIP DURING SESSION 1934–1935.

FELLOWS OF THE SOCIETY ELECTED.

JAMES LAW BROWNLIE.
ROBERT SELBIE CLARK.
SAMUEL HUNTER COWAN.
CHARLES FINDLAY DAVIDSON.
MAXWELL DAVIDSON.
BHIMBHAI NICHHABHAI DESAI.
RONALD GRANT.
ALEXANDER MILLAR MEEK GRIERSON.
ARTHUR CYRIL WILLIAM HUTCHINSON.
JOHN HENRY KENNETH.
PETER MACCALLUM.
WALTER ALAN MOZLEY.
JAMES MUNNOCH.
BASUDEVA NARAYANA.
CHARLES SOLOMON PICHAMUTHU.
THOMAS ROWATT.
MAURICE GEORGE SAY.
ERIC STEVENSON.
JOHN DERG SUTHERLAND.
JOHN EDWARD TOUCHE.
SIR WILLIAM EDWARD WHYTE.

FOREIGN HONORARY FELLOW ELECTED.

RICHARD ANSCHÜTZ.

FELLOWS DECEASED.

FRANCIS JAMES BLIGHT.
HENRY BRIGGS.
DAVID LAWRENCE BRYCE.
CECIL CARUS-WILSON.
JAMES CHATHAM.
SHEPHERD DAWSON.
T. CUTHBERT DAY.
JAMES MAIN DIXON (1933).
REV. WILLIAM GALLOWAY DONALDSON.
SIR JAMES ALFRED EWING.
JAMES GORDON GRAY.
WILLIAM RICHARD HODGKINSON.
HECTOR MUNRO MACDONALD.
SIR LESLIE MACKENZIE.
JOHN JAMES RICKARD MACLEOD.
JAMES MUSGROVE.
ROBERT ALEXANDER ROBERTSON.
SIR JOHN (SMITH) SAMUEL.
SIR EDWARD ALBERT SHARPEY-SCHAFER.
ROBERT RATTRAY TATLOCK.
SIR JAMES WALKER.
ALFRED C. WILSON.
BERTRAM MARTIN WILSON.
JOHNSTONE CHRISTIE WRIGHT.

FOREIGN HONORARY FELLOWS DECEASED.

THEOBALD SMITH.
HUGO DE VRIES.

BRITISH HONORARY FELLOW DECEASED.

SIR HORACE LAMB.

FELLOWS RESIGNED.

THOMAS WILLIAM NAYLOR BARLOW.
JAMES HYSLOP.
JOSEPH PARKER.

FELLOWS REMOVED FROM THE ROLL.

CHARLES LAWRENCE ABERNETHY.
ANDREW CORRIE M‘CANDLISH.
BASIL ALEXANDER PILKINGTON.

LAWS OF THE SOCIETY.

Adopted July 3, 1916; *amended December* 18, 1916.

LAW	I,	amended	February 5, 1934.	LAW	IX,	amended	May 3, 1920.
,,	VI,	,,	,, 7, 1921.	,,	XIII,	,,	May 3, 1920.
,,	,,	,,	July 2, 1928.	,,	XIX,	,,	June 16, 1924.
,,	VIII,	,,	May 3, 1920.				

I.

THE ROYAL SOCIETY OF EDINBURGH, which was instituted by Royal Charter in 1783 for the promotion of Science and Literature, shall consist of Ordinary Fellows (hereinafter to be termed Fellows) and Honorary Fellows. The number of Honorary Fellows shall not exceed sixty-six, of whom not more than twenty-two may be British subjects, and not more than forty-four subjects of Foreign States.

Fellows only shall be eligible to hold office or to vote at any Meeting of the Society.

ELECTION OF FELLOWS.

II.

Each Candidate for admission as a Fellow shall be proposed by at least four Fellows, two of whom must certify from personal knowledge. The Official Certificate shall specify the name, rank, profession, place of residence, and the qualifications of the Candidate. The Certificate shall be delivered to the General Secretary before the 30th of November, and, subject to the approval of the Council, shall be exhibited in the Society's House during the month of January following. All Certificates so exhibited shall be considered by the Council at its first meeting in February, and a list of the Candidates approved by the Council for election shall be issued to the Fellows not later than the 21st of February.

III.

The election of Fellows shall be by Ballot, and shall take place at the first Ordinary Meeting in March. Only Candidates approved by the Council shall be eligible for election. A Candidate shall be held not elected, unless he is supported by a majority of two-thirds of the Fellows present and voting.

IV.

On the day of election of Fellows two scrutineers, nominated by the President, shall examine the votes and hand their report to the President, who shall declare the result.

V.

Each Fellow, after his election, is expected to attend an Ordinary Meeting, and sign the Roll of Fellows, he having first made the payments required by Law VI. He shall be introduced to the President, who shall address him in these words:—

> *In the name and by the authority of THE ROYAL SOCIETY OF EDINBURGH, I admit you a Fellow thereof.*

PAYMENTS BY FELLOWS.

VI.

Each Fellow shall, before he is admitted to the privileges of Fellowship, pay an admission fee of three guineas, and a subscription of three guineas for the year of election. He shall continue to pay a subscription of three guineas at the beginning of each session so long as he remains a Fellow.

Each Fellow who was elected subsequent to December 1916 and previous to December 1920 shall also pay a subscription of three guineas at the beginning of each session so long as he remains a Fellow.

Each Fellow who was elected previous to December 1916, and who has not completed his twenty-five annual payments, shall, at the beginning of each session, pay three guineas until his twenty-five annual payments are made. Each Fellow who has completed or shall complete his payments shall be invited to contribute one guinea per annum or to pay a single sum of ten guineas.

A Fellow may compound for the annual subscriptions by a single payment of fifty guineas, or on such other terms as the Council may from time to time fix.

VII.

A Fellow who, after application made by the Treasurer, fails to pay any contribution due by him, shall be reported to the Council, and, if the Council see fit, shall be declared no longer a Fellow. Notwithstanding such declaration all arrears of contributions shall remain exigible.

ELECTION OF HONORARY FELLOWS.

VIII.

Honorary Fellows shall be persons eminently distinguished in Science or Literature. They shall not be liable to contribute to the Society's Funds. Personages of the Blood Royal may be elected Honorary Fellows at any time on the nomination of the Council, and without regard to the limitation of numbers specified in Law I.

IX.

Honorary Fellows shall be proposed by the Council. The nominations shall be announced from the Chair at the First Ordinary Meeting after their selection. The names shall be printed in the circular for the last Ordinary Meeting of the Session, when the election shall be by Ballot, after the manner prescribed in Laws III and IV for the Election of Fellows.

EXPULSION OF FELLOWS.

X.

If, in the opinion of the Council, the conduct of any Fellow is injurious to the character or interests of the Society, the Council may, by registered letter, request him to resign. If he fail to do so within one month of such request, the Council shall call a Special Meeting of the Society to consider the matter. If a majority consisting of not less than two-thirds of the Fellows present and voting decide for expulsion, he shall be expelled by declaration from the Chair, his name shall be erased from the Roll, and he shall forfeit all right or claim in or to the property of the Society.

XI.

It shall be competent for the Council to remove any person from the Roll of Honorary Fellows if, in their opinion, his remaining on the Roll would be injurious to the character or interests of the Society. Reasonable notice of such proposal shall be given to each member of the Council, and, if possible, to the Honorary Fellow himself. Thereafter the decision on the question shall not be taken until the matter has been discussed at two Meetings of Council, separated by an interval of not less than fourteen days. A majority of two-thirds of the members present and voting shall be required for such removal.

MEETINGS OF THE SOCIETY.

XII.

A Statutory Meeting for the election of Council and Office-Bearers, for the presentation of the Annual Reports, and for such other business as may be arranged by the Council, shall be held on the fourth Monday of October. Each Session of the Society shall begin at the date of the Statutory Meeting.

XIII.

Meetings for reading and discussing communications and for general business, herein termed Ordinary Meetings, shall be held, when convenient, on the first and third Mondays of each month from November to July inclusive, with the exception that in January the meetings shall be held on the second and fourth Mondays.

XIV.

A Special Meeting of the Society may be called at any time by direction of the Council, or on a requisition to the Council signed by not fewer than six Fellows. The date and hour of such Meeting shall be determined by the Council, who shall give not less than seven days' notice of such Meeting. The notice shall state the purpose for which the Special Meeting is summoned; no other business shall be transacted.

PUBLICATION OF PAPERS.

XV.

The Society shall publish Transactions and Proceedings. The consideration of the acceptance, reading, and publication of papers is vested in the Council, whose decision shall be final. Acceptance for reading shall not necessarily imply acceptance for publication.

DISTRIBUTION OF PUBLICATIONS.

XVI.

Fellows who are not in arrear with their Annual Subscriptions and all Honorary Fellows shall be entitled gratis to copies of the Parts of the Transactions and the Proceedings published subsequently to their admission.

Copies of the Parts of the Proceedings shall be distributed by post or otherwise, as soon as may be convenient after publication; copies of the Transactions or Parts thereof shall be obtainable upon application, either personally or by an authorised agent, to the Librarian, provided the application is made within five years after the date of publication.

CONSTITUTION OF COUNCIL.

XVII.

The Council shall consist of a President, six Vice-Presidents, a Treasurer, a General Secretary, two Secretaries to the Ordinary Meetings (the one representing the Biological group and the other the Physical group of Sciences),* a Curator of the Library and Museum, and twelve ordinary members of Council.

ELECTION OF COUNCIL.

XVIII.

The election of the Council and Office-Bearers for the ensuing Session shall be held at the Statutory Meeting on the fourth Monday of October. The list of the names recommended by the Council shall be issued to the Fellows not less than one week before the Meeting. The election shall be by Ballot, and shall be determined by a majority of the Fellows present and voting. Scrutineers shall be nominated as in Law IV.

XIX.

The President may hold office for a period not exceeding five consecutive years; the Vice-Presidents, not exceeding three; the Secretaries to the Ordinary Meetings, not exceeding five; the General Secretary, the Treasurer, and the Curator of the Library and Museum, not exceeding ten; and ordinary members of Council, not exceeding three consecutive years; provided that the Treasurer may be re-elected for more than ten successive years in cases where the Council declares to the Society that an emergency exists.

XX.

In the event of a vacancy arising in the Council or in any of the Offices enumerated in Law XVII, the Council shall proceed, as soon as convenient, to elect a Fellow to fill such vacancy for the period up to the next Statutory Meeting.

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* The Biological group includes Anatomy, Anthropology, Botany, Geology, Pathology, Physiology, Zoology; the Physical group includes Astronomy, Chemistry, Mathematics, Metallurgy, Meteorology, Physics.

POWERS OF THE COUNCIL.

XXI.

The Council shall have the following powers:—(1) To manage all business concerning the affairs of the Society. (2) To decide what papers shall be accepted for communication to the Society, and what papers shall be printed in whole or in part in the Transactions and Proceedings. (3) To appoint Committees. (4) To appoint employees and determine their remuneration. (5) To award the various prizes vested in the Society, in accordance with the terms of the respective deeds of gift, provided that no member of the existing Council shall be eligible for any such award. (6) To make from time to time Standing Orders for the regulation of the affairs of the Society. (7) To control the investment or expenditure of the Funds of the Society.

At Meetings of the Council the President or Chairman shall have a casting as well as a deliberative vote.

DUTIES OF PRESIDENT AND VICE-PRESIDENTS.

XXII.

The President shall take the Chair at Meetings of Council and of the Fellows. It shall be his duty to see that the business is conducted in accordance with the Charter and Laws of the Society. When unable to be present at any Meetings or attend to current business, he shall give notice to the General Secretary, in order that his place may be supplied. In the absence of the President his duties shall be discharged by one of the Vice-Presidents.

DUTIES OF THE TREASURER.

XXIII.

The Treasurer shall receive the monies due to the Society and shall make payments authorised by the Council. He shall lay before the Council a list of arrears in accordance with Rule VII. He shall keep accounts of all receipts and payments, and at the Statutory Meeting shall present the accounts for the preceding Session, balanced to the 30th of September, and audited by a professional accountant appointed annually by the Society.

DUTIES OF THE GENERAL SECRETARY.

XXIV.

The General Secretary shall be responsible to the Council for the conduct of the Society's correspondence, publications, and all other business except that which relates to finance. He shall keep Minutes of the Statutory and Special Meetings of the Society and Minutes of the Meetings of Council. He shall superintend, with the aid of the Assistant Secretary, the publication of the Transactions and Proceedings. He shall supervise the employees in the discharge of their duties.

DUTIES OF SECRETARIES TO ORDINARY MEETINGS.

XXV.

The Secretaries to Ordinary Meetings shall keep Minutes of the Ordinary Meetings. They shall assist the General Secretary, when necessary, in superintending the publication of the Transactions and Proceedings. In his absence, one of them shall perform his duties.

DUTIES OF CURATOR OF LIBRARY AND MUSEUM.

XXVI.

The Curator of the Library and Museum shall have charge of the Books, Manuscripts, Maps, and other articles belonging to the Society. He shall keep the Card Catalogue up to date. He shall purchase Books sanctioned by the Council.

ASSISTANT SECRETARY AND LIBRARIAN.

XXVII.

The Council shall appoint an Assistant Secretary and Librarian, who shall hold office during the pleasure of the Council. He shall give all his time, during prescribed hours, to the work of the Society, and shall be paid according to the determination of the Council. When necessary he shall act under the Treasurer in receiving subscriptions, giving out receipts, and paying employees.

ALTERATION OF LAWS.

XXVIII.

Any proposed alteration in the Laws shall be considered by the Council, due notice having been given to each member of Council. Such alteration, if approved by the Council, shall be proposed from the Chair at the next Ordinary Meeting of the Society, and, in accordance with the Charter, shall be considered and voted upon at a Meeting held at least one month after that at which the motion for alteration shall have been proposed.

Additions to the Library—Presentations, etc.—1934-1935.

Acta Physicochimica U.R.S.S. Vol. I, No. 1–. 8vo. Moscow, 1934. (*Purchased.*)

Adachi, B. Venensystem der Japaner. Fol. Kyoto, 1933. (*Presented.*)

Annales de l'Institut des Mines à Leningrade. T. 8. La. 8vo. Leningrad, 1934. (*Presented.*)

Annals of the Tadjik Astronomical Observatory. Vol. I, Part 1–. 8vo. Stalinbad, 1935. (*Presented.*)

Archiv für die Naturwissenschaftliche Landesdurchforschung von Böhmen. 1869–. 4to. Prague, 1869. (*Exchange.*)

Archiwum Hydrobiologji i Rybactwa. Published by the Hydrobiological Station of Lake Wigrach (Wigry). T. 8–. La. 8vo. Suwalki, Poland, 1934. (*Exchange.*)

Assam Forest Records: (Botany): Some New Species of Assam. Edited by A. Das. Vol. I. 8vo. Shillong, Assam, 1934. (*Presented.*)

Astrophysica Norvegica. Vol. I, No. 1–. Edited by the Institute of Theoretical Astrophysics of Oslo University. 8vo. Oslo, 1934. (*Exchange.*)

B.B.C. Annual, 1935. 4to. London, 1935. (*Presented.*)

British Museum (Natural History):—

British Antarctic ("Terra Nova") Expedition, 1910: Natural History Report: Zoology. Vol. IV, No. 7, Penguin Embryos. By C. W. Parsons. 4to. London, 1934.

Catalogue of the Fossil Cephalopoda in the British Museum (Natural History). Part IV. The Ammonoidea of the Trias. By L. F. Spath. La. 8vo. London, 1934.

Diptera of Patagonia and South Chile. Part VI, fasc. 5. Acalyptrata (concluded). By John R. Malloch. 8vo. London, 1934.

Economic Series No. 4*a*: British Mosquitoes and their Control. By F. W. Edwards and S. P. James. Second (revised) edition. 8vo. London, 1934.

Great Barrier Reef Expedition, 1928–1929: Scientific Reports.

Vol. II, No. 6. The Zooplankton. By F. S. Russell and J. S. Colman.

Vol. IV, No. 15. The Silicoflagellata and Tintinnoinea. By Sheina M. Marshall. 4to. London, 1934.

Norman, J. R. Systematic Monograph of the Flatfishes (Heterosomata). Vol. I. Psettodidæ, Bothidæ, Pleuronectidæ. 4to. London, 1934.

Parker, H. W. Monograph of the Frogs of the Family Microhylidæ. 4to. London, 1934.

Report on Cetacea stranded on the British Coasts from 1927–1932. No. 11. By F. C. Fraser. 4to. London, 1934.

(*Exchange.*)

British Regional Geology:—

Scotland: The Tertiary Volcanic Districts. By J. E. Richey. 8vo. 1935.

The Grampian Highlands. By H. H. Read. 8vo. 1935.

The South of Scotland. By J. Pringle. 8vo. 1935.

Northern England. By T. Eastwood. 8vo. 1935.

The Welsh Borderland. By R. W. Pocock and T. H. Whitehead. 8vo. 1935.

Bristol and Gloucester District. 8vo. London, 1935.

London and Thames Valley. 8vo. London, 1934.

The Wealden District. 8vo. London, 1935.

(Published by the Geological Survey and Museum.) (*Exchange.*)

Bulletin de l'Académie des Sciences Mathématiques et Naturelles: Series A, Sciences Mathématiques et Physiques. No. 1–. (Published by l'Académie Royale Serbe.) La. 8vo. Belgrade, 1933. (*Exchange.*)

Bulletin de l'Institut de Mathématique et Mécanique a l'Université Kouybycheff de Tomsk. T. I, fasc. 1–. 8vo. Tomsk, 1935 (*Exchange.*)

Bulletin: Leningrad University Astronomical Observatory. 1934, No. 4. 8vo. Leningrad, 1934. (*Presented.*)

Bulletin de la Société géologique de Normandie et des Amis du Muséum du Havre. T. XXXVII, Anées 1933 et 1934. 8vo. Le Havre, 1935. (*Exchange.*)

Carnegie Institution of Washington:—

No. 415. Contributions to Palæontology. 8vo. Washington, 1934.

No. 436. Contributions to American Archæology. Vol. II, 1934. 4to. Washington, 1934.

No. 449. Wieland, G. R. The Cerro Cuadrado Petrified Forest. 4to. Washington, 1935.

No. 454. Ruppert, Karl. The Caracol at Chichen Itza, Yucatan, Mexico. 4to. Washington, 1935.

(*Exchange.*)

Catania. Storia della Università di Catania dalle origini ai giorni nostri. (Quinto Centenario dell' ateneo di Catania.) 4to. Catania, 1934. (*Presented.*)

—— L'Università di Catania nel suo V Centenario: 19–22 Ottobre 1934. 4to. Catania, 1934. (*Presented.*)

Central Asiatic Expeditions:—

Natural History of Central Asia. Vol. X. The Reptiles of China: Turtles, Crocodilians, Snakes, Lizards. By Clifford H. Pope. 4to. New York, 1935. (American Museum of Natural History.) (*Exchange.*)

Chinese Journal of Physics. Vol. I, Nos. 1, 2. La. 8vo. Shanghai, 1933, 1934. (*Presented.*)

Collected Papers from the Faculty of Science, Osaka Imperial University.

Series A: Mathematics. Vol. I–. La. 8vo. Osaka, 1933.

Series B: Physics. Vol. I–. La. 8vo. Osaka, 1933.

Series C: Chemistry. Vol. I–. La. 8vo. Osaka, 1933. (*Exchange.*)

La Commission Royale d'Histoire, 1834–1934. Livre Jubilaire. Académie Royale de Belgique. 4to. Bruxelles, 1935. (*Presented.*)

Compositio Mathematica. Vol. I–. 8vo. Groningen, 1934. (*Purchased.*)

Comptes-Rendus du Deuxième Congrès des Mathématiciens des Pays Slavs. 8vo. Prague, 1935. (*Presented.*)

Czech Academy of Sciences and Arts:—

Matiegka, J., A. J. P. van der Broek, and Stan. Sochor. Rapport concernant le résultat des recherches entreprises pour retrouver la tombe de J. Am. Comenius et ses Ossements. Fol. Prague, 1933.

Matiegka, J., and Jiří Malý. Les caractères physiques d'Albert de Wallenstein, duc de Frýdlant et de sa première épouse, Lucretia Nekšova de Landek, avec un supplément du J. Bergl: Les sorts du cadavre et des ossements d'Albert de Wallenstein. Fol. Prague, 1934. (Anthropologica: Trida II, 2nd Classe.)

Schebesta, Paul, and Victor Lebzelter. Anthropology of the Central African Pygmies in the Belgian Congo. Fol. Prague, 1933.
(*Exchange.*)

David Lecture: No. 1. Some Founders of Australian Geology. By Professor E. W. Skeats. 8vo. Sydney, 1933. (Australian National Research Council.) (*Exchange.*)

De Montessus de Ballore, R. Probabilités et Statistiques. 8vo. Paris, 1931. (*Presented by Mr John McD. Field.*)

Expedition Antarctique Belge:—

Résultats du Voyage de la Belgica en 1897–99: Rapports scientifiques.

Zoologie: Oligochètes. Par Léon Černosvitov.
Pycnogonides. Par Louis Giltay.

Botanique: Observations sur des Algues. Par E. de Wildeman. 4to. Anvers, 1935. (*Presented.*)

Fossil Non-Marine Mollusca of North America. By Junius Henderson. (Geological Society of America: Special Papers No. 3.) 8vo. New York, 1935. (*Exchange.*)

Fragments of an Unknown Gospel and Other Early Christian Papyri. Edited by H. Idris Bell and T. C. Skeat. La. 8vo. London, 1935. (*Presented by the British Museum.*)

Geological Society of America:—

Bibliographic Contributions: Bibliography and Index of Geology Exclusive of North America. Vol. I, 1933. By John M. Nickles and Robert B. Miller. La. 8vo. New York, 1934.

Geological Society of America, 1888–1930: A Chapter in Earth Science History. By Herman LeRoy Fairchild. La. 8vo. New York, 1932.

Memoir 1. Stratigraphy of Western Newfoundland. By C. Schuchert and C. O. Dunbar. 8vo. New York, 1934.

—— 2. Paleozoic Plankton of North America. By Rudolf Ruedemann. La. 8vo. New York, 1934.

Proceedings. No. 46–. (Now published separately from the Bulletin.) La. 8vo. New York, 1934.

Special Papers. No. 1. Bibliographic Index of Paleozoic Ostracoda. By R. S. Bassler and Betty Kellett. La. 8vo. New York, 1934.
(*Exchange.*)

Gesta Gothorum: Werner Victor Rosicrucianus, 1837–1903. La. 8vo. Stockholm. No Date. (*Presented.*)

Golding, Arthur. A Tragedie of Abraham's Sacrifice. Written in French by Theodore Beza and translated into English by Arthur Golding. 4to. Toronto, 1906. (*Exchange, University of Toronto.*)

Greaves, R. H. Chromium Steels. 8vo. London, 1935. (*Presented by the Dept. Scient. and Indust. Res.*)

Grierson, H. J. C. Letters of Sir Walter Scott, 1823–25. Vol. VIII. 8vo. London, 1935. (*Presented.*)

Hall, C. R. A Scientist in the Early Republic: Samuel Latham Mitchill, F.R.S.E., 1764–1831. 8vo. New York, 1934. (*Presented.*)

Handbook: The Norman Lockyer Observatory. 2nd Edition, 1935. Compiled by William J. S. Lockyer. 8vo. Sidmouth, 1935. (*Presented.*)

Hanneford-Smith, W. The Architectural Works of Sir Banister Fletcher. 4to. London, 1934. (*Presented by the Author.*)

Kanjilal, U. N., and P. C. and A. Das. Flora of Assam. Vol. I, Part 1. Ranunculaceæ to Elæocarpaceæ. 8vo. Shillong, Assam, 1935. (*Presented.*)

Lorentz, H. A. Collected Papers. Vols. VII and VIII. 4to. The Hague, 1934 and 1935. (*Purchased.*)

Matsuo, Iwao. Biologische Untersuchungen über Farbstoffe. Band I. 4to. Kyoto, 1934. (*Presented.*)

Merriman, A. D. The Romance of Engineering. 8vo. London, 1934. (*Presented.*)

Metz, Rudolf. David Hume: Leben und Philosophie. 8vo. Stuttgart, 1929. (*Presented.*)

Modern Russian-English Dictionary. 8vo. Moscow, 1932. (*Purchased.*)

Murray, Lt.-Col. Hon. A. C. Memorials of Sir Gideon Murray of Elibank and His Times (1560–1621). 8vo. Edinburgh, 1932. (*Presented.*)

Niépce. Commemoration du centenaire de la mort de Joseph-Nicéphore Niépce, Inventeur de la Photographie. Manifestations organées à Chalon-sur-Saône en Juin 1933. 4to. Paris, 1933. (Published by La Société française de Photographie et de Cinématographie.) (*Presented.*)

Notes and Queries: General Indexes to the 1st, 3rd, 4th, 5th, 6th, and 7th Series. 1850–1891. 8vo. London, 1856–1892. (*Purchased.*)

Pearson, Karl. Tables for Statisticians and Biometricians. Part I. 2nd Edition. 4to. London, 1924. (*Presented by Mr J. McD. Field.*)

Pictet, Raoul. La Frigothérapie: ses origines—son but et les résultats déja obtenus. 8vo. Paris, 1898. (Author's presentation copy to Lord Kelvin.) (*Presented by Dr J. H. Kenneth.*)

Polar Year 1932–33.

Meteorological Observations in Nigeria during the Polar Year 1932–33. La. 4to. Lagos, 1934.

Daily Meteorological Observations taken at St Georges, Bermuda, during the Polar Year. La. 4to. St Georges, Bermuda, 1935. (*Exchange, through Meteorological Office, Air Ministry, London.*)

Popular Guide to the Scottish Zoological Park. By T. H. Gillespie. 10th Edition. 8vo. Edinburgh, n.d. (*Presented.*)

Proceedings of the Indian Academy of Sciences. Vol. I, No. 7–. La. 8vo. Bangalore, 1934. (*Exchange.*)

Proceedings of the Institute of Radio Engineers. Vol. XXII, No. 1–. 8vo. New York, 1934. (*Exchange.*)

Publicnaja Biblioteka, Moscow:—

Astronomiceskij zurnal. 1931–. La. 8vo. Moscow, 1931.

Mikrobiologija. 1932–. La. 8vo. Moscow, 1932.

Pocvovedenia (Pedology). 1931–. La. 8vo. Moscow, 1931.

Sovetskaja ethnografia. 1932–. La. 8vo. Moscow, 1932.

Zoologiceskij zurnal. 1933–. La. 8vo. Moscow, 1933.

Zurnal geofiziki. 1932–. La. 8vo. Moscow, 1932. (*Exchange.*)

Quasi Cursores: Portraits of the High Officers and Professors of the University of Edinburgh at its Tercentenary Festival. Drawn and Etched by William Hole, A.R.S.A. 4to. Edinburgh, 1884. (*Purchased.*)

Rathbun, Mary J. Fossil Crustacea of the Atlantic and Gulf Coastal Plain. (Geological Society of America: Special Papers, No. 2.) 8vo. New York, 1935. (*Exchange.*)

Report of the Agricultural Research Council for the Period July 1931–30th September 1933. 8vo. London, 1934. (*Presented.*)

Report of the British Association for 1934 (Aberdeen Meeting). 8vo. London, 1934. (*Presented by Dr James Watt.*)

Reports of the Committee on Contact Catalysis: Division of Chemistry and Chemical Technology. 10th and 11th Reports by Guy B. Taylor and Robert E. Burk. 4to. Washington, 1935. (Published by the National Research Council, U.S.A.) (*Exchange.*)

Resultados del Observatorio Nacional Argentino. Vol. XXV: Cometa Halley, 1910. 4to. Cordoba, 1934. (*Exchange.*)

Runge, C., and H. König. Vorlesungen über numerisches Rechnen, Bd. XI, Die Grundlehren der Mathematischen Wissenschaften in Einzeldarstellungen. 8vo. Berlin, 1924. (*Presented by Mr J. McD. Field.*)

Stephenson, T. A. British Sea Anemones. Vol. II. (Ray Society Publication.) 8vo. London, 1935. (*Purchased.*)

Travaux de la Station Biologique du Caucase du Nord de Gorsky Institute Agronomique. T. I, (4), fasc. 1–. 4to. Ordjonikidze, 1932–1933. (*Exchange.*)

Turner, G. Grey, and W. D. Arnison. The Newcastle-upon-Tyne School of Medicine, 1834–1934. 8vo. Newcastle-upon-Tyne, 1934. (*Presented.*)

INDEX.

Accounts of the Society, 1934–35, 206.
Additions to Library, 253.
Address to H.M. The King, Silver Jubilee, 188, 191.
Aitken (A. C.). On Least Squares and Linear Combination of Observations, 42–48.
——— and Gonin (H. T.). On Fourfold Sampling with and without Replacement, 114–125.
Anderson-Berry Prize. *See* Prizes.
Ashworth (J. H.). Charles Darwin as a Student in Edinburgh, 1825–1827, 97–113.
Auditor, Re-appointment of, 193.
Awards of Prizes, 198–205.

Bhaduri (J. L.). The Anatomy of the Adhesive Apparatus in the Tadpoles of *Rana afghana* Günther, with special reference to the Adaptive Modifications. (*Title only*: published in *Trans.*, vol. lviii.) 188.
Blight (F. J.). Obituary Notice of, 178.
Bond (G.). The Endodermis in Light-grown and Etiolated Shoots of the Leguminosæ: A Contribution to the Causal Study of Differentiation in the Plant. (*Title only*: published in *Trans.*, vol. lviii.) 189.
Bragg (W. L.). The New Crystallography, 62–71.
——— The New Crystallography. (*Address*), Bruce-Preller Lecture Fund, 187.
Briggs (H.). Obituary Notice of, 140.
Bruce Prize. *See* Prizes.
Bruce-Preller Lecture Fund. *See* Prizes.
Bryce (D. L.). Obituary Notice of, 143.

Calder (Mary G.). Further Observations on the Genus *Lyginorachis*, Kidston. (*Title only*: published in *Trans.*, vol. lviii.) 190.
Cameron (A. E.). Awarded Makdougall-Brisbane Prize, 188, 189, 192, 202.
Carnegie Trust, Thanks for Grants, 192–193.
Carus-Wilson (C.). Obituary Notice of, 178.
Charles Darwin as a Student in Edinburgh, 1825–1827, by J. H. Ashworth, 97–113.
Chatham (J.). Obituary Notice of, 144.
Clay Varves, dated as Late-Glacial in Scotland, by G. De Geer, 23–26.
Cockburn (A. M.). The Geology of St Kilda. (*Title only*: published in *Trans.*, vol. lviii.) 188.
Contributions, Voluntary, 214.
Copulation, Effect of Population-Density upon Frequency of, by D. S. MacLagan and E. Dunn, 126–139.
Correlation, Invariant Theory of the, by H. W. Turnbull, 27–41.
Council, 1934–35, 185; 1935–36, 193, 215.
Crystallography, The New (X-ray Analysis), by W. L. Bragg, 62–71.

Darwin, Charles, as a Student in Edinburgh, 1825–1827, by J. H. Ashworth, 97–113.
David Anderson-Berry Prize. *See* Prizes.
Davidson (C. F.). The Tertiary Geology of Raasay (Inner Hebrides). (*Title only*: published in *Trans.*, vol. lviii.) 186.
Davies (L. M.). The Geology of Inchkeith. (*Title only*: published in *Trans.*, vol. lviii.) 186.
Dawson (S.). Obituary Notice of, 146.
Day (T. C.). Obituary Notice of, 148.
Dixon (J. M.). Obituary Notice of, 178.
Donaldson (Rev. W. G.). Obituary Notice of, 178.
Dunn (E.). *See* MacLagan (D. S.).
Dunning Clay, from the Gotiglacial Subepoch, by G. De Geer, 23–26.

Election of Fellows, 188, 192, 244.
——— Honorary Fellows, 189, 192, 244.
Ewing (Sir J. A.). Death of, 186.
——— Obituary Notice of, 150.

Fellows, Deceased and resigned, 244.
——— Elected, 188, 192, 244; Honorary, 189, 192, 244.
——— List of, 216–241.
——— Lists of Honorary, 242–243.
——— Obituary Notices, 140–179.
——— Removed from Roll, 244.
Fourfold Sampling with and without Replacement, by A. C. Aitken and H. T. Gonin, 114–125.

Geer (G. De). Dating of Late-Glacial Clay Varves in Scotland, 23–26.
Gonin (H. T.). *See* Aitken (A. C.).
Grants towards Publications, 192–193.
Gray (J. G.). Obituary Notice of, 154.
Gunning-Victoria Jubilee Prize. *See* Prizes.

Henderson (S. M. K.). Ordovician Submarine Disturbances in the Girvan District. (*Title only*: published in *Trans.*, vol. lviii.) 190.
Hodgkinson (W. R.). Obituary Notice of, 178.
Honorary Fellows, Deceased, 244.
——— Elected, 189, 192, 244.
——— List of, 242–243.

Hydrogen, Secondary Spectrum of.—II, by I. Sandeman, 49–61.
——— III, by I. Sandeman, 72–84.

Integrals of Associated Legendre Functions with respect to their Degrees, by T. M. MacRobert, 85–90.
Invariant Theory of the Correlation, by H. W. Turnbull, 27–41.

Keith Prize. *See* Prizes.
King (H.M. The). Silver Jubilee Address, 188, 191.
Kitching (J. A.). An Introduction to the Ecology of Intertidal Rock Surfaces on the Coast of Argyll. (*Title only*: published in *Trans.*, vol. lviii.) 189.

Lamb (Sir H.). Obituary Notice of, 179.
Laws, 245–252.
Least Squares and Linear Combination of Observations, by A. C. Aitken, 42–48.
Legendre Functions, Associated, Series and Integrals for, by T. M. MacRobert, 85–90.
Levee at Palace of Holyroodhouse (May 22, 1935). R.S.E. represented, 192.
Library, Additions, 253.
Life of Albino Rats, Duration of, by B. P. Wiesner and N. M. Sheard, 1–22.
Linear Combination of Observations, Least Squares and, by A. C. Aitken, 42–48.
Lord High Commissioner's Levee at Holyroodhouse (May 22, 1935). R.S.E. represented, 192.
Lowe (Esther). On the Anatomy of a Marine Copepod *Calanus finmarchicus* (Gunnerus). (*Title only*: published in *Trans.*, vol. lviii.) 187.
Lunar Atmospheric Pressure Inequalities at Glasgow, by R. A. Robb and T. R. Tannahill, 91–96.

Macdonald (H. M.). Obituary Notice of, 156.
Mackenzie (Sir W. L.). Obituary Notice of, 158.
MacLagan (D. S.) and Dunn (E.). The Experimental Analysis of the Growth of an Insect-Population, 126–139.
Macleod (J. J. R.). Obituary Notice of, 160.
MacRobert (T. M.). Some Series and Integrals involving Associated Legendre Functions regarded as Functions of their Degrees, 85–90.
Makdougall-Brisbane Prize. *See* Prizes.
Meetings, Proceedings of Ordinary, 186–190.
——— Proceedings of Statutory, 183, 191.
Mozley (A.). The Freshwater and Terrestrial Mollusca of Northern Asia. (*Title only*: published in *Trans.*, vol. lviii.) 190.
Muir (Sir T.). MSS. bequeathed to the Society, 192.
Musgrove (J.). Obituary Notice of, 163.

Neill Prize. *See* Prizes.
Nicol (T.). The Female Reproductive System in the Guinea-Pig: Intravitam Staining; Fat Production; Influence of Hormones. (*Title only*: published in *Trans.*, vol. lviii.) 189.

Obituary Notices:—Francis James Blight, Henry Briggs, David Lawrence Bryce, Cecil Carus-Wilson, James Chatham, Shepherd Dawson, T. Cuthbert Day, James Main Dixon, Rev. William Galloway Donaldson, Sir James Alfred Ewing, James Gordon Gray, William Richard Hodgkinson, Sir Horace Lamb, Hector Munro Macdonald, Sir William Leslie Mackenzie, John James Rickard Macleod, James Musgrove, Robert Alexander Robertson, Sir John (Smith) Samuel, Sir Edward Albert Sharpey-Schafer, Theobald Smith, Robert Rattray Tatlock, Hugo de Vries, Sir James Walker, Alfred C. Wilson, Bertram Martin Wilson, Johnstone Christie Wright, 140–179.
Oviposition, Relation of Population-Density to Utilisation of Medium for, by D. S. MacLagan and E. Dunn, 126–139.

Papers read, 1934–35, 186–190.
Polar Year, 1932–33, 192.
Population, Internal Resistance to Growth of, by D. S. MacLagan and E. Dunn, 126–139.
Pressure, Atmospheric, Lunar Inequalities in, by R. A. Robb and T. R. Tannahill, 91–96.
Prizes, etc., Bruce, 196, 204.
——— Bruce-Preller Lecture Fund, 187, 192, 196, 205.
——— Davidson Anderson-Berry, 193, 196, 205.
——— Gunning-Victoria Jubilee, 195, 204.
——— Keith, 194, 198.
——— Makdougall-Brisbane, 188, 189, 192, 194, 200.
——— Neill, 195, 202.
——— Scott, 195, 204.
——— Rules and Awards, 194–197, 198–205.
——— *See* also in Accounts.
Proceedings, Ordinary Meetings, 186–290.
——— Statutory Meetings, 183, 191.
Proceedings, *R.S.E.*, Issue increased, 192.

Rat, Duration of Life in the Albino, by B. P. Wiesner and N. M. Sheard, 1–22.
Reports, General Secretary's and Treasurer's, 191, 193.
Representatives, R.S.E., Committees, Congresses, etc., 192.
Robb (R. A.) and Tannahill (T. R.). The Lunar Atmospheric Pressure Inequalities at Glasgow, 91–96.
Robertson (R. A.). Obituary Notice of, 164.
Royal Society, Government Publication Grant, 193.

Sampling, Fourfold, with and without Replacement, by A. C. Aitken and H. T. Gonin, 114–125.
Samuel (Sir J. (S.)). Obituary Notice of, 179.

Sandeman (I.). The Mathematical Representation of the Energy Levels of the Secondary Spectrum of Hydrogen.—II, 49–61.
——— The Mathematical Representation of the Energy Levels of the Secondary Spectrum of Hydrogen.—III, 72–84.
Schafer, (Sir E. A.). *See* Sharpey-Schafer.
Scott Prize. *See* Prizes.
Scott (C. M.). Awarded David Anderson-Berry Prize, 1935, 193, 205.
Series of Associated Legendre Functions, by T. M. MacRobert, 85–90.
Sewell (Lt.-Col. R. B. Seymour). The John Murray Expedition. (*Address*), 186, 192.
Sharpey-Schafer (Sir E. A.). Obituary Notice of, 166.
Sheard (N. M.). *See* Wiesner (B. P.).
Silver Jubilee of Their Majesties The King and Queen, Address, 188, 191.
Smith (T.). Obituary Notice of, 170.
Spectrum of Hydrogen Molecule.—II, by I. Sandeman, 49–61.
——— Secondary, of Hydrogen.—III, by I. Sandeman, 72–84.
Statutory Meetings, 183, 191.

Tannahill (T. R.). *See* Robb (R. A.).
Tatlock (R. R.). Obituary Notice of, 172.
Transactions, *R.S.E.*, Index of Papers, 260.
Turnbull (H. W.). The Invariant Theory of the Correlation, 27.

Voluntary Contributors, 214.
Vries (Hugo de). Obituary Notice of, 173.

Walker (Sir J.). Obituary Notice of, 175.
Walton (J.). Scottish Lower Carboniferous Plants: the Fossil Hollow Trees of Arran and their Branches (*Lepidophloios Wünschianus*, Carruthers). (*Title only*: published in *Trans.*, vol. lviii.) 187.
Wiesner (B. P.) and Sheard (N. M.). The Duration of Life in an Albino Rat Population, 1–22.
Wills (L. J.). Rare and New Ostracoderm Fishes from the Downtonian of Shropshire. (*Title only*: published in *Trans.*, vol. lviii.) 189.
Wilson (A. C.). Obituary Notice of, 179.
Wilson (B. M.). Obituary Notice of, 176.
Wright (J. C.). Obituary Notice of, 179.

X-ray Analysis, a Review, by W. L. Bragg, 62–71.

Index of Papers published in the "Transactions" during Session 1934–35.

Bhaduri (J. L.). The Anatomy of the Adhesive Apparatus in the Tadpoles of *Rana afghana* Günther, with special reference to the Adaptive Modifications, vol. lviii, No. 14, p. 339.
Bond (G.). The Endodermis in Light-grown and Etiolated Shoots of the Leguminosæ: A Contribution to the Causal Study of Differentiation in the Plant, vol. lviii, No. 17, p. 409.

Calder (Mary G.). Further Observations on the Genus *Lyginorachis*, vol. lviii, No. 22, p. 549.
Cockburn (A. M.). The Geology of St Kilda, vol. lviii, No. 21, p. 511.

Davidson (C. F.). The Tertiary Geology of Raasay, Inner Hebrides, vol. lviii, No. 16, p. 375.

Gordon (W. T.). The Genus *Pitys*, Witham, emend, vol. lviii, No. 12, p. 279.

Hamilton (W. J.). The Early Stages in the Development of the Ferret: Fertilisation to the Formation of the Prochordal Plate, vol. lviii, No. 11, p. 251.

Henderson (S. M. K.). Ordovician Submarine Disturbances in the Girvan District, vol. lviii, No. 20, p. 487.

Kitching (J. A.). An Introduction to the Ecology of Intertidal Rock Surfaces on the Coast of Argyll, vol. lviii, No. 15, p. 351.

Lowe (Esther). On the Anatomy of a Marine Copepod *Calanus finmarchicus* (Gunnerus), vol. lviii, No. 23, p. 561.

Mozley (A.). The Fresh-water and Terrestrial Mollusca of Northern Asia, vol. lviii, No. 24, p. 605.

Nicol (T.). The Female Reproductive System in the Guinea-Pig: Intravitam Staining; Fat Production; Influence of Hormones, vol. lviii, No. 19, p. 449.

Walton (J.). Scottish Lower Carboniferous Plants: The Fossil Hollow Trees of Arran and their Branches (*Lepidophloios Wünschianus* Carruthers), vol. lviii, No. 13, p. 313.
Wills (L. J.). Rare and New Ostracoderm Fishes from the Downtonian of Shropshire, vol. lviii, No. 18, p. 427.

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